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PRECIPITATION OF SUPRATHERMAL (100 EV) ELECTRONS BY OBLIQUE WHISTLER WAVES

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Abstract. Electron precipitation induced by oblique whistler waves is investigated using a new test particle simulation code based on gyro-averaged equations of motion. Results indicate that highly oblique waves can efficiently pitch angle scatter suprathermal electrons (10–100 eV). At $L = 3$, for example, in a single encounter with a wave with power density $S \sim 8$ pW/m^2 and propagating at $\psi = 60°$ with respect to the Earth magnetic field B_o, average pitch angle scattering of 100 eV electrons is $\sim 0.1°$. In comparison, the average scattering of energetic (100 keV) electrons by a parallel propagating wave with the same S is $\sim 0.01°$. Estimates indicate that the precipitated electron energy fluxes resulting from the interaction of $\sim$ 100 eV electrons with oblique waves can be up to 30 times larger than that due to the precipitation of 100 keV electrons by parallel propagating waves.

1. Introduction

Precipitation of energetic (> 40 keV) electrons out of the radiation belts in gyroresonant interactions with lightning-generated ducted (i.e., parallel propagating) whistlers is now commonly observed [*Inan et al.*, 1990; *Burgess and Inan*, 1990, and references therein] at mid-to-low latitudes ($L < 4$). Little attention has been paid so far to the role of nonducted, oblique (i.e. propagating at an angle to the B_o) waves in electron precipitation. The bulk of the magnetospheric wave energy from lightning discharges propagates in the nonducted mode and is often found to multiply reflect (up to 8–10 bounces) back and forth between hemispheres [*Edgar*, 1976; *Draganov et al.*, 1992]. Since the wave normal angles of such waves are generally within a few degrees of the resonance cone they can resonantly interact with suprathermal electrons of 10–100 eV energy [*Jasna et al.*, 1990]. Preliminary estimates of the diffusion coefficients at the geomagnetic equator for such highly oblique waves have shown that they are at least as large as those for parallel propagating waves [*Inan and Bell*, 1991]. However, the diffusion coefficients evaluated at the equator are not by themselves sufficient to determine net pitch angle scattering since the effective interaction length (or interaction time) in the general case also depends sensitively on various parameters such as the location along the field line, frequency and obliqueness of the wave, energy and pitch angle of the particle.

2. Theoretical Formulation

The interaction between radiation belt electrons and magnetospherically reflected (MR) whistler waves is schematically illustrated in Figure 1.

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Electrons are considered to be in resonance with the wave when their velocity parallel to B_o is such that

$$v_z \simeq (\epsilon\omega_H - \omega)k_z^{-1} = v_R \tag{1}$$

where $\epsilon = \sqrt{1-(v/c)^2}$, v is the total particle velocity, c is the speed of light, $k_z = (\omega/c)n\cos\psi$ is the wave vector along B_o, ω_H is the electron gyrofrequency, ω is the wave frequency, n is the refractive index and ψ is the wave normal angle. In the following, the refractive index n is determined using the full whistler-mode dispersion relation including a single-ion (H^+) species [*Stix*, 1962].

We adopt a test particle method so that we can include inhomogeneity of the medium and coherent as well as incoherent waves [*Inan*, 1987]. The gyro-averaged equations of motion for a test electron interacting with an oblique whistler-mode wave can be expressed in terms of the components of the particle's momentum in the direction parallel (p_z) and perpendicular ($p_\perp$) to the B_o, the components of the wave electric ($E_{x,y,z}$) and magnetic ($B_{x,y}$) field, and the angle η between $p_\perp$ and $B_\perp$:

$$\dot{\eta} = \epsilon\omega_H - \omega - \frac{\epsilon}{m}k_z p_z \tag{2a}$$

$$\dot{p}_z = \omega_\tau^2 k_z^{-1}\sin\eta - \frac{\epsilon}{m}\frac{p_\perp^2}{2\omega_H}\frac{\partial\omega_H}{\partial z} \tag{2b}$$

$$\dot{p}_\perp = -[\omega_1(\epsilon p_z + mR_1)J_0(\beta) - \omega_2(\epsilon p_z - mR_2)J_2(\beta)]\sin\eta + \frac{\epsilon}{m}\frac{p_\perp p_z}{2\omega_H}\frac{\partial\omega_H}{\partial z} \tag{2c}$$

