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THE PLASMAPAUSE AS A VLF WAVE GUIDE

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Abstract. The properties of the plasmapause as a VLF wave guide are studied. The guidance that occurs is a form of gradient trapping of VLF wave energy. It is shown that guiding is possible at both the inner and outer edges of the plasmapause and that more efficient guiding occurs as the plasmapause gradients become stronger. In the case of strong gradients, waves coming from a wide latitude range (~8°) are focused tightly about the plasmapause field lines, resulting in a wave intensity increase of approximately 3 dB near the magnetic equatorial plane. It is shown that plasmapause-guided waves can be observed on the ground and can echo between hemispheres, precisely as can waves guided in normal whistler ducts. The single distinguishing feature of plasmapause-guided waves is a lowered upper cutoff frequency. The results indicate that the vicinity of the plasmapause represents a natural and readily accessible region of VLF wave guidance and focusing where both passive and active VLF experiments can be studied through ground and in situ satellite measurements.

Introduction

Guidance of whistler mode waves by discrete columns of enhanced ionization in the magnetosphere has long been known. These ionization columns, called ducts, guide whistler mode energy from one hemisphere to the other along the earth's magnetic field lines. The presence of such ducts has been established by numerous indirect ground measurements of natural and artificial magnetospheric whistler mode signals [Helliwell, 1965; Helliwell and Katsufrakis, 1974; Carpenter et al., 1969; Carpenter, 1968a, b, 1970] as well as in situ satellite observations [Angerami, 1970]. The ray theory for guidance of VLF whistler mode waves along linear field-aligned columns of enhanced or depressed ionization is well developed [Smith et al., 1960]. One important property of ducted signals that distinguishes them from the large variety of nonducted magnetospheric signals is the high probability of ground observation of these signals at middle and high latitudes. Since in the ducted mode the wave normals are very nearly aligned with the magnetic field, they generally fall within the transmission cone of the lower ionosphere boundary. They therefore can cross this boundary and can be observed on the ground. This property of ducted signals allows them to be used as efficient diagnostic tools for studying the magnetosphere through passive means. Recent active wave injection experiments in which VLF signals are transmitted from Siple Station, Antarctica, and are received at the conjugate point at Roberval, Canada, also make extensive use of ducted signals [Helliwell and Katsufrakis, 1974; Helliwell, 1974].

A second important feature of the ducted mode is echoing. Signals propagating along the field line are reflected from the ionosphere or the ground and bounce back and forth between the two hemispheres in the same duct [Helliwell, 1965]. This is different from the case of the magnetospherically reflected (MR) whistler [Edgar, 1972], where the reflection process is of the lower hybrid resonance type, occurring generally at altitudes much higher than the ionosphere. Also in the MR case, although the whistler energy bounces back and forth between hemispheres, it rarely is confined to the same discrete path [Edgar, 1972]. In the ducted mode, however, the wave normals at the lower ionospheric boundary are close to vertical, and hence the signal can be reflected back into the same duct [Helliwell, 1965].

In this paper we show how the plasmapause can act as a one-sided duct so as to guide whistler mode waves along the earth's magnetic field lines, in a manner similar to normal ducting. In some respects this guidance is analogous to the 'whispering gallery' mode of guidance discussed by Budden and Martin [1962], Booker [1962], and Walker [1966]. However, these authors pay little attention to how the whispering gallery mode can be excited from the ground, a factor which is of significant importance in our own work. Furthermore, since the modes we study generally propagate with a larger wave normal angle (with respect to the magnetic field) than the lowest-order modes studied by the above authors, they generally represent higher-order whispering gallery modes. Wave guidance in the magnetosphere by one-sided irregularities has also been considered by other authors. For instance, the guidance of HF and VLF electromagnetic waves by one-sided irregularities has been considered by Voge [1961, 1962] . In the ULF range the plasmapause has also been shown to be able to guide very long wavelength waves [Glangeaud and Lacoume, 1971]. The question of plasmapause guidance of VLF waves has also been addressed previously in the literature. For instance, Walter and Scarabucci [1974] claimed, but did not show, that VLF energy could be guided along the plasmapause boundary and give rise to the type of whistler known as the knee whistler [Carpenter, 1963]. In addition, Aikyo and Ondoh [1971] showed how the plasmapause gradients could act to guide VLF hiss along the plasmapause from the magnetic equatorial plane to the ionosphere. However, these brief treatments have not been sufficient to show the general conditions under which the plasmapause can efficiently guide VLF waves between hemispheres.

