# A Comparison of ULF and VLF Measurements of Magnetospheric Cold Plasma Densities

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Equatorial cold plasma density profiles determined from VLF whistlers propagating in magnetospheric ducts are compared with densities computed from the observations of ULF hydromagnetic waves (geomagnetic pulsations). The densities obtained by the ULF technique are based on the identification of resonant geomagnetic field lines, assumed to be driven by a monochromatic source. The ULF results thus obtained agree well with the whistler results throughout the period, June 17-20, 1973, when simultaneous data comparisons could be made. The ULF observations show that at mid-latitudes, shorter-period resonances can be supported in the plasma trough region, while those of longer period can occur inside the plasmasphere.

## INTRODUCTION

In this paper we compare the results of ground-based VLF and ULF techniques for estimating the equatorial cold plasma densities in the magnetosphere. The VLF technique is based on dispersion characteristics of whistlers, which are well understood and well documented in the literature [e.g., *Helliwell*, 1965; *Carpenter and Smith*, 1964; *Angerami*, 1966; *Park*, 1972].

The possibility of probing the magnetosphere with ULF magnetohydrodynamic waves has been significantly advanced by several recent experimental and theoretical results (see the reviews by *Orr* [1975] and *Lanzerotti* [1976]). Few results, however, have yet been reported in which wave characteristics are used simultaneously with measured magnetospheric plasma properties in order to 'calibrate' the ULF technique. *Lanzerotti et al.* [1974*a*, 1975] analyzed a storm time Pc 5 event and estimated a cold plasma density which agreed quite well with a cold plasma density inferred from the pitch angle distributions of hot proton fluxes. *Orr and Matthew* [1971] computed the eigenperiods of the fundamental mode of transverse oscillations of field lines in a range of mid-latitude $L$ values. Using the electron density values deduced by *Angerami and Carpenter* [1966] from whistler measurements and assuming charge neutrality, their predictions of wave resonances supported on field lines in the region of the plasmapause show very good agreement with the (statistical) experimental observations made in the European sector. For the stations that exhibited two peaks in their statistical data the longer-period pulsations were interpreted as arising from standing waves within the plasmasphere. Shorter-period pulsations seen most clearly at higher latitudes were associated with plasma trough resonances [see *Orr*, 1973].

Extension of the statistical work by *Orr and Webb* [1975] supported these suggestions by showing associations between geomagnetic activity (as measured by the $Kp$ index), the statistical plasmapause position, and the position of the maximum occurrence of ~20- to 100-mHz (Pc 3) pulsations. Some individual results presented by *Webb and Orr* [1975] also show agreement with these statistical results.

Recent work at Bell Laboratories has been concerned with relationships of the characteristics of ULF waves to plasma density gradients (see the review by *Lanzerotti et al.* [1976]). The earlier work of *Fukunishi and Lanzerotti* [1974*a*, *b*], using 1 day of data, showed that local time and latitudinal changes occurred in the wave amplitudes and polarization characteristics. *Maclennan et al.* [1974], using spectral analysis techniques on data obtained for 5 days during a magnetic storm, demonstrated a good association between the location of the peak amplitudes in the 10- to 15-mHz frequency band and simultaneous satellite determinations of the location of the plasmapause. However, relationships between ULF wave characteristics and latitude profiles of topside electron density observed by the Isis 2 satellite near the same local and universal times [*Lanzerotti and Fukunishi*, 1975] suggested that the density gradients needed for the coupling of the external driving force to a local resonant field line may also occur away from the plasmapause position as measured by Explorer 45. Thus it would appear that the equatorial plasma density could be determined by the ULF technique at different $L$ values, both inside and outside the plasmapause.

ULF pulsation data for this study were obtained from the Bell Laboratories flux-gate magnetometer stations, shown in Figure 1. Cold plasma densities inferred from these data are compared with nearly simultaneous density measurements calculated from whistlers recorded at Siple, Antarctica. Siple is approximately conjugate to Lac Rebours, as indicated by the open circles in Figure 1. The period of observation and analysis is June 17-20, 1973, an interval when the dynamic behavior of the plasmapause as deduced from whistler measurements could be well documented [*Park and Seely*, 1976]. In the following sections the ULF and VLF techniques will be briefly discussed; then a comparison of the results obtained by the two techniques and a discussion will follow.

## COLD PLASMA DENSITY DETERMINATIONS

*ULF.* Theoretical considerations regarding ULF waves have recently been reviewed by *Lanzerotti* [1976]. Much work in the past considered possible standing resonances of the earth's magnetic field lines and/or resonances of the geomagnetic cavity as an entirety [e.g., *MacDonald*, 1961; *Radoski and McClay*, 1967; *Radoski*, 1967*a*; *Carovillano and Radoski*,

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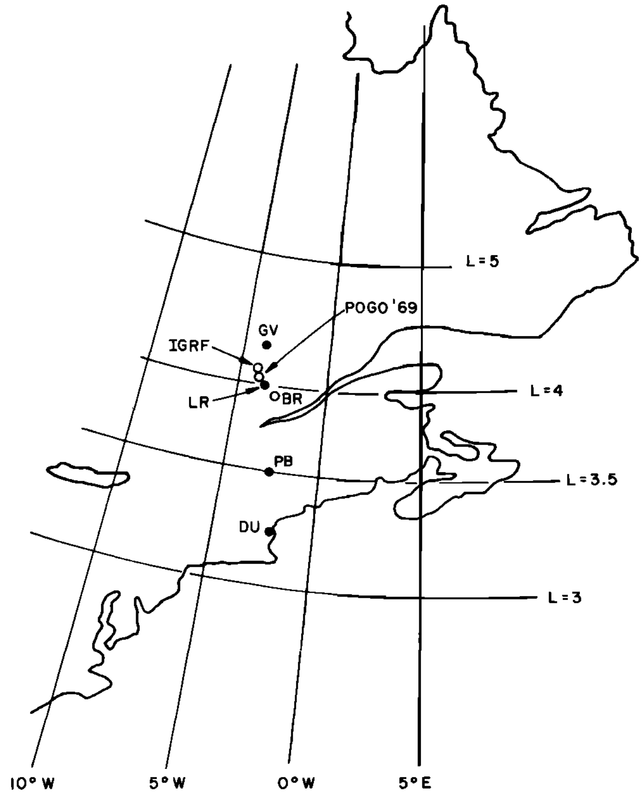


Fig. 1. Location in geomagnetic coordinates of the northern hemisphere stations used in the ULF study (GV, Girardville; LR, Lac Rebours; DU, Durham). The locations of the Siple (SI) conjugate point (open circles) are also shown, according to three field model calculations: BR (F. D. Barrish and J. R. Roederer, private communications, 1971, 1972) and IGRF and Pogo '69 (model calculations by *Stassinopoulis and Mead* [1972]).