where
$\omega_\tau^2 = \omega_{\tau o}^2[J_0(\beta) - \alpha_1 J_2(\beta) + \epsilon^{-1}\alpha_2 J_1(\beta)]$, $\omega_{\tau o}^2 = \epsilon\omega_1 k_z p_\perp$, $\alpha_1 = (B_x^w - B_y^w)/(B_x^w + B_y^w)$, $\alpha_2 = eE_z^w/(\omega_1 p_\perp)$, $R_{1,2} = (E_x^w \pm E_y^w)/(B_x^w \pm B_y^w)$, $\omega_{1,2} = (e/2m)(B_x^w \pm B_y^w)$ and $\beta = \epsilon k_\perp p_\perp/(m\omega_H)$, with J_i, $i = 0, 1, 2$ being the Bessel function of the first kind and order i, and m being electron's rest mass. Equations 2a,b,c are integrated in time to determine the velocity space trajectories of individual test particles.

We consider interactions at $L = 3$, between electrons with initial equatorial pitch angle in the vicinity of the equatorial loss cone angle ($\alpha_{eq_{lc}} \cong 9°$) and a whistler wave propagating at an angle ψ (assumed constant along the field line) to the B_o. The wave Poynting flux is held constant during the interaction and taken to be $S = 8.1$ pW/m^2, corresponding to that of a 10 kHz wave with $\psi = 0°$ and an intensity at the equator of $B^w = 1$ pT.

Figure 2 indicates that gyroresonance with 100 eV electrons can occur at $L = 3$ for wave frequencies up to $\sim$ 25 kHz, although we only consider waves for which $f < f_{Heq}/2$ or $f <$ 16.17 kHz. Raytracing with typical model magnetospheres, such as that shown in Figure 3, indicates that for signals originating on the ground (such as VLF transmitter signals and lightning-generated whistlers), which upon entry

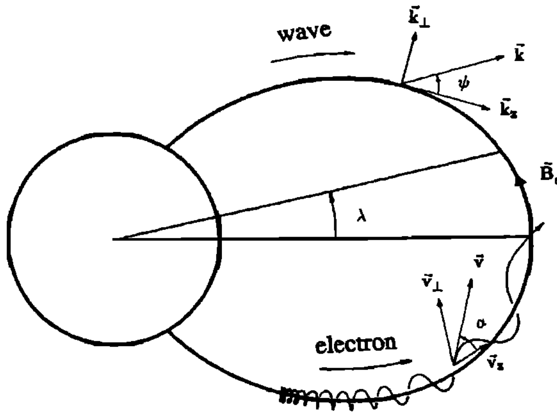

Fig. 1. The motion of radiation belt electrons is fully described by any three motion coordinates that can be derived from the electron velocity $\vec{v}$. We use the equatorial pitch angle α_{eq} and energy E to initially identify the test particle. The equations of motion are expressed in terms of the velocity parallel to the B_o field line v_z, equatorial pitch angle α_{eq} and gyrophase η. Position in space is defined by specifying the field line (L-shell) and geomagnetic latitude λ. The electromagnetic wave is specified by the value of its wave vector k, wave normal angle ψ with respect to the B_o, and power density S (Poynting flux).

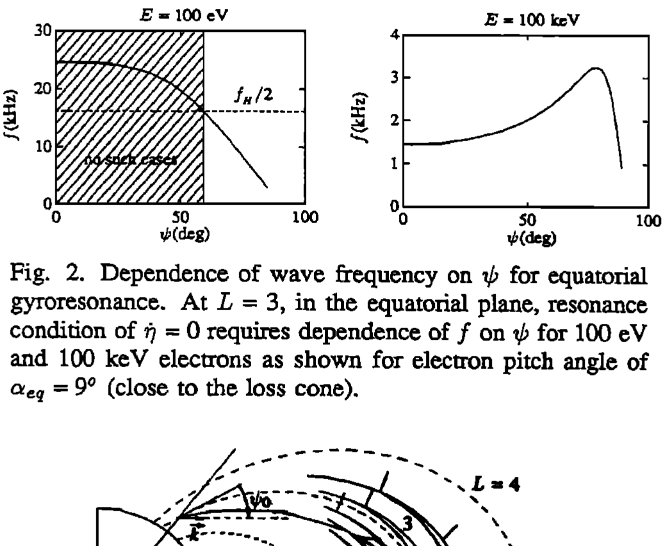

Fig. 2. Dependence of wave frequency on ψ for equatorial gyroresonance. At $L = 3$, in the equatorial plane, resonance condition of $\dot{\eta} = 0$ requires dependence of f on ψ for 100 eV and 100 keV electrons as shown for electron pitch angle of $\alpha_{eq} = 9^o$ (close to the loss cone).