In the present paper we examine the ducting properties of the plasmapause in detail. We show that the vicinity of the plasmapause represents a natural and readily accessible region of VLF wave guidance and focusing where both passive and active VLF experiments can be studied through in situ satellite measurements in addition to ground observations. For ground-

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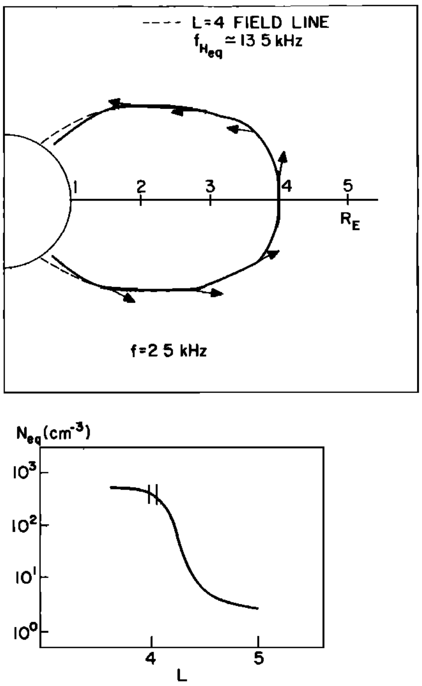

Fig. 1. Typical ray path for a ray guided by the inner edge of the plasmapause. The electron density profile used for ray tracing calculations is also shown. The region that the ray is confined to during most of its path is indicated by two vertical bars on the profile.

based or satellite-based wave injection experiments the plasmapause can be a particularly useful ducting surface, since the location is reasonably predictable [Carpenter and Park, 1973] in comparison with the highly random location of normal whistler ducts.

Our work was originally inspired by experimental data from the Imp 6 satellite that showed evidence of plasmapause guiding of whistler mode waves. The detailed analysis of these data is reported in other papers [Inan et al., 1977 a, b]. In brief, it was found that whistler echo trains were guided along the earth's magnetic field lines by cold plasma density gradients located at the inner edge of a field-aligned density depression just inside the plasmapause. The fact that the ray path could be ducted by density gradients at the inner edge of a density depression suggested that the plasmapause itself should also be able to provide such ducting, since a large plasma density gradient is the defining feature of the plasmapause.

As we shall demonstrate below, general ray tracing with a variety of plasmapause distributions shows that the plasmapause acts as a one-sided duct and guides VLF wave energy along the earth's magnetic field lines. Furthermore, we show that rays can be guided by both the inner edge and the outer edge of the plasmapause.

The ray paths were found by using the Stanford VLF ray tracing program. This program has its origins in the pioneer work of Yabroff [1961], Smith et al. [1960], and Kimura [1966]. The ray tracing program was written by Walter [1969] and modified by Burtis [1974]. Description of the program and the density models that are used in our calculations are given in the appendix. The program performs ray tracing in a two-dimensional meridional plane, using a centered dipole model for the earth's magnetic field. One of the main components of the input data to the ray tracing program is a field-aligned electron density profile. By adjusting the various input parameters of the program it is possible to input a field-aligned density profile which at the equator will give any desired profile. The ionosphere is also simulated by appropriate vertical density gradients. In our calculations we have assumed that there are no horizontal gradients other than the plasmapause, which for simplicity is assumed to extend down to ionospheric altitudes. The assumption that the sharp plasmapause gradients extend down to ionospheric heights is not generally borne out by experimental results. However, the assumption does not affect our results, since almost all of the plasmapause-guided rays deviate from the plasmapause field line at around 3000 to 4000-km altitude and reach the ionosphere at latitudes lower than the latitude of the plasmapause inner edge. It will also be seen in what follows that the rays we consider in this analysis are confined to a latitude range of at most 6°-7° centered around ±55° latitude in both hemispheres. Although the ionospheric electron density shows daily and seasonal variations, data over a 1-year period show that there are many times when the ionosphere density does not vary appreciably in such a small latitude range [Chan and Colin, 1969]. Therefore we think that our assumption of no low-altitude horizontal gradients (other than the plasmapause itself) is a reasonable one for our calculations.

