VLF Pulse Propagation in the Magnetosphere

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***Abstract*—Pulse distortion is important in controlled experiments on very low frequency (VLF) wave particle interaction (WPI) processes in the magnetosphere. Whistler-mode (WM) propagation of RF pulses through a homogeneous magnetoplasma as well as through a duct in the magnetosphere has been investigated by using the fast Fourier transform technique. This technique can be applied in both homogeneous and slowly varying media. As far as we know this is the first time that the distortion of a VLF pulse propagating in the magnetosphere has been calculated. In a homogeneous medium at frequencies below $f_H/4$ (f_H is the electron gyrofrequency) the high frequency components arrive prior to the main body of the pulse while the low frequency components lag behind. This sequence is reversed when the carrier frequency exceeds $f_H/4$. The distortion increases as the frequency departs from $f_H/4$. In the magnetosphere it is found that the frequency of minimum distortion is the "nose" frequency f_N, as expected. When the carrier frequency is below f_N the high frequency components of a pulse always arrive first at the equator and the distortions increase as the carrier frequency departs from f_N. Above f_N, on the other hand, there is always a location along the path where the pulse distortion is minimum. It is also found that the stretching of the front end of a pulse is large enough (~30 ms for a pulse at 4 kHz traveling through a duct at 4 R_E) to require compensatory pre- and postprocessing of signals in certain wave-injection experiments. An equalizer to compensate for the phase distortion introduced by the medium has been designed. Computer simulation results show that when the preprocessed signals arrive at the interaction region they have the prescribed waveforms. By the same principle the propagation distortion developed between the interaction region and the receiving site can also be removed.**

I. INTRODUCTION

PULSE PROPAGATION in a dispersive medium has been widely discussed in the literature [1]-[6]. However, as far as we know, the present paper is the first to treat the distortion of a very low frequency (VLF) pulse propagating in an inhomogeneous medium, the magnetosphere.

Recently investigations on wave-particle and wave-wave interactions have been carried out experimentally by injecting VLF pulses into the magnetosphere [7]. As illustrated in Fig. 1, a VLF pulse injected from a 21.2-km dipole antenna at Siple, Antarctica, propagates in the whistler mode (WM) along a field-aligned duct of enhanced ionization. Through cyclotron resonance the VLF waves interact with high energy electrons streaming in the opposite direction. Near the equatorial region this interaction becomes especially strong, often causing the signal to be amplified and emissions to be triggered at frequencies other than those of the input signals. These signals then may interact with one another through these high energy electrons. Energy coupling among the signals and suppression of signal growth are commonly observed.

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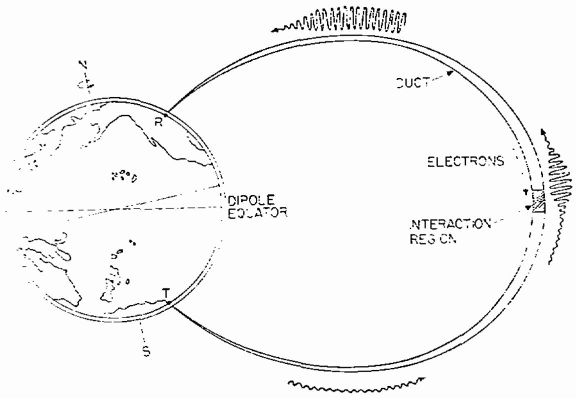

Fig. 1. Sketch showing propagation of transmitter signals between Siple Station and Roberval. Signals injected from Siple travel along a field-aligned duct of enhanced ionization. The signals interact with high energy electrons traveling in the opposite direction in an interaction region near the equator (after [13]).

After the interaction the signals travel through the remainder of the duct and arrive at the Siple conjugate near Roberval, Quebec.

The dynamic spectrum in Fig. 2 shows an example of wave amplification and emission triggering. The blackness and trace width both increase with the wave intensity. A 2-s pulse at 3 kHz is injected into the magnetosphere and arrives at Roberval at $t = 0$. It is observed that the pulse is amplified and triggers two emissions: one at about 200 ms after the beginning of the pulse and the other at the end of the triggering wave.

It is generally believed that the amplification and the triggering mechanisms are due to cyclotron resonance between energetic electrons and VLF waves, but there is as yet no general agreement on the detailed mechanisms. VLF wave-injection experiments carried out between Siple and Roverval are providing new data on these phenomena.

