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The role of the Gendrin mode of VLF propagation in the generation of magnetospheric emissions

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Abstract. Impulsive VLF wave packets and electron beams may be formed in the magnetosphere from the superposition of Gendrin mode components defined by the condition $\cos\theta = \frac{2f}{f_H}$, where θ=wave normal angle measured from the Earth's static magnetic field $\bar{B}_o$, f=frequency of the wave component and f_H=electron gyrofrequency. The frequency range of the Gendrin mode spectrum is defined by $f_L < f < f_H/2$, where f_L=lower hybrid resonance frequency $\approx \sqrt{f_H f_{H_i}} = f_H/43$ and f_{H_i}=proton gyrofrequency. Since the group ray velocity of all Gendrin components is given by $v_G = \frac{c}{2}\frac{f_H}{f_N}$ where f_N=plasma frequency, and is aligned with $\bar{B}_o$ and since the longitudinal (parallel to $\bar{B}_o$) component of the phase velocity is the same as v_G, a Gendrin wave packet will travel in a homogeneous medium without distortion, constituting a kind of soliton. An electron whose parallel velocity $v_{||}$ is close to v_G (i.e., near longitudinal resonance) may become trapped by the $E_{||}$ of the Gendrin wave packet, giving up energy to the packet as it propagates in a region of decreasing v_G, such as a typical field line path approaching the magnetic equator. As the packet grows $E_{||}$ increases causing more trapping and accordingly more wave growth, giving rise to an "Impulsive Wave Instability" (IWI). Wave packet growth is limited by wave loss due to drift away from the main packet of previously generated components that no longer satisfy the Gendrin condition. As the packet leaves the equator growth ceases, the packet collapses and the previously trapped electrons become transient electron beams that may themselves trigger more emissions. It is suggested that the impulsive VLF emissions observed by DE1 and other satellites may be caused by the IWI mechanism.

The purpose of this letter is to show that the Gendrin mode of propagation [*Gendrin*, 1961] is capable of supporting a new kind of wave particle interaction involving longitudinal resonance, which we shall call the Impulsive Wave Instability (IWI). Here a short duration whistler-mode wave packet (essentially an impulse) consisting of superposed Gendrin components (frequencies ranging from the lower hybrid resonance frequency $f_L \approx \sqrt{f_H f_{Hi}} = \frac{f_H}{43}$, where f_{Hi} =proton gyrofrequency, to one-half the electron gyrofrequency $f_H/2$) traps passing electrons that are close to longitudinal (i.e., Landau) resonance, where the electron parallel velocity $v_{||}$ equals the parallel component of the wave phase velocity $v_{p||}$ which is the same as the group ray velocity. For typical models of the cold plasma, it is shown that the velocity of such a wave packet is slowly reduced as it approaches the equator, causing continual trapping of electrons whose parallel velocity $v_{||}$ slightly exceeds the parallel component of the wave phase velocity $v_{p||}$. As these trapped electrons are slowed by the wave their lost energy is transferred to the wave causing it to grow [*Brice*, 1960; *Helliwell, EOS*, 1989], resembling the traveling wave tube where an electron beam is slowed by a slightly slower circuit wave with an axial component of electric field.

This hypothesis was motivated by a proposed theory of whistler precursors [*Park and Helliwell*, 1977], supported by calculations [*Tkalcevic*, 1982; *Tkalcevic et al.*, 1984] showing that longitudinally resonant energetic electrons can be trapped by the $E_{||}$ of a strong upgoing whistler. These trapped electrons augment the background flux of cyclotron-resonant electrons sufficiently to trigger rising chorus emissions often starting at harmonics of powerline radiation. Such chorus emissions travel oppositely to the initiating whistler and arrive at the ground as precursors ahead of the main part of the two-hop whistler echo.