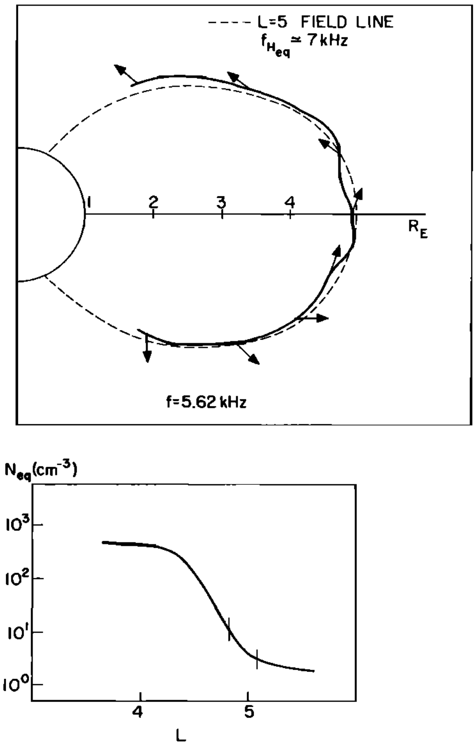

Fig. 2. Typical ray path for a ray guided by the outer edge of the plasmapause. Format is same as that for Figure 1.

In all our calculations except for those in connection with Figure 9, rays are injected at 100-km altitude with vertical wave normal angles.

General Properties of Plasmapause Guiding

Figure 1 shows a typical ray path for a ray guided by the inner edge of the plasmapause. The equatorial density profile that is used for the ray tracing calculations is also shown. The inner edge of the plasmapause in this case is located at about L = 4. The mode of guiding involved has been called 'gradient trapping' [Helliwell, 1965]. The large negative radial density gradients deflect the ray inward, but as the ray moves in and encounters markedly reduced gradients, the curvature of the earth's field deflects the ray outward. Upon entering the region of high gradient the ray will again be refracted inward, and the process will be repeated. The result is that the ray is trapped by the density gradient and its path oscillates about the direction of earth's field. The typical refractive index surface for the case where $f < f_H/2$ is shown in Figure 3a. The ray direction oscillates back and forth between positions 1 and 2 as indicated in the figure. One general characteristic of this mode of propagation is that rays starting with vertical wave normal angles at latitudes lower than the latitude of the plasmapause (in this case the plasmapause is located at L = 4) gradually converge to the plasmapause field line, becoming parallel to it at about 4000 to 8000-km altitude. As the rays descend to the same altitudes in the conjugate hemisphere, they diverge inward from the plasmapause, as shown. Apart from the deviations at low altitudes these rays stay close to the L = 4 field line with maximum deviations in L shell as small as 1-2%. The region to which the ray is confined during most of its path is also indicated with two vertical bars on the density profile. This type of propagation can be shown to be possible for a variety of plasmapause density distributions having different density gradients and inside-outside density ratios.

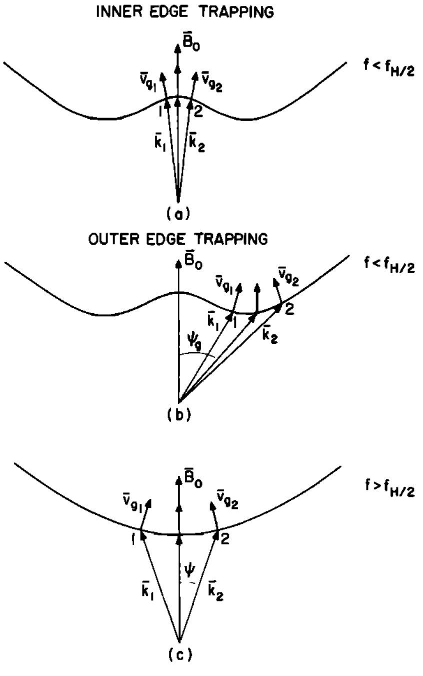

Fig. 3. Refractive index surfaces for ray guidance at both (a) the inner edge and (b,c) the outer edge of the plasmapause. Wave normal direction is indicated by the vector $\bar{k}$, while the associated ray direction is indicated by the vector $\bar{V}_g$. While trapping takes place, the wave normal direction oscillates between positions 1 and 2.