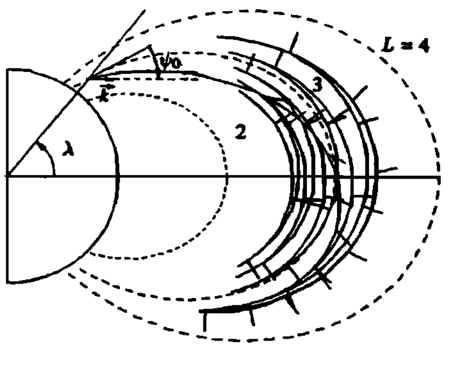

Fig. 3. A sample raypath in a magnetic meridional plane for a wave injected at 1000 km altitude at $L = 3$, with frequency $f = 2$ kHz. The wave normal direction is shown along the raypath, and its angle with respect to the magnetic field line is defined at the point of injection as ψ_0. Several magnetospherical reflections and eventual settlement at $L \sim 3$ are evident.

into the ionosphere have nearly vertical ψ, the equatorial plane crossings occur at L such that $f < f_{H_{eq}}/2$ [*Inan and Bell*, 1991]. This fact is also consistent with experimental data which indicates that signals from ground-based sources are confined to L-shells corresponding to $f < f_{H_{eq}}$ [*Bell et al.*, 1981]. Thus, our analysis is particularly applicable to interactions involving waves generated on the ground. Note that for 100 keV electrons the frequency of gyroresonant waves for all ψ is less than 3.25 kHz (0.1 f_{Heq}).

The results of Figure 2 are used below to select f and ψ so that the 100 eV and 100 keV test electrons would be gyroresonant with the wave at the geomagnetic equator at $L = 3$.

3. Test Particle Trajectories

Trajectories of sample equatorially resonant 100 eV and 100 keV test electrons calculated under the conditions described above are shown respectively in Figures 4 and 5.

For each case, 12 different test electrons initially distributed uniformly in Larmor phase are used since we see from Figures 4 and 5 that the net scattering is strongly dependent on initial phase [*Inan et al.*, 1978].

The variation of v_R with geomagnetic latitude λ represents the inhomogeneity of the medium which ultimately restricts the duration of the interaction. As is evident from Figures 4 and 5, significant interaction (i.e., significant pitch angle scattering) occurs only when $v_z \simeq v_R$. We note that the length of the available interaction region rapidly decreases with increasing ψ, which is why the net scattering is roughly the same for different ψ, although the diffusion rate itself rapidly increases with ψ for $\psi \to \psi_r$, where ψ_r is the resonance cone angle [*Inan and Bell*, 1991].

For 100 eV electrons (Figure 4) $\overline{\Delta\alpha_{eq_f}}$ slightly decreases with increasing ψ, being $\overline{\Delta\alpha_{eq_f}} = 0.2797^o$ for $\psi = 60^o$ and 0.1997^o for $\psi = 80^o$. For 100 keV electrons (Figure 5) we see a more rapid dependence on ψ with $\overline{\Delta\alpha_{eq_f}} = 0.0107^o$ for $\psi = 0^o$ (parallel propagation), while $\overline{\Delta\alpha_{eq_f}} = 0.0039^o$ for $\psi = 80^o$ (highly oblique propagation).

Comparison of $\overline{\Delta\alpha_{eq_f}}$ for 100 eV and 100 keV electrons shows, for the parameters considered here, that equatorial pitch angle scattering of suprathermal electrons by oblique waves produces higher scattering than even the most efficient scattering of 100 keV electrons by a wave with $\psi = 0^o$ and the same S. We note that scattering of energetic electrons by parallel propagating waves at $L = 3$ has been extensively studied [e.g., *Chang and Inan*, 1985] and that electron fluxes have been measured on satellites [*Voss et al.*, 1984], rockets [*Goldberg et al.*, 1986] and via ground-based methods [e.g., *Inan et al.*, 1990]. Thus, a preliminary estimate of precipitation fluxes of suprathermal electrons can be obtained by means of comparisons of scattering of 100 eV and 100 keV electrons as shown in Figures 4 and 5.