1967*a*, *b*; *McClay*, 1970]. The basic resonance modes correspond to the shear Alfvén (or anisotropic) wave

$$1 - k_z^2 v_A^2/\omega^2 = 0 \tag{1}$$

and the magnetosonic (or isotropic) wave

$$1 - k^2 v_A^2/\omega^2 = 0 \tag{2}$$

as obtained from the linearized cold plasma magnetohydrodynamic equations [e.g., *Stix*, 1962]. In (1) and (2), $k$ is the wave number, $\omega$ is the wave angular frequency, and $v_A$ is

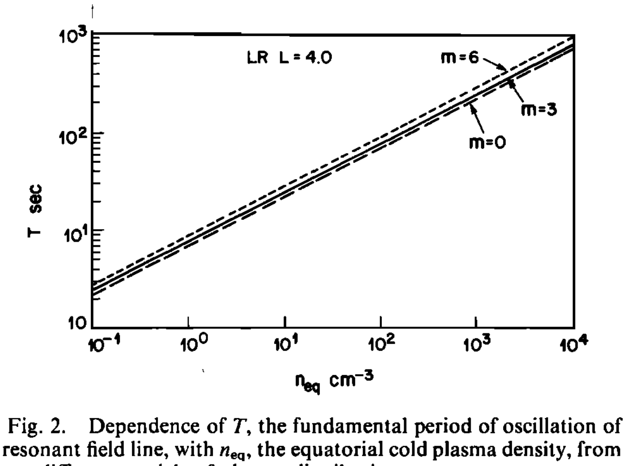


Fig. 2. Dependence of $T$, the fundamental period of oscillation of a resonant field line, with $n_{eq}$, the equatorial cold plasma density, from three different models of plasma distribution.

the Alfvén (group) velocity

$$v_A = B/(4\pi n)^{1/2} \tag{3}$$

where $B$ is the magnetic field intensity and $n$ is the cold plasma density.

The plasma and magnetic field nonuniformities in the magnetosphere must produce mode coupling. Such coupling makes a treatment of the theoretical problem intractable. However, recent observations demonstrating significant wave localizations in latitude ($L$ value) of individual magnetic pulsation events [e.g., *Kiselev et al.*, 1969; *Samson et al.*, 1971; *Gudkova et al.*, 1971; *Orr*, 1973; *Fukunishi and Lanzerotti*, 1974*a*, *b*; *Lanzerotti et al.*, 1976; *Dmitrieva et al.*, 1975] gave impetus to new theoretical treatments of the coupled equations and the production of hydromagnetic resonances in the magnetosphere [*Southwood*, 1974; *Hasegawa and Chen*, 1974; *Radoski*, 1974].

These considerations invoke a driving force composed of nearly monochromatic waves. The theoretical solutions predict the excitation (by a driving wave of frequency $\omega$) of a field line whose resonant frequency $\omega$ corresponds to the shear Alfvén wave,

$$\omega^{-1} = \frac{1}{\pi}\int \frac{ds}{v_A(s)} \tag{4}$$

where $v_A(s)$ is the Alfvén wave velocity along the resonant field line. Both $B$ and $n$ in (3) vary with distance $s$. At resonance the wave is linearly polarized; away from the resonant field line the solution exhibits surface waves (the evanescent compressional mode) with elliptical polarizations. The polarizations are opposite on either side of the resonance.

Given an observed resonant field line and frequency, the cold plasma distribution along the field line can be determined from (4). In practice, assumptions have been made as to this plasma distribution even before the observations indicated the validity of the resonant field line assumption [e.g., *Kitamura and Jacobs*, 1968; *Cummings et al.*, 1969; *Namgaladze and Brunelli*, 1970; *Troitskaya and Gul'elmi*, 1970; *Orr and Matthew*, 1971]. The hydrogen plasma distribution model assumed in most works takes the form

$$n = n_{eq}(r_{eq}/r)^m \tag{5}$$

where $r_{eq}$ is the geocentric distance to the equatorial crossing point of the field line and $n_{eq}$ is the proton number density at $r_{eq}$. Figure 2 shows values of the fundamental field line resonance period $T = 2\pi/\omega$ as a function of $n_{eq}$ for three model values of $m$ at an $L$ value of 4.0 (corresponding to the position of the station at Lac Rebours). A simple dipole field model was used because all of the magnetic observations were made at $L < 4.4$. In addition, for the data time period under discussion the magnetic station spacing was not as small as would be desired. Thus the largest uncertainties in the following determinations arise from the estimation of the latitude of the resonance field lines based on wave polarization characteristics.

In the analyses the wave resonance position is taken to be the latitude where the ULF wave polarization reverses rather than the location of the wave amplitude maximum. This is because it is expected that the ionosphere screening and ground effects can produce an amplitude versus latitude distribution different from that existent in the magnetosphere [e.g., *Hughes and Southwood*, 1976].

*VLF.* Lightning-generated whistlers propagate through the magnetosphere along discrete field-aligned paths known as 'whistler ducts.' A typical duct may measure several hundred kilometers in diameter at the equator and have density enhancements of the order of 10% or more above the background level [*Smith and Angerami,* 1968]. There may be as many as 30 or more ducts that are illuminated by the same lightning sferic and that are within the 'viewing' range of a single whistler receiver. Thus a well-placed receiver can monitor a large volume of the magnetosphere between $L \cong 2$ and 6 and $\sim\pm20°$ longitude.