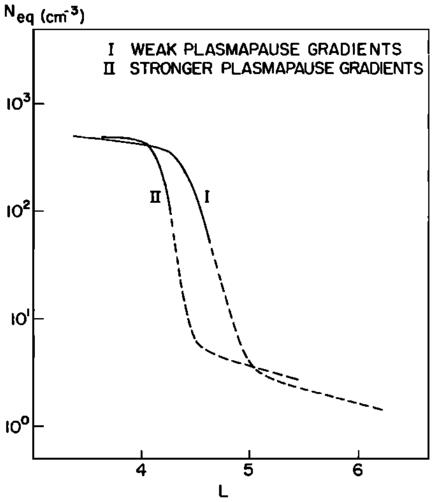

Fig. 4. Electron density profiles used for ray tracing calculations for guiding by the inner edge of the plasmapause.

Whistler mode waves can also be guided in the outer edge of the plasmapause. Figure 2 shows such a ray path for a 5.62-kHz signal, together with the corresponding density profile. For this mode of propagation the wave normal angle (the angle between the wave normal and the geomagnetic field) ψ must be large at low altitudes where the wave frequency f is less than $f_H/2$, where f_H is the local gyrofrequency. To be precise $\psi \simeq \psi_g = \cos^{-1}[2f/f_H]$, where ψ_g is the Gendrin angle [Gendrin, 1960, 1961] . At higher altitudes the wave frequency becomes larger than $f_H/2$, and guiding is possible only for small wave normal angles, $\psi \simeq 0$. The whistler mode refractive indices for the cases of $f < f_H/2$ and $f > f_H/2$ are shown in Figures 3b and 3c. At high altitudes, where $f > f_H/2$, the ray direction and the wave normal direction oscillate around the static magnetic field direction (points 1 and 2 in Figure 3c). At low latitudes, however, where $f < f_H/2$, the wave normal stays on one side of the static magnetic field direction, and the wave normal angle oscillates around the Gendrin angle ψ_g. As in the case of inner edge trapping, in this mode of propagation the curvature of the earth's field and the one-sided density gradient work together to keep the ray path aligned with the earth's field. The guiding mechanism here is a form of gradient trapping which is analogous to the 'trough' trapping discussed by Smith et al. [1960]. It can be seen that as the ray reaches the conjugate hemisphere, it deviates outward. This occurs because the curvature of the earth's field is not strong enough to deflect the ray inward again. Unlike the inner edge trapping case,