In traveling between transmitter and receiver, the signals must traverse a dispersive path both before and after arriving at the equatorial interaction region. Without knowing how the signals are distored by dispersion, we may draw incorrect conclusions about the interaction mechanism. Therefore it is essential to study the problem of VLF pulse propagation in the magnetosphere.

In this paper we calculate the pulse distortion as a function of carrier frequency. Then we show how to compensate for the distortion so that the signal will have a prescribed waveform at a given location along the path.

Our method of simulating VLF pulse propagation in a magnetoplasma is similar to that used by Seylor *et al.* [6]. We use the fast Fourier transform (FFT) technique to decompose a pulse train into its Fourier components, allowing each component to propagate through the medium separately. At each point of interest we sum all components by taking the inverse Fourier transform to obtain the amplitude *versus* time of the

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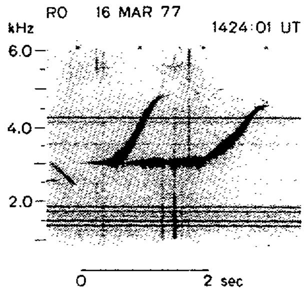

Fig. 2. Frequency–time spectrogram showing typical examples of amplification and triggering of WM waves.

distorted pulses. This method requires only a knowledge of the phase delays and a simple modification of the amplitudes of these frequency components. In a lossless but spatially slowly varying medium the amplitude modification is a local effect [3].

We are interested in the propagation of a whistler-mode (WM) pulse along a duct in the magnetosphere. By using a well-developed model for electron concentrations along a duct [8] and by assuming a dipole model for the geomagnetic field, the phase velocity v_p at every point along the path for each frequency component in a whistler mode can be calculated from the dispersion relationship [9]

$$\frac{c^2}{{v_p}^2} = 1 - \frac{{f_p}^2}{f(f - f_H)} \tag{1}$$

where f_p and f_H are the plasma frequency and the gyrofrequency, respectively. The phase delay can then be obtained from

$$t_{\text{ph}} = \int_0^z \frac{ds}{v_p}. \tag{2}$$

We have assumed that the spatial variation is so small that the phase velocity is constant within a wavelength.

In the next section we describe the method of study in detail. In Section III WM pulse propagation in a homogeneous magnetoplasma is analyzed. Some effects of dispersion on a WM wave can be seen easily. It is observed that the distortion of a pulse depends on its carrier frequency. In Section IV the problem of pulse propagation along a duct in the magnetosphere is analyzed. The pulse shape is monitored at six locations along the path. The results show how to choose the carrier frequency such that at a particular location along the path the pulse envelope shows a minimum distortion. Discussions and conclusions are presented in Sections IV and V, respectively.

II. METHOD OF STUDY

The basic scheme of simulating VLF pulse propagation in a lossless and linear magnetoplasma is to decompose a pulse into its Fourier components via FFT and then to allow each frequency component to pass through the dispersive medium separately. Because of the dispersion, the phase retardation at a point of interest is not linearly proportional to the frequencies of the components. As a result of this nonlinear phase retardation the temporal shape of the pulse, which can be obtained by summing all components via the inverse Fourier transform, is altered. In addition to the nonlinear phase retardation, there is a change in the relative amplitudes of the components of a pulse which propagates in an inhomogeneous dispersive medium. In a slowly varying medium this change is a local effect; it is discussed in Section IV.

In a dispersive medium the phase velocity varies with frequency and can be found through the dispersion relationship for the medium. The amplitude and phase components of a pulse in the frequency domain, after traveling from $Z = 0$ to $Z = Z_1$, can be written as

$$A_m(f, z_1) = A_m(f, 0) m(f, z_1) \tag{3a}$$

$$A_{\text{ph}}(f, z_1) = A_{ph}(f, 0) - \int_0^{z_1} 2\pi f \frac{dz}{v_p(f, z)} \tag{3b}$$

where $A_m(f, z)$ and $A_{\text{ph}}(f, z)$ are, respectively, the amplitude and the phase of the frequency components at f when the pulse is located at z. $v_p(f, z)$ is the corresponding phase velocity of the frequency component at f and $m(f, z)$ is the amplitude modulation function. In a slowly varying medium $m(f, z_1)$ depends only on the refractive index at z_1. In a homogeneous medium it is unity.