The theory of whistler propagation is well understood [e.g., *Helliwell,* 1965], and the observed frequency-time characteristics can be used to deduce the path latitude and electron concentrations along the path in a straightforward way [e.g., *Park,* 1972]. The travel time $t$ is given by

$$t(f) = \int \frac{ds}{v_g} = \frac{1}{2cf^{1/2}} \int \frac{f_p}{f_H^{1/2}[1 - (f/f_H)]^{3/2}}\, ds \qquad (6)$$

where $f$ is the wave frequency, $v_g$ is the group velocity, $c$ is the speed of light in free space, $f_p$ is the plasma frequency, $f_H$ is the electron gyrofrequency, and $ds$ is an element of length along a field-aligned path. The integral is taken over the entire path above the lower edges of the ionosphere. If we adopt any reasonable field line distribution models of $f_p$ and $f_H$, the resulting $t(f)$ curve accurately describes the shape of nose whistlers. The nose frequency $f_n$ (the frequency of minimum travel time) is roughly proportional to the minimum $f_H$ along the path ($f_{Hm}$), the 'constant' of proportionality being somewhat dependent on the path latitude as well as the field line distribution models of $f_p$ and $f_H$. This $f_n$, together with the $f_H$ model, specifies the path latitude.

For a given path the travel time at the nose ($t_n$) provides a scale factor for the assumed field line distribution model of $f_p$. Therefore in principle, electron densities can be estimated everywhere along the path. However, for reasons discussed below, electron densities near the equator are much less model dependent than densities at low altitudes.

The equatorial density ($n_{eq}$) estimates obtained from the whistler observations were made by approximating the geomagnetic field by a centered dipole. A diffusive equilibrium model was used inside the plasmapause, and a collisionless model was used outside. The collisionless model is closely approximated by (5) with $m = 4$. *Angerami* [1966] found that outside the plasmapause, collisionless models are more appropriate than diffusive equilibrium models. However, since collisions must occur at some finite rate, collisionless models represent idealized situations that may be approached only if electron densities are extremely low. In most cases, including those presented here, the use of a collisionless model probably underestimates equatorial densities by factors of up to about 2.

In (6) the integral is heavily weighted near the equator because of the factor $f_H^{1/2}$ in the denominator. For $L \gtrsim 2$ it is found that about 80% of the whistler time delay comes from within about $\pm30°$ of the equator, where both $f_p$ and $f_H$ vary slowly with distance along field lines. Thus the equatorial parameters deduced from whistlers are remarkably insensitive to assumed field line distributions of $f_p$ and $f_H$. Inside the plasmapause, typical model-dependent errors in $f_{Hm}$ and $n_{eq}$ are estimated to be less than 1% and 10%, respectively. Outside the plasmapause the use of a collisionless model may underestimate $f_{Hm}$ and $n_{eq}$ by as much as 10% and 50%, respectively. A detailed analysis of the whistler technique, including model-dependent and other sources of error, is given by *Park* [1972].

## RESULTS

During the period of study the plasmapause position was determined from whistler observations [*Park and Seely,* 1976], and a number of latitudinal density profiles were available. Figure 3 shows the results of plasmapause position plotted against local time for the Siple/magnetometer chain meridian (LT = UT − 5 h). The triangles with the apex pointing upward

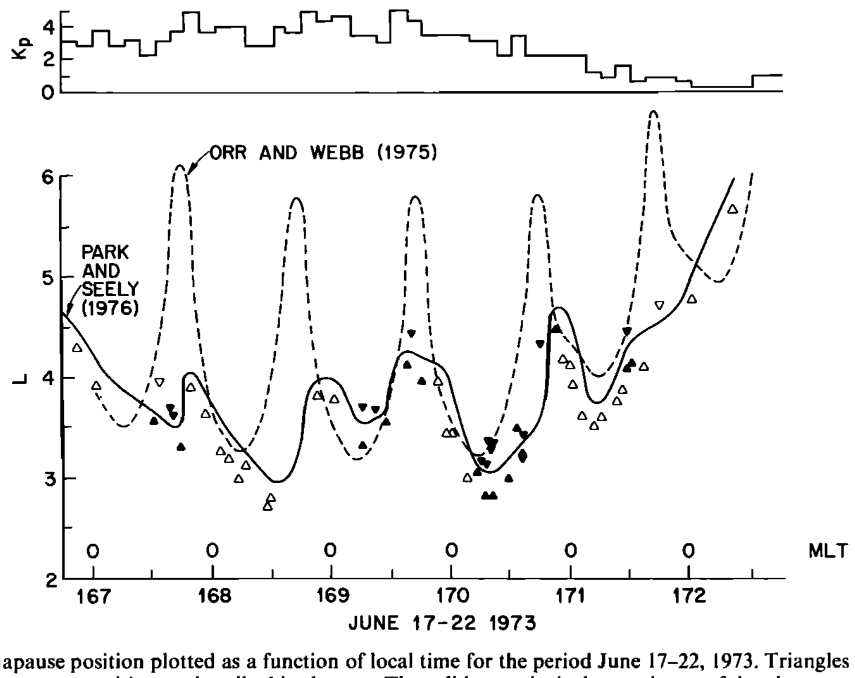


Fig. 3. Plasmapause position plotted as a function of local time for the period June 17–22, 1973. Triangles indicate the limits of the plasmapause position as described in the text. The solid curve is the best estimate of the plasmapause position from *Park and Seely* [1976], and the dashed curve represents the locations estimated by a statistical method [*Orr and Webb,* 1975].