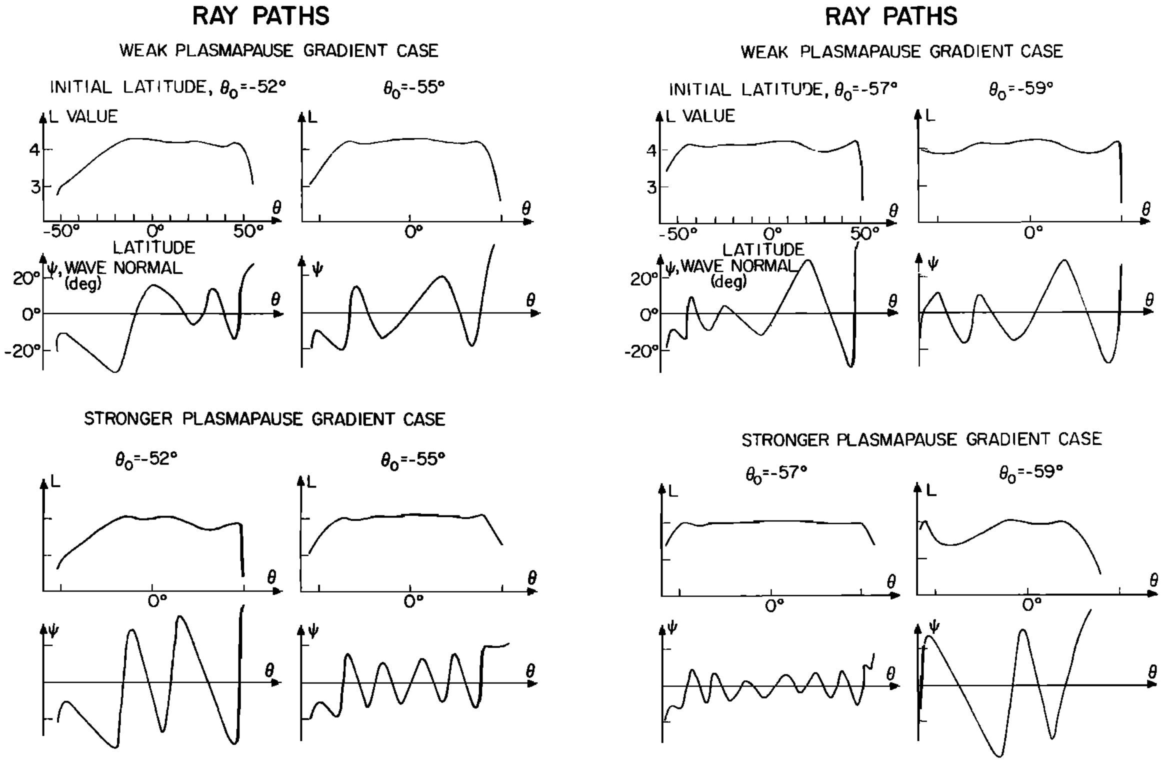

Fig. 5. Ray path behavior for different initial latitudes for both the strong and weak gradient cases. For each ray the L value and wave normal is plotted versus latitude along the ray path. Frequency is 2.5 kHz.

the ray shown in Figure 2 is not strongly confined to its initial field line. The region to which the ray is confined is indicated by two vertical bars on the density profile. Since this type of propagation requires high wave normals at low altitudes, the possibility of such rays being excited from the ground is low unless strong horizontal gradients are involved. However, these waves could be generated by satellite-borne VLF transmitters like those planned for the AMPS (Atmospheric and Magnetospheric Plasmas in Space) Laboratory in the Shuttle Bus program.

The above description gives the basic physics of VLF wave guidance near the plasmapause. For a general analytic treatment of the role of density gradients and magnetic field curvature in wave guidance the reader is referred to the works of Cerisier [1967], Walter [1969], and Scarabucci [1969].

In the remainder of this paper we analyze the properties of wave guiding by the inner edge of the plasmapause.

Guiding at the Inner Edge of the Plasmapause

In this section we show the dependence of plasmapause guiding on the gradients involved. Figure 4 shows two equatorial density profiles with different gradients. Since here we are only concerned with trapping by the inner edge of the plasmapause, we can ignore the parts of the profiles shown by the dotted lines. Only the local gradients determine the effectiveness of guiding. It should be noted here that both the profiles shown are realistic and commonly observed plasmapause density variations [Angerami and Carpenter, 1966; Chappell et al., 1970].

Figure 5 shows the behavior of ray paths for 2.5-kHz rays with different injection latitudes for both the weak and the strong plasmapause gradient cases. For each ray the L value and wave normal angle ψ are plotted versus latitude. Negative latitude indicates south. The scales for the axes are shown for the top left figures only. We see that for all cases the rays start with lower L values and slowly converge to the plasmapause field line. Until they descend to low altitudes in the conjugate hemisphere, they stay very close to the plasmapause ($L \simeq 4$). At about 4000 to 6000-km altitude they deviate inward and reach the ionosphere at lower L values. The wave normal direction oscillates about the field line direction, just as in the case of two-sided ducts. The general shape of the ray path in all these cases is as shown in Figure 1.

The wave normal variations for the rays with initial latitudes of -55° and -57° clearly illustrate that stronger plasmasphere gradients produce more effective guiding. The wave normal angle deviates less from the field line direction for the strong gradient case than for the weak gradient case.

An important aspect of this mode of propagation is illustrated by the ray behavior for $\theta_o = -59°$ in the strong gradient case. When the rays start close to the plasmapause, they are deflected inward by the plasmapause gradients at low altitudes, where the curvature of the earth's field is not strong enough to bend them back. As a result the rays deviate inward and are no longer guided by the plasmapause. The wave normal direction for these rays shows wide oscillations. Hence effective guiding is only possible for rays injected at sufficiently low latitudes. On the other hand, if the initial latitude is too low, the rays will not reach the plasmapause field line before they reach the equator and hence are not trapped and guided by the large gradients. Thus there exists a band of initial latitudes for which plasmapause guiding is effective. In our case this range of initial latitudes is -51° to -58°.