It is seen that the phase retardation

$$\int_0^{z_1} 2\pi f (dz/v_p(f, z))$$

is not a linear function of frequency in general. The spectrum at $z = z_1$ can be expressed as

$$A(f, z_1) = m(f, z_1) A(f, 0) \exp\left(-i \int_0^{z_1} 2\pi f \frac{dz}{v_p(f, z)}\right). \tag{4}$$

The corresponding signal in the time domain is the inverse transform of (4) given by

$$a(t, z_1) = \int_{-\infty}^{\infty} m(f, z_1) A(f, 0) \cdot \exp\left[i 2\pi f \left(t - \int_0^{z_1} \frac{dz}{v_p(f, z)}\right)\right] df. \tag{5}$$

Recalling that $A(f, 0)$ is the spectrum of the input pulse and that $v_p(f, z)$ can easily be obtained from the dispersion relation, we can obtain $a(t, z_1)$ numerically by taking advantage of the available fast Fourier transform technique (FFT) algorithm. For the detailed calculation the reader is referred to [10].

Because of the discrete nature of the FFT we are physically monitoring a pulse train instead of a single pulse. The period of this pulse train is fixed in our simulation at 512 ms. Therefore, by monitoring the signals at every station for only 512 ms, we obtain all of the necessary information regarding distortion of the pulse. Similarly, the bandwidth of the signal is assumed to be less than 1 kHz. For a square pulse with a duration greater than 10 ms the energy in the 1-kHz bandwidth centered on the carrier frequency is more than 98 percent of its total energy.

III. PROPAGATION IN A HOMOGENEOUS MODEL

Simulation of WM pulses propagating longitudinally is carried out in a homogeneous model with a plasma density of 400 electrons per cubic centimeter and a gyrofrequency of 13 kHz for various carrier frequencies. These values are appropriate to the equatorial region near $4R_E$ in the magnetosphere. The results have been translated down in frequency so that the carrier frequency of the input pulse is always displayed at 500 Hz. The actual carrier frequency may be several kilohertz.

According to the whistler theory [9] the group velocity can be written as

$$v_g = 2C \frac{f^{1/2}(f_H - f)^{3/2}}{f_p f_H} \tag{6}$$

The group velocity *versus* frequency is sketched in Fig. 3. The maximum group velocity occurs at $f = f_H/4$ and is given by

$$v_{\max} = \frac{\sqrt{27}}{8} C \frac{f_H}{f_p}. \tag{7}$$

In the model the maximum group velocity occurs at $f =$ 3.25 kHz, the so-called "nose" frequency, and has a value of 14073 km/s. Fig. 4 shows the dispersion of six 20-ms pulses at different frequencies over a fixed path of length 25 000 km. The square pulses were injected into the magnetoplasma at −1.526 s. The carrier frequencies are 4.25, 3.75, 3.25, 2.75, 2.25, and 1.75 kHz. At the nose frequency, 3.25 kHz, the pulse propagates at a maximum group velocity and exhibits the least distortion.

In the simulation we use phase velocities to calculate the phase retardations for all frequency components. We have not used the concept of group velocity. However, our simulation result shows the *center*[1] of an RF pulse indeed propagates with the group velocity (within a measurement error of less than one percent).

Using the first detectable part of the pulse as a reference, the arrival time may be shorter than the "group delay time" by as much as a few pulsewidths. For example, the arrival time of the first part of the distorted pulse in the lowest panel in Fig. 4 is ~1.846 s while the group delay time is found with the aid of (6) to be 1.953 s. Thus there is a time discrepancy of about 100 ms. On the other hand, the arrival time of the "center" is measured to be 1.955 ms. The discrepancy here is less than one percent.

The timing of the first detectable part of the pulse is related to the choice of the bandwidth in the simulation. Had a larger bandwidth, say 3 kHz, been employed the first detectable part of the pulse at 1.75 kHz in Fig. 4 would be the component at the nose frequency. There are no other componemts arriving earlier than the nose frequency component in the whistler mode.

It is important to realize that the group velocity evaluated at the carrier frequency can predict the arrival time of the "center," not the leading edge of the pulse. This statement applies at frequencies not too close to either the gyrofrequency or zero frequency, where the phase velocity is zero. In the model we use here, the valid frequencies of the carrier of a pulse longer than 10 ms range from about 500 Hz to 12.5 kHz. Other important features depicted in Fig. 4 are as follows.