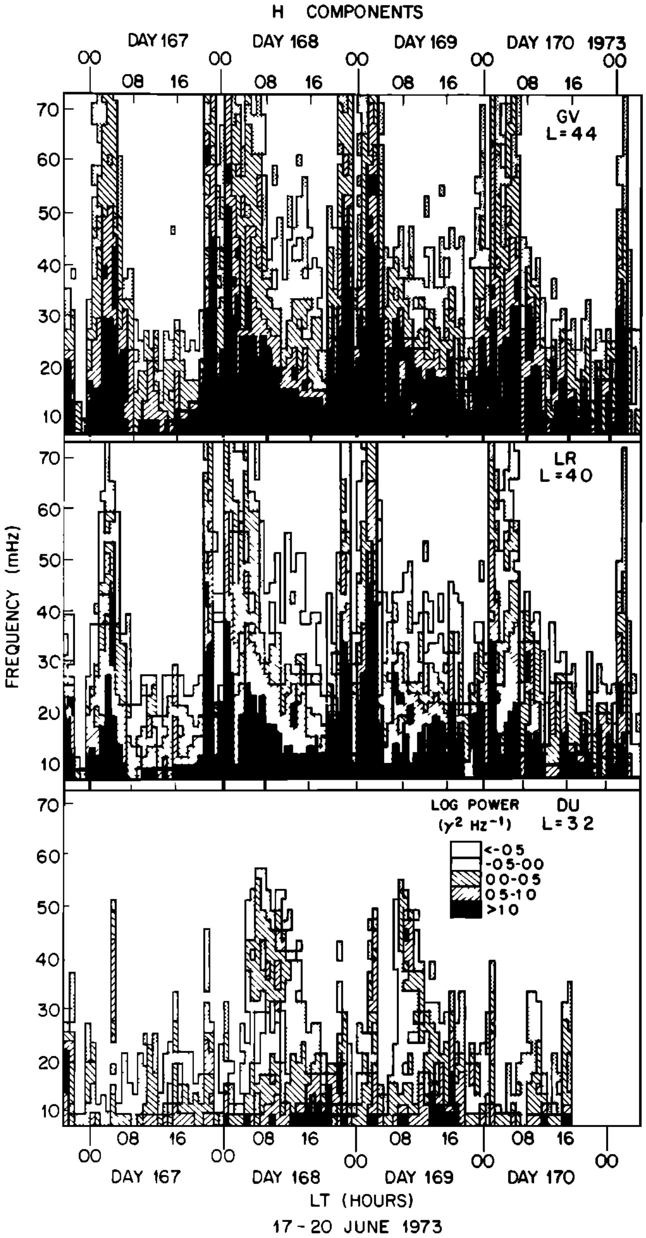


Fig. 4. Hourly average dynamic power spectra computed from the *H* components of the three magnetometer stations for the period June 17-20, 1973.

represent the inner limit of the plasmapause, while those with the apex pointing downward represent the outer limit. Solid triangles are used when whistlers were observed to propagate just inside and just outside the plasmapause, defining the plasmapause to within $\pm 0.2\ R_E$. For the open triangles, uncertainties in the plasmapause position may be $\sim 0.5\ R_E$. The solid curve represents the best estimate of the plasmapause position by *Park and Seely* [1976].

Numerous workers have in the past examined the relationship of various geophysical observations, including ULF waves, to the statistical location of the plasmapause. Probably the most thorough study of this nature [*Orr and Webb*, 1975] involved the relationship of the statistical plasmapause position determined from 150 satellite crossings [*Chappell et al.*, 1971] to the average nighttime *Kp* value (2100–0600 LT) and a best fit curve from data of *Kp* versus plasmapause position [see *Roth and Orr*, 1975]. This statistical plasmapause, as a function of local time, is plotted as the dashed curve in Figure 3. The principal differences between the statistical (dashed) and measured (solid) plasmapause positions occur in the region of the afternoon plasma (bulge) (~1800 LT). The discrepancies in this local time region are very large. These comparisons point out the difficulties and dangers that can be involved with the use of statistical plasmapause relationships in the search for physical understandings.

The ULF observations for the period June 17–20, 1973, are shown in Figures 4 and 5. In Figure 4, hourly average dynamic power spectra computed from the *H* components of the magnetic data recorded at the three stations are shown. It can be seen that the power level is enhanced at higher latitudes, especially during local night.

The ULF waves throughout the time period were selected as those events of sufficient amplitude exhibiting a quasi-monochromatic signal for at least five cycles. The period of each event was measured from the digital *H* and *D* component plots. These measurements determined the digital filter bandwidths used in making the hodogram polarization plots.

The polarization direction (right handed (+) or left handed (−) when viewed along the geomagnetic field line) for every ULF wave event observed at each of the three northern hemisphere stations throughout this period is plotted in Figure 5. Each of these waves exhibited a measurable polarization on hodograms plotted every 3 min throughout the 4-day period. For those events where a polarization reversal was observed an asterisk indicates the approximate expected latitude of the linear polarization (resonance position). The estimates of the plasmapause position from Figure 3 are shown here on an expanded scale. The plasma density computed from the wave period and *L* value of the resonance position (Figure 2) is plotted above each set of wave polarizations where reversals were seen. The error bars indicate the uncertainty in the exact position of the field line resonance (usually lying somewhere between two of the magnetometer stations). Comparison of the times of VLF data occurrences (triangles) and ULF data occurrences (asterisks) in Figure 5 shows that there are a number of occasions when plasma densities computed by the two techniques can be compared.

Plotted in Figure 6 are ULF wave ellipse hodograms for the three magnetometer stations for three time intervals 0500 UT on June 17, 0550 on June 19, and 0740 on June 19. In each case the data are filtered in the period ranges shown. The plasma densities computed from the ULF information are compared with whistler measurements in Figure 7.

Whistler measurements made during the time intervals 0510–0511 UT and 0425–0426 UT on June 19 are compared in Figure 7*a* with the ~50-s ULF wave event occurring at 0500 UT. The wave polarization ellipses as observed at the three magnetometer stations indicate a maximum in wave amplitude (in the horizontal plane) near, or at higher latitudes than, Girardville (GV; $L = 4.4$). However, the polarization reversal occurs at or near Lac Rebours (LR; $L = 4.0$), and the corresponding estimated density from the ULF waves is ~40 $cm^{-3}$. Whistler density data obtained in the time interval 0550–0551 UT are compared in Figure 7*b* with the density derived from the ULF wave event shown in Figure 6*b* at 0550 UT. This ~55-s-period wave with a maximum ellipse magnitude at or near Durham (DU; $L = 3.2$) changes polarization between LR ($L = 4.0$) and GV ($L = 4.4$); the derived density is 30 $cm^{-3}$. Figure 7*c* compares whistler density data of 0735–0736 UT and 0705–0706 UT with the ~30-s-period ULF wave event in