In the IWI a feedback loop is established in which the growing impulse traps more electrons which in turn cause more Gendrin wave growth until the equator is reached. A sketch of the wave packet envelope is shown in Figure 1, including a suggested location of the trapped electrons. As the wave packet and its ensemble of previously trapped electrons leave the equator the sign of the inhomogeneity reverses causing the flow of energy to reverse resulting in decay of the wave packet and detrapping of the electrons. Decay is caused by the dispersion of the different frequency components of the wave packet (that is no longer growing by the addition of new Gendrin components) that travel along different paths and at different velocities because they no longer satisfy the Gendrin condition. They may echo back and forth, at high wave normal angles, between lower hybrid reflection points much like an MR whistler [*Edgar*, 1976], leading to a buildup of plasmaspheric hiss. This mechanism could exist in parallel with those previously proposed to explain plasmaspheric hiss [*Muzzio and Angerami*, 1972; *Thorne et al.*, 1973; *Storey et al.*, 1991; *Draganov et al.*, 1992]. It might also play a role in the generation of the band-limited electromagnetic impulses observed on DE-1 and ISEE 1, 2 outside the plasmasphere [*Reinleitner et al.*, 1983; *Ondoh et al.*, 1989; *Sonwalkar et al.*, 1990].

The Gendrin Mode

Following *Gendrin* [1961] we assume the whistler-mode refractive index μ to be given by

$$\mu^2 = 1 + \frac{f_N^2}{f(f_H\cos\Theta - f)} \approx \frac{f_N^2}{f(f_H\cos\Theta - f)} \tag{1}$$

where Θ=angle between the wave normal and the Earth's static magnetic field $\bar{B}_o$ (in direction of the z axis), f_N=plasma frequency= $\sqrt{81N}$ Hz, N=electron concentration in el/m^3, f_H=electron gyrofrequency in Hz and f=wave frequency in Hz, assumed to lie in the range f_L to $f_H/2$. For $\cos\Theta = \frac{2f}{f_H}$ in (1), it is then readily shown [*Gendrin*, 1961; *Sturrock*, 1962; *Helliwell*, 1965] that the group ray velocity is directed along $\bar{B}_o$ and its magnitude is given by

$$v_{gr} = \frac{cf_H}{2f_N} \tag{2}$$

In this paper any whistler-mode wave for which $\cos\Theta = 2f/f_H$, $f_L < f < f_H/2$, will be called a Gendrin mode or

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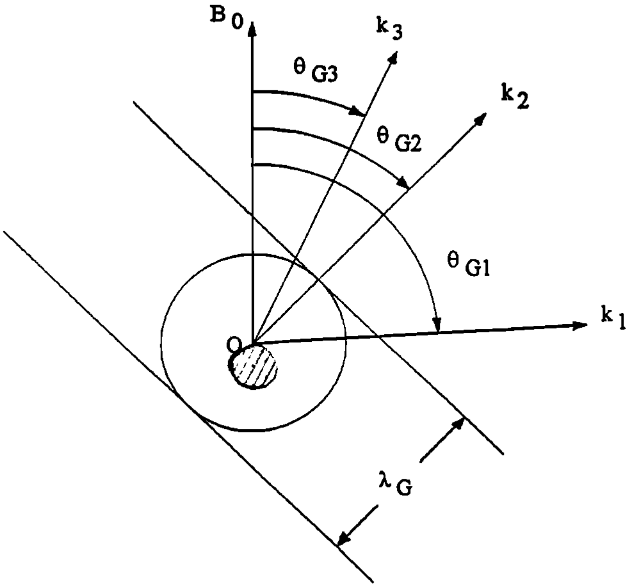

Figure 1. Gendrin-mode wave packet, showing wave normals of three representative plane wave components, at $f_1 = 1.05 f_L = 0.333$ kHz, $f_2 = 0.354 f_H = 4.83$ kHz (frequency of max $E_{\|}$ for constant $B_{\perp}$) and $f_3 = 0.45 f_H = 6.14$ kHz, where $f_L \cong \sqrt{f_H f_{H_i}} \cong \frac{f_H}{43}$. Wave normal angles Θ_G, shown for vectors k_1, k_2 and k_3, are given by the Gendrin condition $\cos\Theta_G = 2f/f_H$; $\Theta_{G_1} = 87.2°$, $\Theta_{G_2} = 45°$, $\Theta_{G_3} = 25.9°$. All wavefronts are tangent to spherical surfaces centered at O and all waves are in phase at O. All wave vectors k have the same magnitude, as required by Eq. (4). The diameter λ_G marks the locus of negative maxima of the Gendrin components comprising the wave packet. Wave vectors k_1 and k_2 mark the effective boundaries in θ_G of the most significant component waves. The circular cross-hatched region is the crudely estimated location of the trapped electrons whose retardation by $E_{\|}$ causes the wave packet to grow. Definition of this trapping region as well as the spatial distribution of the wave packet fields themselves requires a yet-to-be accomplished self-consistent solution of wave generation and loss as well as resonant electron energy exchange using representative distribution functions.