[1] The center of a distorted pulse is *defined* as the center of the interval within which the signal exceeds 30 percent of the peak value.

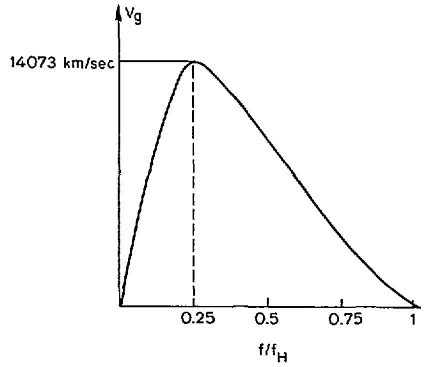

Fig. 3. Group velocity *versus* frequency of a WM signal propagating in a homogeneous magnetoplasma in which f_p = 180 kHz and f_H = 13 kHz.

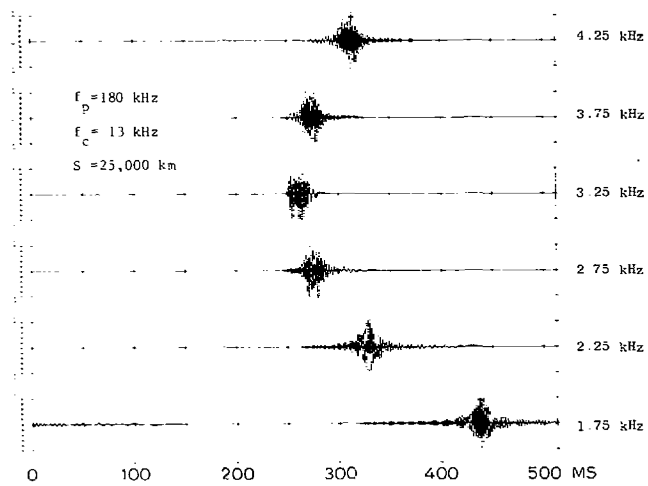

Fig. 4. Propagation of six 20-ms pulses at 1.75, 2.25, 2.75, 3.25, 3.75, and 4.25 kHz through homogeneous magnetoplasma over a 25 000-km. Pulses were injected at −1.526 s. Stairstep jumps in the plots are due to the finite resolution of the Calcomp. plotter.

1) The distortion of a pulse depends on its carrier frequency; the further away from the nose frequency, the more the distortion. It is generally true that the distortion is larger below the nose frequency than above.
2) When the carrier is above the nose frequency, the low frequency components arrive before the main wave packet, and the high frequency components arrive behind the packet. When the carrier is below the nose frequency, this sequence is reversed.

IV. PULSE PROPAGATION IN THE MAGNETOSPHERE

In general, a wave propagating in an inhomogeneous dispersive medium changes both in its amplitude and in its phase. We assume the magnetosphere at VLF to be a spatially, slowly varying, and lossless medium, so that the Wentzel-Kramer-Brillouin (WKB) approximation will apply. The amplitudes of the E and H fields then vary as $n^{-1/2}$ and $n^{1/2}$, respectively, so that the Poynting vector $|E \times H|$ remains constant, where n is the refractive index [5]. It is clear that the amplitudes of E and H depend only on the local refractive index n, not on the propagation path.

The refractive indices at various frequencies at a particular location of observation have been normalized locally so that the refractive index at the carrier frequency is unity in calculating differential amplitude changes. The absolute amplitudes are lost in the normalization process, but the ratios between the various components are preserved.

It will be shown later that the differential change in amplitudes of the frequency components in the magnetosphere has very little effect on the distortion of a pulse. The major contribution to the distortion comes from the nonlinear phase retardation in the frequency components.

The key to the solution is to calculate the phase delay time for each frequency component. The phase velocity of any frequency component at a location where the gyrofrequency and plasma frequency are specified is given by (1).

A diffusive equilibrium (DE) model [8] is used to describe the distribution of electrons in the magnetosphere in the simulation, with the equatorial density assumed to be 400 electrons/cm^3 while a dipole magnetic field is used to model the geomagnetic field. In this model the gyrofrequency at every point is given once the equatorial distance of a duct is known.