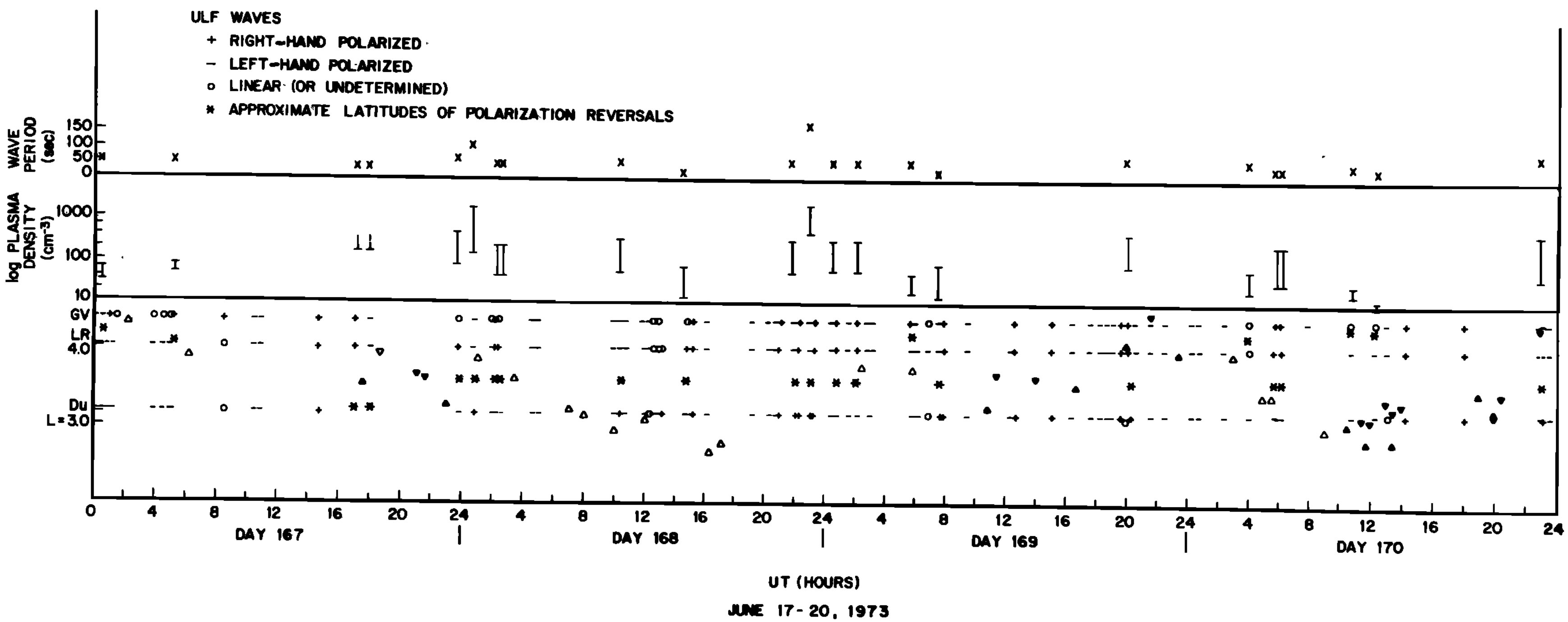


Fig. 5. Plot of ULF wave events in the time interval June 17–20, 1973. VLF estimates of the plasmapause position taken from Figure 2 are shown together with the ULF wave period and the polarization direction (right handed (+) or left handed (–) when viewed along the field line) at the three magnetometer stations. Estimates of the equatorial cold plasma density $n_{eq}$ are obtained from field line resonances marked by asterisks.

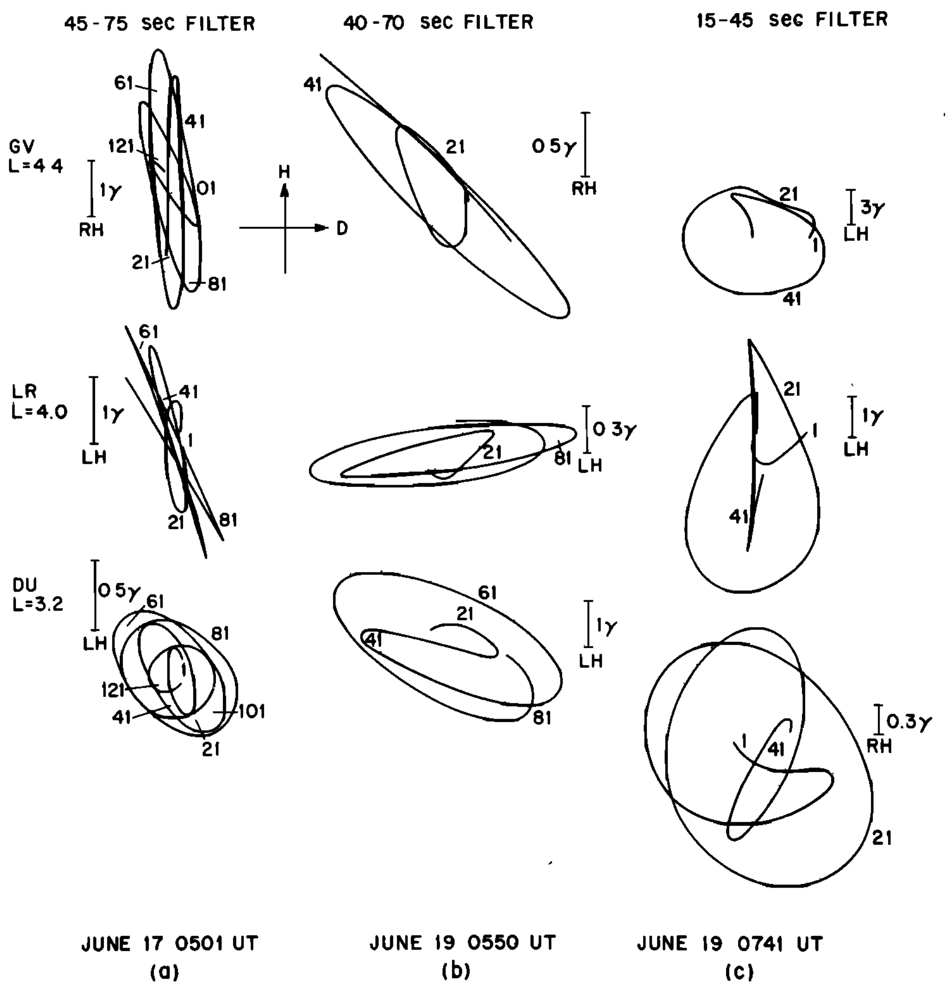


Fig. 6. Polarization hodograms in the *H-D* plane, obtained from the three magnetometer stations, showing reversal in polarization direction (LH, left-handed rotation; RH, right-handed rotation). The latitude where the polarization is linear is the position of field line resonance. In those cases where linear polarization is not observed an approximate latitude for the resonance position is assumed. The numbers on the hodograms represent simultaneous time values from the start of the event.