wave, and the corresponding wave normal angle Θ_G. At the Gendrin angle Θ_G, it is clear from (1) and (2) that the phase velocity has a longitudinal component given by

$$v_{p\|} = \frac{v_p}{\cos\Theta_G} = \frac{c f_H}{2 f_N} = v_{gr} \equiv v_G \qquad (3)$$

and a wavelength given by

$$\lambda_G = \frac{v_p}{f} = \frac{c}{f_N} \qquad (4)$$

Thus for the Gendrin mode, the wavelength (and therefore the wave number k) is a constant of the medium and both the ray and the longitudinal component of the phase velocity have the same numerical value v_G. Both are independent of the wave frequency and are aligned with the static magnetic field $\bar{B}_o$, which is the guiding center for geomagnetically trapped electrons and ions. It is suggested therefore that a wave packet consisting of a spectrum of Gendrin waves may interact with electrons or ions that are in longitudinal resonance (i.e., $v_{\|} = v_G$) with the wave packet, causing either growth or attenuation of the packet, depending on the relative spatial variations of v_G and the $v_{\|}$ of the charged particles.

To illustrate we show in Figure 2 the equatorial values of v_G and λ_G versus L value for a typical model of the magnetosphere [*Carpenter and Anderson*, 1992]. Inside the plasmapause, located at $L \sim 4.1$, $\lambda_{G_o} \cong 1.6$ km whereas outside it jumps to 10.6 km mainly because of the rapid drop, by a factor of 6.6, of the plasma frequency f_N. The Gendrin wavelength continues to increase with L, reaching 33 km at $L = 8$. The equatorial value of the Gendrin velocity, v_{G_o}, on the other hand, tends to decrease with L, except at the plasmapause, owing to the more rapid fall off of f_{H_o} compared with f_{N_o} (see Eq. (3)). Just inside the plasmapause $v_{G_o} = 1.1 \times 10^7$ m/s, corresponding to a parallel resonant electron energy $W_{\|} = 334$ eV, while just outside, $v_{G_o} = 6.5 \times 10^7$ m/s, corresponding to $W_{\|} = 11$ keV. Thus for a given concentration of trapped electrons those outside the plasmapause carry about 35 times more energy per electron than those inside. In addition the larger wavelength outside results in a correspondingly larger wave packet that can therefore trap a larger number of these more energetic electrons, leading to more total wave power generated outside the plasmapause by this mechanism.

Since the Gendrin wave packet travels along $\bar{B}_o$ (the z axis), the growth conditions must depend on $v_G(z)$. Using a dipole magnetic field model and typical electron density variations along z (diffusive equilibrium (DE) model inside the plasmapause and collisionless model outside [*Park*, 1972]), it can be shown that the Gendrin velocity is well represented by a function of the form

$$v_G = v_{G_o}(1 + a z^n) \text{ m/s} \qquad (5)$$

where

v_{G_o} = equatorial value of v_G, as given in Figure 2;
a, n = empirically derived constants;
$n = 2.5$, $a = 2.70 \times 10^{-18}$, inside pp;
$n = 2.0$, $a = 2.69 \times 10^{-15}$, outside pp.

For each value of v_G calculated from Eq. (5) we obtain the parallel energy $W_{\|} = 1/2 m v_G^2$ of a longitudinally resonant electron, where m =electron mass, as shown in Figure 3 for $L = 4$.