Using (1) the phase delay time at $z = z_1$ is given by

$$t(f, z_1) = \int_0^{z_1} \frac{dz}{v_p(f, z)} \tag{8}$$

The distorted pulse at the "monitoring station" at $z = z_1$ is obtained by modifying the amplitude and phase of each frequency component according to the refractive index and the phase delay at z_1 and then adding all the components together by taking the inverse Fourier transform via the FFT.

The sketch in Fig. 5 shows the locations of the six monitoring stations along a duct that rises to four earth radii ($4R_E$). Pulses are injected into the magnetosphere at location 1 at the rate of one pulse every 512 ms. The arc length from location 1 to location 6 is about 30 000 km.

The distortion of a pulse as it propagates from location 1 to location 6 is shown in Fig. 5. A 10-ms pulse at 5065 kHz is injected at the 10th ms at location 1. As it propagates along the duct its shape changes. The high frequency components appear to reach the first five monitoring stations prior to the main body of the wave packet, while the low frequency components lag behind. It should be pointed out that the pulse shown in locations 1–4 is the one injected at the 10th ms at location 1 and it arrives at locations 5 and 6 after the 512th ms. The pulse shown in locations 5 and 6 is actually the one injected into the magnetosphere at –502 ms from location 1.

It is interesting to see that the distortion is minimum at the equator (location 6) at this particular frequency and that the travel time from location 1 to the equator is about 942 ms. According to Park's result [8] the one-hop minimum group delay for a whistler propagating at $4R_E$ in the same model magnetosphere occurs at about 5063 Hz (nose frequency) and is approximately 1.88 s. Traveling from one end of the duct to the equator requires half of the one-hop time, assuming that the duct is symmetrical about the equator. There is very little discrepancy between our result (943 ms) and Park's (940 ms). In his study group velocities were employed to calculate the group delays for whistlers.

It should be pointed out that before the pulse arrives at location 5, the high frequency components arrive at the monitoring stations first, and the low frequency components lag behind. In the region between locations 5 and 6 the low frequency components speed up and the high frequency components slow down. The result is that the pulse shows minimun distortion at the equator. Thus it is not generally true that the longer the pulse travels the more the distortion.

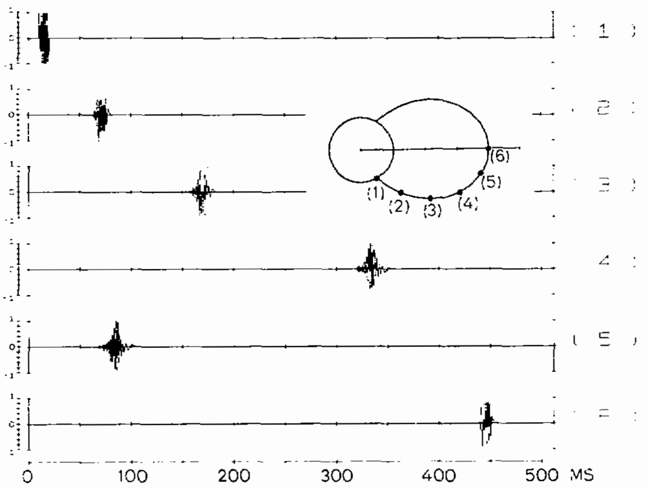

Fig. 5. Propagation of 10-ms pulses at 5.065 kHz through a duct at $4R_E$ in a DE-1 model magnetosphere. Pulses are monitored at six stations from 1000 km above earth to the equator.

Pulses injected at other carrier frequencies f_c show different distortions. Examples of 10-ms pulses at 7.075 and 3.065 are illustrated in Figs. 6 and 7, respectively. It can be seen that the delay time from location 1 to the equator is indeed a minimum for the pulse at 5.065 kHz.

When f_c is greater than the nose frequency (f_{ns}), the low frequency components arrive at the equator prior to the main wave packet and the high frequency components lag behind. There is always a location at which the distortion is minimum, as illustrated by the waveform at location 5 in Fig. 6. The arrival sequence of high and low frequency components at location 4 is the reverse of that of location 6.

When $f_c < f_{ns}$ the high frequency components arrive at the equator first and the low frequency components last. The greater the difference between the carrier frequency and f_{ns} the greater the distortion.