Figure 6*c* and also with that in Figure 6*b*, which occurs only 2 hours earlier in local time. The ULF-derived density is ~30–40 $cm^{-3}$.

In each of Figures 7*a–c* the estimated plasmapause position as obtained from Figure 2 has been represented by an arrow. It should be noted that the error bars on the ULF plasma density determination in both coordinates (i.e., in density and in *L* value) arise from the spacing of the magnetometer stations (or the inaccuracy in determining the exact resonance position) and could be minimized by closer station spacing (although, at

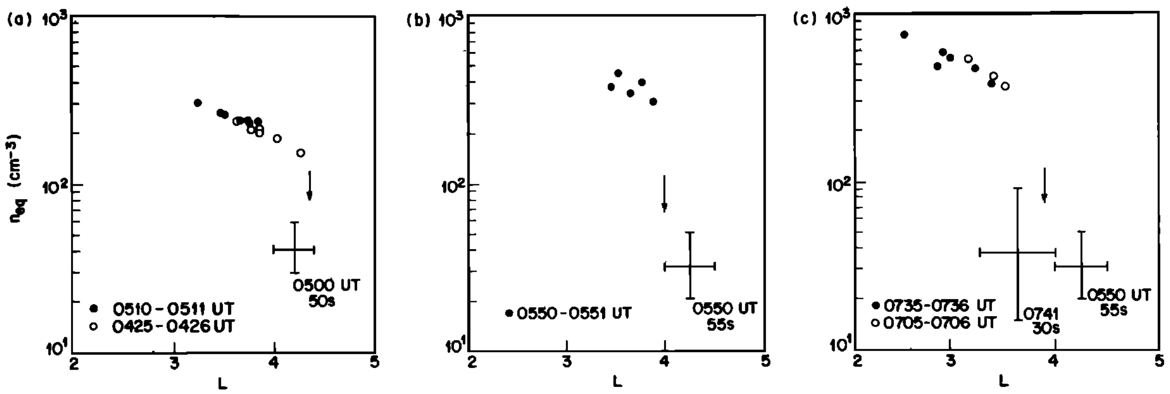


Fig. 7. Three comparisons of the equatorial cold plasma density $n_{eq}$ obtained from whistler measurements (circles) and from ULF waves (crosses). (*a*) June 17, 1973. (*b, c*) June 19, 1973. The arrows represent the position of the plasmapause taken from Figure 2 [*Park and Seely*, 1976]. The time observation of each ULF and VLF event is given along with the period (in seconds) of the ULF wave.

present, insufficient experimental data exist to determine optimum station spacing). In each of the comparisons shown in Figure 7 it appears that the ULF wave events occur close to or at the plasmapause position, the resonances corresponding to plasma densities typical of the plasma trough region. However, the positions of the field line resonance (polarization reversal) do not necessarily correspond to latitudes where the wave amplitude, in the horizontal plane, is a maximum.

Two ULF wave events, occurring at 2345 UT on June 17 and at 2003 UT on June 19, are shown in Figure 8, corresponding plots of plasma density profiles from VLF and ULF being given in Figure 9. The event of 2345 UT on June 17 shows a polarization reversal between Durham (right hand) and Lac Rebours (left hand), while the ellipse at Girardville is essentially undetermined (Figure 8*a*). The plasma density computed from the assumption of a 70-s-period field line resonance between $L = 3.2$ and $L = 4.0$ (Figure 9*a*) is in very good agreement with the whistler-produced density profile, the resonance occurring in the plasmasphere. Also plotted on the figure is another estimate of the plasma density at a similar $L$ value taken from a 120-s-period pulsation event at 0041 UT on June 18. If it is assumed that the plasma density conditions do not change significantly in this time period of about 1 hour, then Figure 9*a* shows that the plasma density estimate obtained from the overlap of the points plotted for 0041 and 2345 UT lies in the range ~250–500 $cm^{-3}$, which is in very good agreement with the values obtained from the VLF measurements.

The density determination from the event at 2003 UT on June 19 (Figure 8*b*) is compared with VLF whistler density data in Figure 9*b*; a good agreement is observed in the density estimate. For this time period the plasmapause position is estimated to be at $L \simeq 4.3$; this means that this resonance occurs inside the plasmasphere at an $L$ value of ~3.2–4.0.

Figure 10 illustrates the whistler profiles obtained during the period 1035–1221 UT on June 20. For these times, two ULF wave events were observed at 1046 UT and 1216 UT with periods of 50 and 35 s, respectively. The characteristics of these two events were left hand polarized at DU and LR and appeared to be almost linear at GV in both cases. A linearly polarized wave event at GV or at a somewhat higher latitude would set a limit on the ULF-determined plasma density as shown in Figures 10*a* and 10*b*. In each case the arrows indicate that the estimate obtained gives a maximum in plasma density at the minimum latitude of the resonance ($L = 4.4$). Values of plasma density obtained if the actual resonance position is at a higher latitude will be correspondingly smaller. The density values plotted in Figure 10, although only approximate upper limits of the plasma density, also show good agreement with the density values obtained for the plasma trough region by the VLF whistler technique.