A critical factor in the model is the parallel electric field $E_{\|}$ of the wave packet which acts to trap and decelerate longitudinally resonant electrons. To keep trapped electrons from escaping from the wave packet's potential well as it approaches the equator the loss of parallel energy of these trapped electrons caused by $E_{\|}$ must

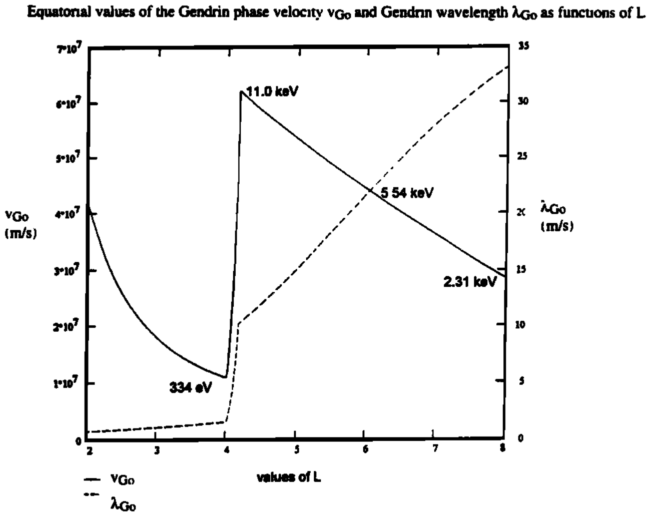

Figure 2. Equatorial values of the Gendrin parameters v_{G_o} (Eq. (3)) and λ_{G_o} (Eq. (4)) as functions of L for a model magnetosphere (*Carpenter and Anderson*, [1992]). Parallel resonant electron energies at the inner and outer edges of the plasmapause (at $L = 4.15$) are shown and at $L = 6$ and 8 on the $v_G(z)$ curve.

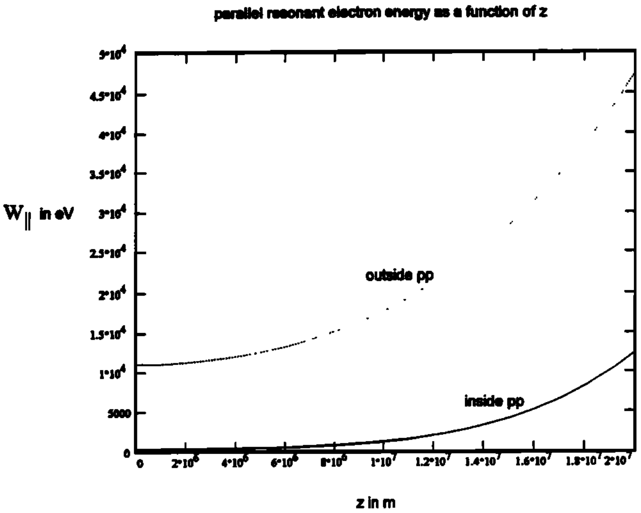

Figure 3. Parallel resonant electron energy $W_{\|} = \frac{1}{2}mv_G^2$ as a function of distance in kilometers from the equator along a dipole field line at $L = 4$ for a) a diffusive equilibrium model inside the plasmapause ($n = 2.5$, $a = 2.697 \times 10^{-18}$) and b) a collisionless model outside the plasmapause ($n = 2$, $a = 2.69 \times 10^{-15}$). See Eq. (5).

equal or exceed the change in the parallel resonance energy for the Gendrin wave packet as shown in Figure 3. Thus for non-relativistic electrons the critical value of the parallel electric field is given by

$$E_{\|} = \frac{m}{q} naz^{n-1}(1 + az^n)v_{Go}^2 \tag{6}$$

where m and q are the mass and charge, respectively, of the electron. From Eq. (6) we see that $E_{\|}$ will increase with z, both inside and outside the plasmapause, because of the increasing slopes of $v_G(z)$. Thus any trapped electrons (small pitch angles assumed) will remain trapped until the packet reaches the equator and each electron will have transferred to the wave an amount of energy $\Delta W_{\|} = W_{\|}(z) - W_{\|}(o)$, which can be found from the curves of Figure 3. The corresponding power density at the equator is then $\Delta W_{\|}$ (in Joules/m^3)$\times v_{G_o} = \Delta P_{\|}$ watts/m^2, a quantity that we shall call the available power density per trapped electron. Thus the weaker the initial applied $E_{\|}$, the lower the available power per electron. In addition $E_{\|}$ controls the concentration of trapped electrons since the range of $v_{\|}$ that can be trapped increases with $E_{\|}$. A full self-consistent solution to this non-linear problem that includes the continuous feedback between the trapped electrons and the wave as well as the effect of ions has not yet been carried out and is beyond the scope of this paper. However, significant electron trapping by $E_{\|}$ has been demonstrated in the case of a ducted whistler [*Tkalcevic*, 1982; *Tkalcevic et al.*, 1984] and for chorus bursts [*Reinleitner et al.*, 1983]. It should be noted that untrapped charged particles also exchange energy with the wave through the same longtudinal resonance, depending on the distribution function of the interacting particles. Their contributions must be evaluated in any self-consistent modeling of the IWI.