V. DISCUSSION

In addition to the "slowly varying" assumption employed in the simulation of pulses propagating along a duct in the magnetosphere, we have assumed that all frequency componets propagate strictly parallel to a geomagnetic field line. Exact longitudinal propagation seems unlikely to occur. In fact, various frequency components follow different snakelike paths inside a duct. However, in light of the theory of whistler propagation in ducts [1] this assumption appears to be reasonable at wave frequencies below half the local gyrofrequency. On the other hand, exact ray tracing for all frequency components inside a duct can, in principle, be included in the simulation if necessary.

A duct can trap WM waves just as an optical fiber can confine optical beams. The cross-sectional area of a duct is not uniform and is assumed to vary inversely with the local geomagnetic flux density. Therefore, the wave intensity should reach a minimum on the equator since the largest "illumination area" occurs there. Since there is no differential amplitude change among the frequency components in a

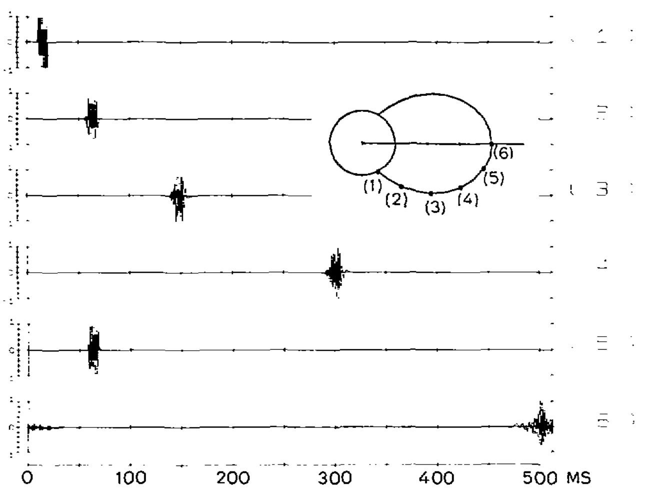

Fig. 6. Propagation of 10-ms pulses at 7.065 kHz through a duct at $4R_E$ in model magnetosphere.

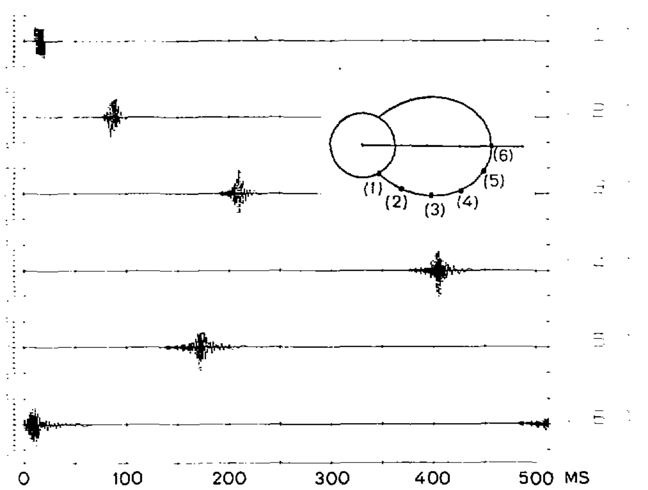

Fig. 7. Propagation of 10-ms pulses at 3.065 kHz through a duct at $4R_E$ in model magnetosphere.

particular location due to the change in the cross section of a duct, it does not affect the pulse shape. Hence, it is not included in the simulation.

As noted in Section IV the amplitudes of E and H are controlled entirely by the local parameters of the medium. As we mentioned before, normalization has been performed so that $n(f_c)$ equals unity in calculating the wave amplitudes at a particular location. The absolute wave amplitudes are lost in the normalization process. A true amplitude can be obtained by scaling the pulse amplitudes up or down by a factor of $\sqrt{n(f_c)}$ while the shapes remain the same.

The contribution to the pulse distortion of the differential amplitude changes in the WKB approximation is small in comparison with that from the cumulative phase retardations. A 10-ms pulse at 3.065 Hz propagating along a duct at $4R_E$ has been simulated for two cases: one with and one without the differential amplitude changes. The results (not shown) show that the difference is negligible. This is because most energy (about 90 percent) of a 10-ms pulse is contained between $f_c \pm$ 100 Hz. Even at the equator, where the maximum dispersion occurs, the amplitude corrections introduced by the WKB approximation within this 200-Hz bandwidth are always less than 0.5 percent. In general, if the pulse is not too short (say, >5 ms) and the carrier frequency is far from the local gyrofrequency and greater than, say 500 Hz, the amplitude corrections can be neglected.