## DISCUSSION

The above results from the analysis of ULF waves and VLF whistler waves observed at the ground near $L = 4$ have shown good agreement between equatorial cold plasma densities determined by the two techniques. Certain parts of the theory of ULF field line resonance phenomena still require experimental verification. In particular, some observations show that few or no amplitude enhancements accompany distinct polarization reversals (e.g., Figure 6); similar results have been shown by *Bjornsson et al.* [1971], *Lanzerotti and Fukunishi* [1975], *Fukunishi* [1975], and *Dmitrieva et al.* [1975] Most of these observations, however, concern irregular nighttime pulsations

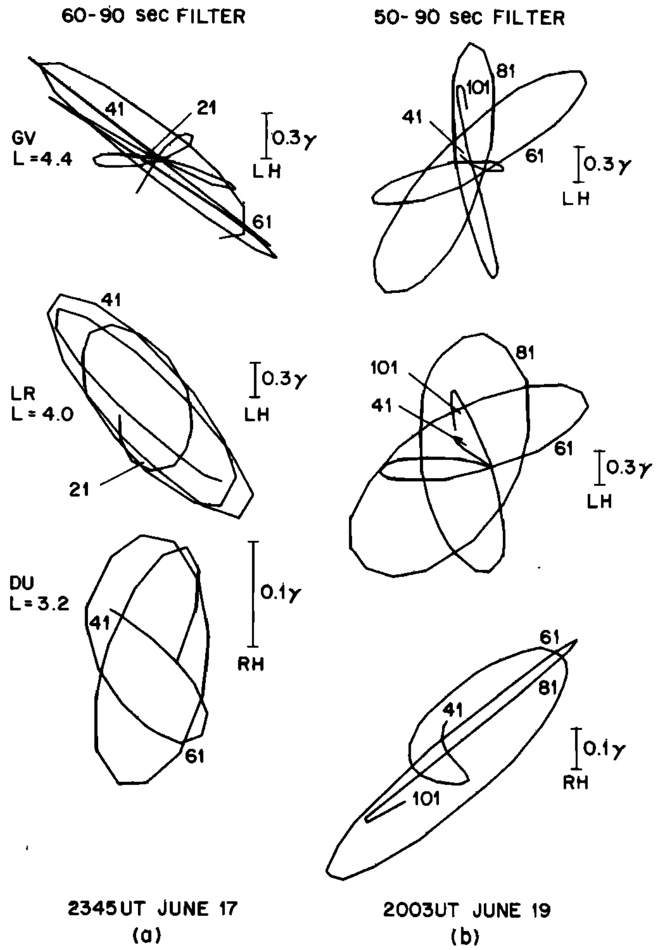


Fig. 8. Polarization hodograms for two events: 2345 UT on June 17 and 2003 UT on June 19, 1973. Details are the same as those in Figure 6.

which may be related to very different source characteristics than dayside waves, which have been examined in the context of the resonance theory by *Lanzerotti et al.* [1974*b*]. It is of interest to note in this connection that the polarization reversals on the nightside near $L = 4$ may be related to the plasmapause position [*Fukunishi*, 1975]. In the results presented here the reversals of wave polarization for the local nighttime events shown in Figure 6 occur close to the position of the plasmapause as estimated by the VLF whistler measurements.

Other important questions concerned with ULF waves and field line resonances include ionosphere/atmosphere/ground conductivity effects and azimuthal wave propagation. *Hughes* [1974] has used models of the ionosphere and ground conductivities and has predicted that the wave ellipse of a pulsation event will be rotated through 90° in propagating through the ionosphere/atmosphere to the ground [see *Hughes and Southwood*, 1976]. Recent comparisons of statistical results from the near-geomagnetic-equatorial synchronous satellite ATS 6 and ground measurements near $L = 4$ suggest that this may be the case [*Arthur et al.*, 1977].

The azimuthal wavelength is very important in understanding the coupling of solar wind energy into hydromagnetic waves in the magnetosphere [e.g., *Lanzerotti*, 1976]. The wave polarizations on the ground are determined by the ratio of the magnetic field components, $H/D$; in the northern hemisphere,

$$H/D = ik_x\xi_y(d\xi/dy)^{-1} \quad (7)$$

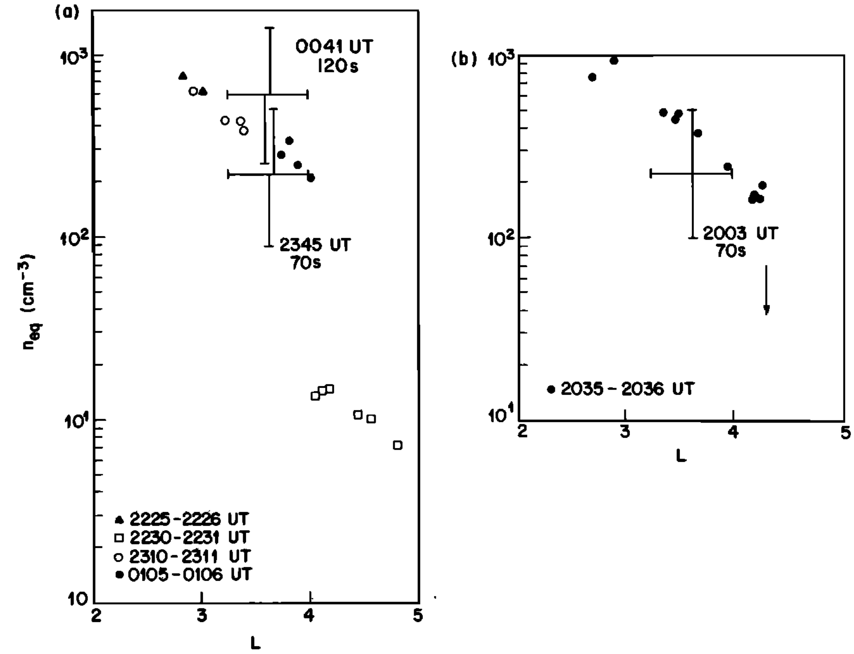


Fig. 9. Estimates of the equatorial cold plasma density $n_{eq}$ obtained from whistler and ULF waves. Details are the same as those in Figure 7. (*a*) June 17–18, 1973. (*b*) June 19, 1973.

where $\xi_y$ is the plasma displacement in the radially inward direction and $k_x$ is the wave number in the azimuthal direction. Obviously, the wave ellipse ratio $H/D$ depends directly upon the sign of $k_x$.

In a statistical study of the azimuthal propagation between three stations spaced over ~645 km in the United Kingdom, *Green* [1976] reported that most daytime pulsation events (~5–50 mHz) are not well localized in longitude. Somewhat surprisingly, *Green* [1976] did not find the expected reversal in propagation direction around local noon, a reversal expected from past statistical reports of polarization reversals around local noon and the theoretical considerations of a Kelvin-Helmholtz generation mechanism at the magnetopause for an evanescent source wave. Green did not report results of wave polarizations for the events that he used.