Having established the fact that stronger gradients provide stronger guiding, we now consider propagation for the case of a strong plasmapause density variation as given by profile II in Figure 4.

Figure 6 shows results for 2.5-kHz rays injected at different initial latitudes. For each ray starting with some initial latitude θ_0 the arrival latitude θ_F (F used for final value), wave normal at the equator (ψ_{eq}), and angle from upward vertical (Δ) at the conjugate point are shown. The coordinate system used in the ray tracings is also shown. The circled points indicate the results for individual rays. The points that are connected with dotted lines represent rays that are not strongly guided, and hence the ray path endpoint latitude is highly dependent on initial latitude. The solid lines indicate the region of efficient and strong trapping. Thus rays starting with initial latitudes of -54° to -58° reach the conjugate hemisphere in the same latitude range and with very nearly vertical wave normal angles. These rays will therefore be within the transmission cone of the lower ionosphere boundary and hence should be observed on the ground. Also, since their arrival latitude is in the same range as their starting latitude and both the arrival and starting wave normals are very nearly vertical, some energy will be reflected and will travel back to the opposite hemisphere. Hence multihop propagation is possible in this mode for a wide latitude (~4°) range. Since our model is symmetric about the equatorial plane, a sufficient condition for wave penetration of the conjugate ionosphere is just that the wave normal in the equatorial plane be parallel to the dipole magnetic field lines. The physics here is that of time reversal. Thus our results can be understood by noting in Figure 7 that the wave normals of some of the 2.5-kHz rays are closely parallel to the magnetic field lines in the equatorial plane, and consequently on the basis of time reversal arguments the wave normals of these waves should be closely perpendicular to the ionosphere boundary in the conjugate hemisphere, ensuring transmission. In this regard our work is similar to that of Singh [1976], who has presented a theoretical model which explains the problem of how whistlers are observed on the ground at low latitudes in the absence of suitable ducts in the ionosphere. However, although Singh's model also exhibits the physics of time reversal, the latitudinal gradients of ionization employed in his model are rather small in comparison with typical plasmapause gradients, and the L shell values of interest in his model are much smaller than typical plasmapause L shell values. Thus his results do not have direct relevance to the present study.

Figure 6 shows results for a single frequency but different initial latitudes. Figure 7 shows ray path parameters for rays with different frequencies all starting at -57° latitude. Note that -57° is about the center of the initial latitude band that gave effective guiding. The format of Figure 7 is very similar to that of Figure 6, except that ray path parameters are plotted against frequency. We see that for frequencies of 1-4 kHz the rays reach the conjugate hemisphere in the same latitude range as they come from and with very nearly vertical wave normals. Therefore using arguments similar to those in connection with Figure 6, we conclude that the signals can be observed on the ground and can echo between hemispheres over a wide frequency (1-4 kHz) as well as latitude (~4°) range.

The results given so far apply to rays injected from the ground into the magnetosphere, starting with vertical wave normal angles at an altitude of 100 km. But satellite transmitters such as those planned for the AMPS mission will be able to excite waves with a wide range of wave normals. Figure 8 shows ray path parameters for rays starting at 500-km altitude with wave normal directions of -4° to 4° away from vertical (i.e., Δ = -4°, Δ being as defined in the coordinate system given in Figure 6). We have plotted the angle from the upward vertical and the arrival latitude in the conjugate hemisphere against the wave normal measured from the upward vertical at the starting hemisphere. All rays are injected at a latitude of -57°. From Figure 8 we see that rays with initial wave normals of 0°-4° reach the conjugate hemisphere in the same latitude range and with almost