4. Precipitation Energy Flux Estimates

We use the results obtained above to comparatively estimate differential energy fluxes for 100 eV and 100 keV electrons, pitch angle scattered by the parallel propagating or oblique waves. The differential scattered energy flux dQ is given by

$$dQ \sim E v_z^3 f(E) dv_z d\alpha_{eq} \quad (3)$$

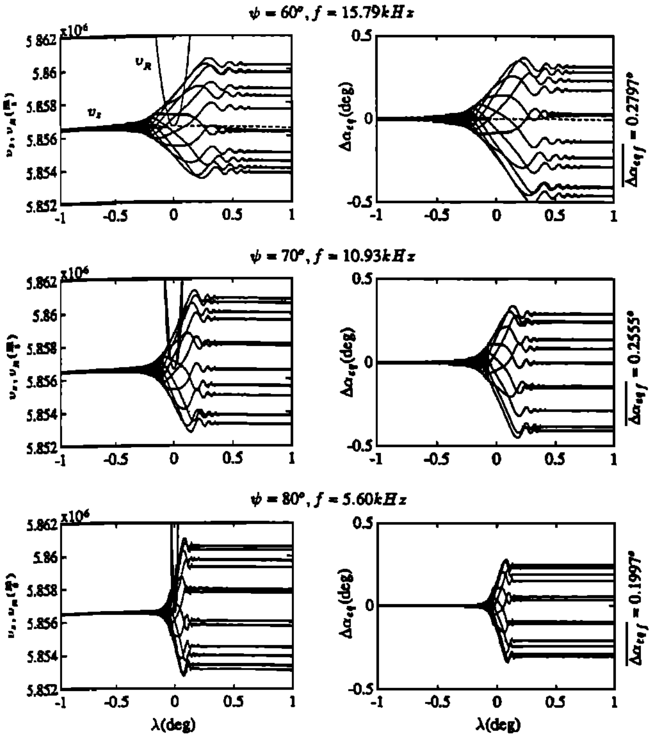

Fig. 4. Sample trajectories for suprathermal (E = 100 eV) electrons interacting at L = 3 with waves having a power density of $S = 8.1$ pW/cm^2 and three different wave normal angles $\psi = 60^o, 70^o, 80^o$ and corresponding equatorial resonance frequencies f = 15.79 kHz, 10.93 kHz, 5.60 kHz. The trajectories are shown as the variation with geomagnetic latitude (λ) of the electron's velocity parallel to the Earth's magnetic field v_z and equatorial pitch angle change $\Delta\alpha_{eq}$, for 12 test electrons with different, equally spaced initial gyrophases $\eta(\lambda_i) = n\pi/6$, $(n = 1, 2, ...12)$. $\overline{\Delta\alpha_{eq_f}}$ is the root-mean-square value of the final pitch angle change at the end of the resonant interaction, namely $\overline{\Delta\alpha_{eq_f}} = \sqrt{\langle\Delta\alpha_{eq}^2(\lambda_f)\rangle}$, where the averaging ($\langle\rangle$) is over the 12 test particles. In the absence of a perturbing wave, v_z follows an adiabatic variation while $\Delta\alpha_{eq}$ remains zero, as indicated with a dashed line in the top panel. v_R is the resonant velocity defined by (1) and is shown on the same graph with the $v_z(\lambda)$ for easy assessment of the resonant condition $v_z \simeq v_R$.

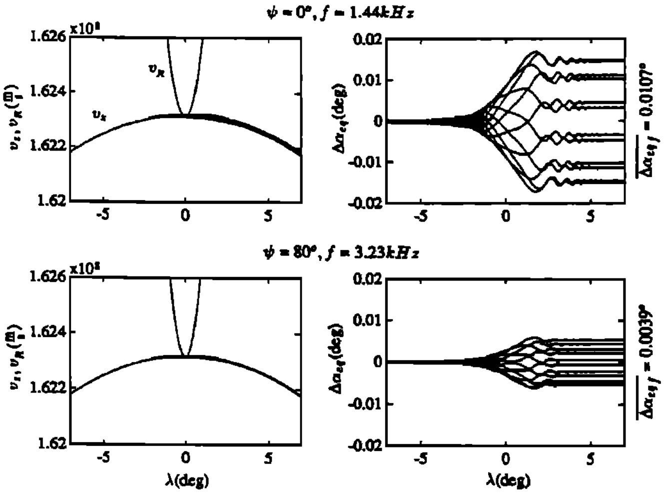

Fig. 5. Sample trajectories for energetic (100 keV) electrons. The format is identical to that of Figure 4.