Previous discussions by *Lanzerotti and Fukunishi* [1975] point out that the orientation angle of the polarization ellipse may be significantly affected by structures in the $L$ shell profile distribution of cold plasma near the plasmapause (i.e., the differential term in (7)). Therefore even though ground effects at these stations may not be significant in determining the major axis orientations of the wave ellipses [*Anderson et al.*, 1976; *Anderson,* 1975], the wave polarization, together with a knowledge of the field line resonance location, is probably the best method of estimating the azimuthal wave propagation direction.

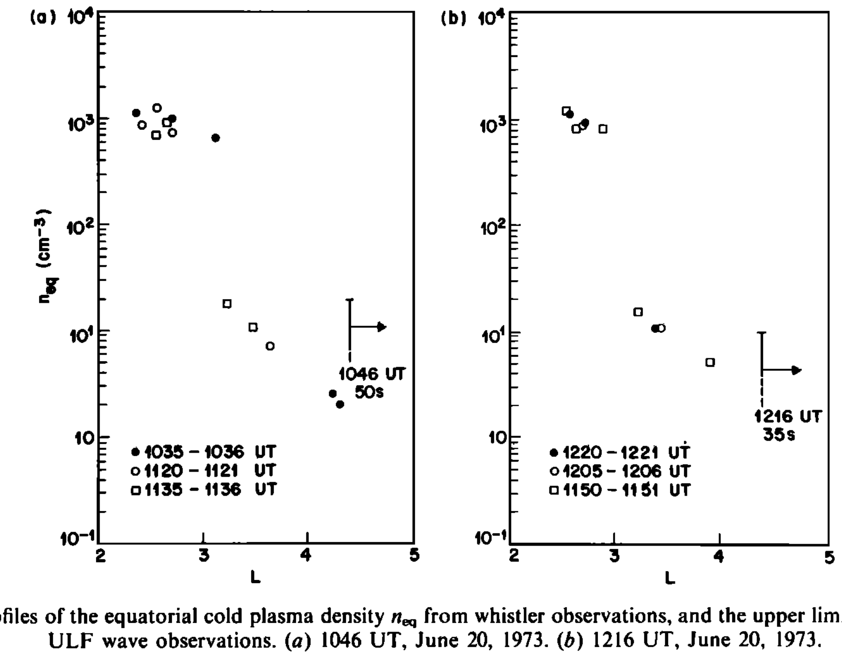


Fig. 10. Profiles of the equatorial cold plasma density $n_{eq}$ from whistler observations, and the upper limit obtained from ULF wave observations. (*a*) 1046 UT, June 20, 1973. (*b*) 1216 UT, June 20, 1973.

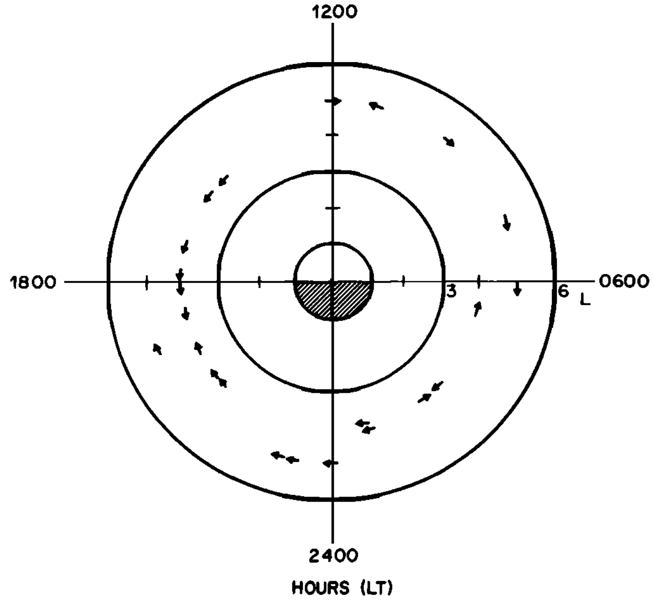


Fig. 11. ULF wave source propagation directions inferred from the wave polarization direction at latitudes higher than the resonance position (linear polarization). For convenience, waves with field line resonances in the range $3.2 < L < 4.0$ are plotted at $L = 4.0$, and those with resonances at $4.0 < L < 4.4$ are plotted at $L = 5.0$.

Figure 11 shows the source propagation directions inferred for each of the ULF wave events for which a field line resonance at $3.2 < L < 4.4$ was found (Figure 5). Since the exact location of the source is not known, for resonances in the range $3.2 < L < 4.0$ the direction arrows are plotted at $L = 4.0$, and for resonances at $4.0 < L < 4.4$ the direction arrows are plotted at $L = 5.0$. The inferred source propagation directions are in good agreement with previous results reported (inferred) for nighttime events [*Fukunishi,* 1975; *Lanzerotti and Fukunishi,* 1975] and for daytime events with a reversal at local noon [e.g., *Samson et al.,* 1971; *Lanzerotti et al.,* 1974*a*; *Dmitrieva et al.,* 1975], and, as was noted above, these inferred local day source propagation directions are not consistent with the results of *Green* [1976].

The use of VLF whistlers for deducing cold plasma profiles is based upon well-established theoretical considerations. However, determination of such profiles from whistlers depends upon the diurnal, geographic, and seasonal occurrences of terrestrial thunderstorm activity and the production of suitable ionospheric conditions and magnetospheric ducts. The occurrence of mid-latitude ULF magnetic wave activity depends upon the driving sources, which are nonterrestrial in origin. For dayside events the existence of ULF wave sources depends upon solar wind conditions; the interplanetary magnetic field appears to be a strong modulator (or controller) of ULF waves [*Troitskaya et al.,* 1974; *Webb and Orr,* 1976; *Greenstadt and Olson,* 1976; *Nourry,* 1976; *Gul'elmi,* 1974]. Thus unlike VLF activity, ULF activity does not have the strong seasonal (weather dependent) variations. In this respect the two techniques may prove to be quite complementary for use in remote sensing of the magnetosphere.

In summary, this first comparison of equatorial plasma density determinations by ULF and VLF techniques is very encouraging for the continued development of the ULF technique. As more experimental and theoretical investigations of the ULF polarization characteristics are carried out, it can be expected that more extensive cold plasma details, including more latitudinal profile details, will result in the future. Continued VLF/ULF comparison studies are certainly warranted.

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