Growth Conditions

Growth of the Gendrin wave packet occurs when the trapped electrons are slowed by the wave packet [*Brice*, 1960]. The amount of wave growth per electron equals the decrease in the trapped electron's parallel kinetic energy $W_{\|}$ caused by the $E_{\|}$ of the wave, which can be found from Figure 3. As new Gendrin components are created in this way, previously generated Gendrin components that no longer satisfy the Gendrin condition given by (3), as a result of slowly changing values of f_N and f_H, will propagate away from the centroid of the main wave packet. These "lost" components then follow different ray paths that depend on frequency and initial wave normal angle, experiencing repeated magnetospheric reflections near the locations where $f = f_L$ [*Edgar*, 1976] assuming non-ducted propagation. Since the attenuation rate of the magnetospherically reflected (MR) waves, whose wave normal directions are near the resonance cone ($\Theta = \cos^{-1} f/f_H$), have been shown experimentally to be small [*Edgar*, 1976], the magnetosphere tends to fill up with MR waves, whose equilibrium power level is determined by equating the production rate (as yet unknown) to the loss rate. Such a process could contribute to the background plasmaspheric VLF/ELF hiss that is seen throughout much of the magnetosphere and which has been attributed to growth processes (cyclotron resonance [*Thorne et al.*, 1979] and longitudinal resonance [*Muzzio and Angerami*, 1972] as well as echoing MR whistlers [*Draganov et al.*, 1992].

Nature of the Source

To account for significant wave generation by the mechanism described above there must be an adequate reservoir of energetic electrons trapped in the wave packet. To estimate this quantity we need to know the volume of space within which the $E_{\|}$ required for trapping exists. We start by assuming an angular spectrum of plane Gendrin components that add together in phase at the center of the wave packet, sketched in Figure 1. To estimate the wave packet volume we use the wavelength λ_G, given by (4). For example, on the equator outside the plasmasphere at $L \cong 4$, where $f_N \cong 2.8$ kHz, $\lambda_G = 11$ km (see Figure 2). This means that regardless of the bandwidth of the wave packet (noting that $f_L < f < 0.5 f_N$), there is a roughly spherical volume of about 5.5 km diameter within which $E_{\|}$ is of one polarity. Following *Tkalcevic* [1982], we use a maximum density of resonant trapped electrons of $N_c = 0.5$ el/m^3 as estimated from the debunching effect of the internal space charge fields created by the excess of electric charge caused by trapping. Using a one-dimensional application of Gauss' Law [*Tkalcevic*, 1982] estimated that the internal field due to the electron bunch is less than 10% of the wave field if the density of trapped electrons, given by $N = -\epsilon_o E/q\Delta s$, is less than roughly 0.5 el/m^3, where ϵ_o=permittivity of free space (8.854×10^{-12} F/m), E=parallel electric field in V/m, q=electron charge and Δs=total displacement (500 m) of the trapped charge.

The next step is to show that a "blob" of such trapped electrons ($N_c = 0.5$ el/m^3) can supply enough energy to produce significant wave growth. Using the case assumed above, a crude estimate of the available energy can be made by assuming that the electron blob is confined to the first half of the wave packet's retarding electric field, $0 < E_{\|} < E_{\|max}$, a distance of $\sim\lambda_G/4$ along $\bar{B}_o$ as shown by the cross-hatched region of Figure 1. For simplicity we represent this volume by a sphere of diameter $\lambda_G/4 = 2.75$ km. Hence the number of electrons in this $\lambda_G/4$ "blob" of electrons is

$$N = \frac{\pi}{6}D^3 \times N_c = \frac{\pi}{6}(2.75 \times 10^3)^3 \times 0.5 \approx 5.4 \times 10^9 \tag{7}$$