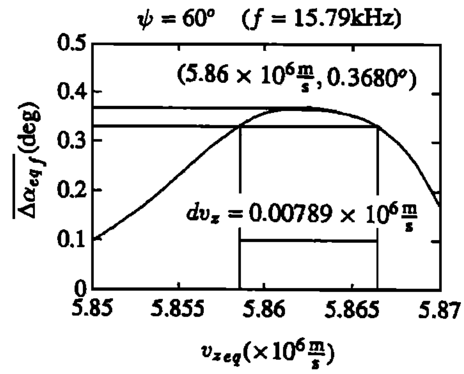

Fig. 6. Scattering as a function of initial equatorial electron velocity in the direction of the B_o field, v_{zeq} for suprathermal (100 eV) electrons. Each point on the curve represents ensemble averaged scattering of 12 test particles with the same initial v_{zeq} but different gyrophases. The result shown is for test electrons with initial equatorial pitch angle $\alpha_{eq} = 9^o$ and energy $\sim$ 100 eV interacting with a wave with $\psi = 60^o$, f = 15.79 kHz, and $S = 8.1$ pW/cm^2.

where $f(E)$ is the electron distribution function, E is the electron energy and units of dQ are ergs/cm^2-sec. Based on experimental data [*Schield and Frank*, 1970] a typical electron distribution function at $L = 3$ can be taken to be

$$f(E) \sim E^{-m} \quad \text{where } 2.5 \leq \text{m} \leq 3$$

To compare the ratio of dQ for 100 eV electrons precipitated by oblique ($\psi = 60^o$) wave to that for 100 keV electrons precipitated by a ducted ($\psi = 0^o$) wave, we use $\overline{\Delta\alpha_{eq_f}}$ for $d\alpha_{eq}$. To estimate dv_z, test particle simulations were used to determine $\overline{\Delta\alpha_{eq_f}}$ for different initial v_{zeq} as shown in Figures 6 and 7 respectively for 100 eV and 100 keV. Using the velocities corresponding to peak $\overline{\Delta\alpha_{eq_f}}$ as v_z and dv_z as determined

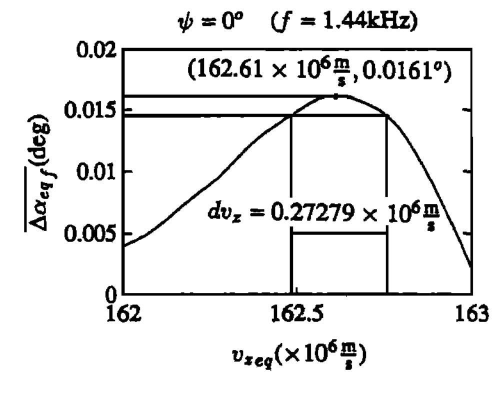

Fig. 7. Same as Figure 6 but for energetic (100 keV) electrons.

by a 10% reduction in $\overline{\Delta\alpha_{eqf}}$ on both sides of the peak in (3) we find:

$$1 \leq \frac{dQ(E = 100 \text{ eV}, \psi = 60^o)}{dQ(E = 100 \text{ keV}, \psi = 0^o)} \leq 34$$

It thus appears that, at $L = 3$, the differential precipitated energy flux of 100 eV electrons precipitated by an oblique wave would be higher than that of 100 keV electrons precipitated by parallel propagating waves.

To put this result in perspective, we note that lightning-induced electron precipitation bursts have been measured to have peak flux levels of 10^{-3}–10^{-2} ergs/cm^2-sec [*Voss et al.*, 1984], consistent with the ionospheric disturbances produced by such bursts [*Inan et al.*, 1985]. On this basis, we can expect lightning-induced whistlers propagating in the oblique mode to precipitate fluxes of $> 10^{-2}$ ergs/cm^2-sec of $\sim$ 100 eV electrons. Note that the power density in an oblique wave may be expected to be lower, but this effect would be compensated by the illumination of much larger spatial regions.

5. Conclusions

Our results indicate that on typical mid-latitude field lines (e.g., $L = 3$) pitch angle changes of 100 eV electrons gyroresonantly scattered by highly oblique whistler-mode waves is generally higher than those of 100 keV electrons scattered by parallel propagating waves having the same power density. Estimates indicate that precipitated energy fluxes resulting from such interactions can be up to 30 times larger than those due to the precipitation of 100 keV electrons, which are commonly observed in ground based experiments by means of their effects in the lower ionosphere. In view of the omnipresence of highly oblique magnetospherically reflected waves at midlatitudes [*Edgar*, 1976], it appears that lightning-induced whistler wave energy can precipitate substantial fluxes of 100 eV electrons which would be deposited at 200-300 km altitude and produce secondary ionization enhancements. In view of the relatively large scale heights at such altitudes, the upward field-aligned diffusion from such ionization enhancements can contribute to the formation of whistler-mode 'ducts' as suggested by *Inan and Bell* [1991].

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