Knowing how a pulse is distorted by dispersion in the magnetosphere is essential in interpreting the observed data of many VLF wave experiments. An important feature of a distorted pulse is that it has been stretched in the front in such a way that it exhibits "exponential growth" at the receiving site. For example, in one wave-injection experiment [12] phase alternating signals at 6.6 kHz were injected into the magnetosphere to investigate the effect on wave-particle interactions (WPI) of the phase reversal. The data show a large decrease in signal intensity whenever the phase of the triggering wave was reversed. The duration of this decrease was roughly 55 ms (measured from [12, fig. 3]), more than ten times longer than the antenna current response time. Koons *et al.* [12] concluded that the recovery in amplitude was due to temporal amplification of the signal in the magnetosphere. The apparent "growth rate" was about 260 dB/s, which was significantly higher than those commonly observed [7]. The growth time (~33 ms) was much shorter than that obtained from other experiments. Furthermore, there was no other evidence of amplification or triggering of emissions at that period. These facts suggest that the apparent "growth" might actually be caused by dispersion in the cold background plasma.

A phase-reversing signal consists of several sections of RF pulses with the same carrier frequency. Any two successive pulses are 180° out of phase. As the phase-reversing signal propagates in the magnetosphere the front and rear ends of each section will be distorted and stretched. Therefore interference between the tail end of one pulse and the front end of the next can be expected, resulting in amplitude fading.

We have simulated a phase-reversing signal at 6.6 kHz traveling through a duct at $3.1R_E$, having an equatorial plasma density of 1000 electrons/cm^3. These parameters are chosen to fit the nose frequency and minimum delay time reported by Koons *et al.* [11]. The result is shown in Fig. 8. The phase is reversed at $t = 0$ on a 100 ms pulse injected at -50 ms at location 1. As the pulse propagates along the duct a "gap" is developed near the time of phase alternation. When it reaches the equator the gap is about 10 ms wide. We expect that the gap would be about 20 ms wide when it reaches the conjugate point of location 1. By considering the finite antenna current response time (~3.3 ms) and the finite time constant associated with the analyzers, we suggest that the gap could appear to be 30 to 40 ms long, consistent with the observations [12]. Under these circumstances we conclude that propagation distortion is an acceptable alternative explanation of the apparent growth effect observed in the Koons *et al.* experiment.

Another important result from these studies is that the stretching of a pulse is large enough by the time a VLF pulse arrives at the interaction region to significantly alter the details of the nonlinear WPI processes. For example, a pulse at 4 kHz propagating from an altitude of 1000 km up to the equator along a duct at $4R_E$ may be stretched about 30 ms in the front. A scheme for compensating for this dispersion effect is shown in Fig. 9. A phase equalizer $h_e(t)$ is a device

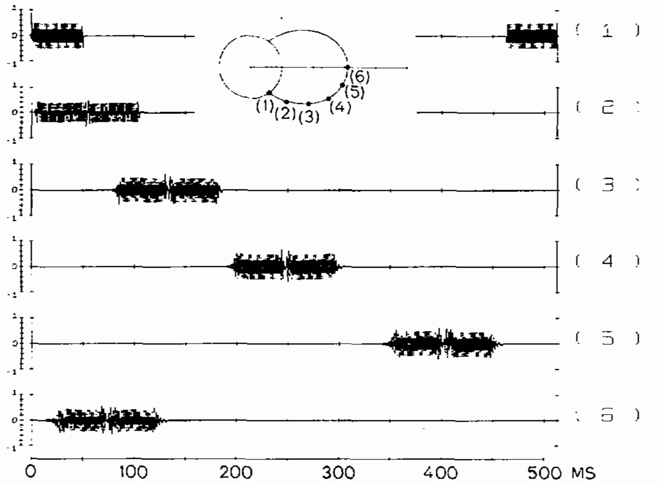

Fig. 8. Propagation through a duct at $3.1R_E$ of a 100-ms pulse at 6.6 kHz with a phase discontinuity at $t = 0$. Stairstep jumps in amplitude plots are due to the finite resolutions of the Calcomp plotter. It is noted that the wave amplitude at location 2 appears to be smaller. This is because of the lack of sufficient samples (we have four samples in a cycle) to produce a full range of oscillations in these amplitude plots. However, the main features of the waveforms at these monitor stations can still be seen clearly.