At $v_G = 6.3 \times 10^7$ m/s, this blob carries a kinetic energy density of $W = 0.5(\frac{1}{2}mv_G^2) = 0.5\frac{1}{2}9.1 \times 10^{-31} \times (6.3 \times 10^7)^2 =\sim 9 \times 10^{-16}$ J/m^3. The corresponding power density is $P_d = Wv_G = 9 \times 10^{-16} \times 6.3 \times 10^7$ m/s$\cong 6 \times 10^{-8}$ watts/m^2 which is $\sim$250 times greater than the DE1 impulse wave flux [*Sonwalkar et al.*, 1990] of

$$\bar{E} \times \bar{H} = 30 \times 10^{-6} \times \frac{10^{-11}}{4\pi \times 10^{-7}} \approx 2.4 \times 10^{-10} \text{ watts/m}^2 \tag{8}$$

This result shows that the maximum kinetic energy carried by electrons trapped in a Gendrin mode wave packet is more than adequate to provide the impulse wave energy measured by DE1. From (5) we see that this blob must be fully formed at $z \cong 1000$ km to provide the required wave energy. The corresponding $E_{||}$ (Eq. 6) is 130μV/m which is a reasonable value for the relatively intense chorus emissions outside the plasmapause [*Burtis and Helliwell*, 1976].

Discussion and Conclusions

We have shown that the IWI can be an effective converter of radiation belt particle energy into VLF wave energy. We have suggested that the mechanism of the IWI might explain the generation of the band limited impulses observed with DE-1 and other satellites, which resemble solitons [*Al'pert*, 1983]. We can test the idea by comparing theoretical and observed values of the refractive index $\mu = cB/E$. From DE1 [*Sonwalkar et al.*, 1990], $\mu \approx 5$, while the theoretical value for the Gendrin mode at $f = 0.354 f_H$ (freq. of maximum $E_{||}/B$) outside the plasmapause, where most impulses are seen, is ~6, in good agreement with the data.

When the electron blob and wave packet pass the equator and travel down towards the ionosphere the packet collapses, with its component waves drifting away in different ray directions at different group velocity, because it is no longer being renewed by new Gendrin components, and the bunched resonant electrons are released from their wave trap. Although these electrons necessarily have nearly the same parallel velocity at the instant of release, they will tend to drift apart along the z axis because of their spread in pitch angle and the mutually repulsive coulomb force. In this proposed mechanism of beam formation the source blob is formed by deceleration of trapped electrons (raises the pitch angles) whereas in the [*Reinleitner et al.*, 1983] model the trapped electrons are accelerated by the $E_{||}$ of the wave and hence would have their pitch angles lowered, not raised. Release from the trap results in the formation of many transient electron beams, similar to that postulated to account for precursors [*Park and Helliwell*, 1977]. Although these beams would eventually diffuse into the background radiation belt population, their temporary presence could serve to trigger instabilities (as in the precursor mechanism mentioned earlier) that produce new emissions. Likewise, the banded chorus seen outside the plasmasphere [*Burtis and Helliwell*, 1976], might be related to such electron beams. Furthermore the elements of chorus could trap longitudinally resonant electrons [*Reinleitner et al.*, 1983]. These could generate coherent Cerenkov radiation and hence excite a Gendrin mode wave packet. Electrons trapped by this packet would upon their release constitute a particle beam that could trigger backward traveling narrow band doppler shifted cyclotron radiation (i.e., a chorus element) [*Carlson et al.*, 1990], which could then trap more longitudinally resonant electrons, thus forming a feedback loop. Such a self-sustaining process is needed outside the plasmapause where triggering signals from ground based sources are seldom present. Beam formation of this type could be expected to occur in other solar system magnetospheres as well as any cosmic plasma containing a magnetic field.

Confirmation of the proposed IWI requires a self-consistent numerical simulation in which non-trapped as well as trapped electrons are included. Free parallel energy would be available from electrons near the loss cone, noting that the trapping mechanism described here has the effect of removing some longitudinally resonant electrons from the loss cone by raising their pitch angles.

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