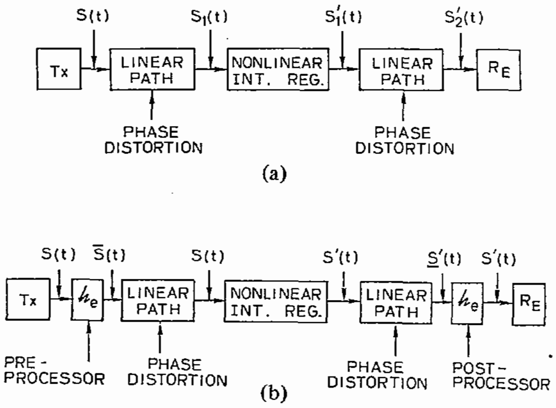

Fig. 9. (a) Block diagram representing configuration of VLF wave injection experimentation between Siple and Roberval. (b) Block diagram illustrating how equalizers are fitted into experimentation. Preprocessor compensates for distortions developed in linear path between transmitter and interaction region. Postprocessor equalizes phase distortion introduced by dispersive path between equatorial interaction region and receiving site.

that compensates for propagation distortion so that the phase change (in the frequency domain) will become linearly proportional to f. Linearization of the phase variation restores the signal shape in the time domain. Equalization can be performed either at the transmitter, at the receiver, or both. We have designed such a device using the tapped delay line technique. Computer stimulation shows that such a device can completely compensate for the distortion.

The experiment of phase alternations can conceivably be carried out in the future by "preprocessing" the injected signal so that when a phase reversing signal reaches the interaction region the phase of the signal will appear to have been reversed instantaneously.

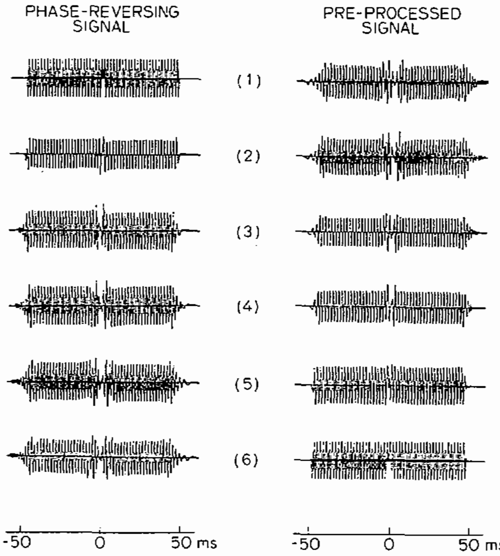

Fig. 10. Illustration of how a preprocessed phase-reversing signal becomes desired waveform when it arrives at the equator. On left are same waveforms shown in the six monitor stations in Fig. 8. Group delays are subtracted from time scale so that waveforms at the six stations are aligned with one another. A gap is developed near the time of phase reversal when the signal arrives at the equator (location 6). On right are preprocessed phase reversal signals. When the signal arrives at the equator it becomes the prescribed waveform with an instant phase reversal at $t = 0$.

Fig. 10 illustrates a phase-reversing signal and a preprocessed phase-reversing signal propagating along a duct at $3.1R_E$. On the left are the same waveforms shown at the six monitor stations in Fig. 8. We have removed the time delay for the purpose of illustrating the development of the "gap." On the right are the waveforms of a preprocessed signal at 6.6 kHz at the six monitor stations along the duct at $3.1R_E$. The preprocessed signal is injected at location 1. As it arrives at the equator (location 6) it becomes the desired waveform, with an instant phase-reversal at $t = 0$.

An equalizer can also be implemented for use at the receiver to compensate for the distortion between the interaction region and the receiving site.

VI. CONCLUSION

Among the results of the study of pulse propagation in the magnetosphere are the following.

1) By properly choosing the carrier frequency of injected pulses the propagation effects can be minimized. The optimum frequency is the "nose frequency" (as measured at the equator).
2) In designing wave-injection experiments for the study of WPI processes in the future, such as the phase-reversal experiments, we should include pre- and postprocessing filters at both the transmitter and receiver to remove the distortion due to dispersion.

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