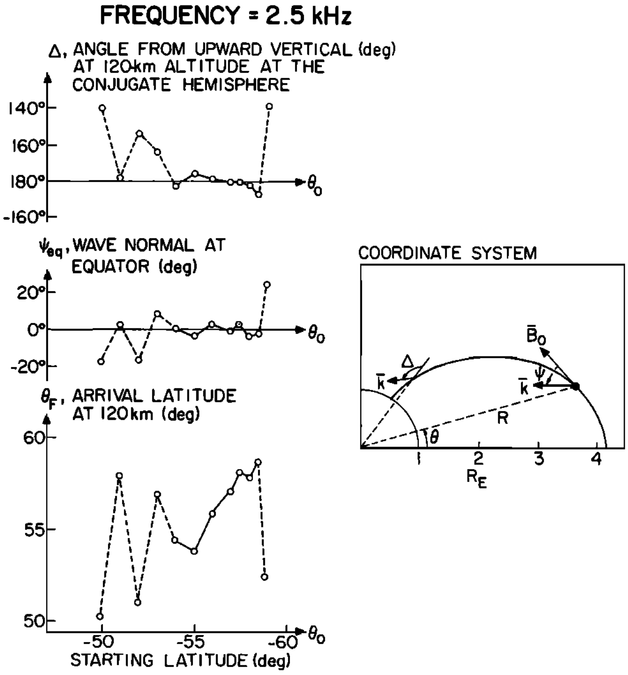

Fig. 6. Ray path parameters for 2.5-kHz rays starting at different initial latitudes. For each ray starting with some initial latitude θ_0, the arrival latitude θ_F, wave normal at the equator ψ_{eq}, and angle from upward vertical (Δ) at the conjugate point are shown. The circled points indicate the results for individual rays. The points that are connected with dotted lines represent rays that are not strongly guided, and hence the ray path endpoints are highly dependent on initial latitude. The solid lines indicate the region of efficient and strong trapping.

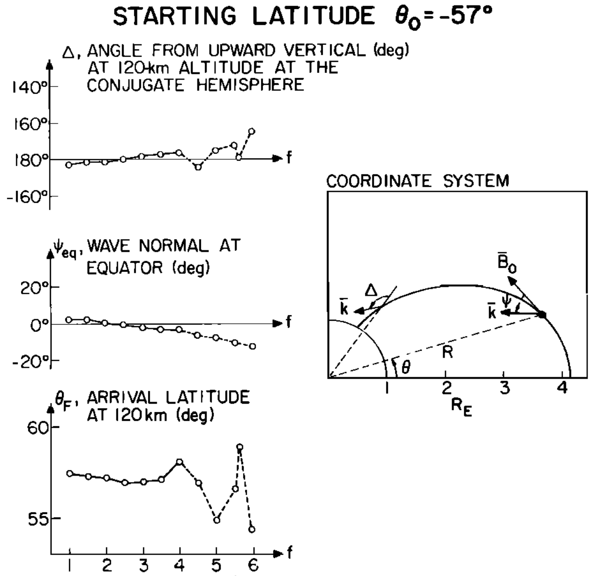

Fig. 7. Ray path parameters for rays at different frequencies all starting at -57° initial latitude. The format is very similar to that of Figure 6.

vertical wave normals. Hence again we conclude that both ground observations and multihop propagation are possible for an initial wave normal range of almost 4°.

Discussion

A discussion of upper cutoff frequency effects is in order here. It is well known that whistlers which propagate in ducts inside the plasmapause have an upper cutoff frequency (f_{uco}) which is generally equal to one half the minimum gyrofrequency (f_{Ho}) along the ray path [Carpenter, 1968b]. This is apparently due to the duct cutoff effect [Smith et al., 1960]. In our centered dipole model this would indicate that a whistler guided by a duct close to the plasmapause ($L \simeq 4$) would have $f_{uco} \simeq 7$ kHz. While Figure 7 might seem to suggest that a plasmapause-guided whistler would have $f_{uco} \simeq 4$ kHz, this is not correct. Figure 7 applies only to inputs at a fixed latitude ($\theta_0 = -57°$). By inputting waves at somewhat different latitudes it can be shown that the spectral form of plasmapause-guided whistlers (in our model) is as depicted in Figure 9. Figure 9 shows the computed spectral characteristics of a plasmapause-guided whistler as it would be observed on the ground. The frequency range covered in Figure 9 is 1-6 kHz. We have computed the ray paths and time delays for frequencies between 1 and 6 kHz with 0.5-kHz steps. The spectral characteristics of the trace in Figure 9 are indistinguishable from those of a ducted whistler which has propagated along the L = 4 field line, with the exception that $f_{uco} \sim 6$ kHz for the plasmapause-guided whistler. Thus according to our model the ratio f_{uco}/f_{Ho} is not the same for duct-guided and plasmapause-guided whistlers. Consequently, measurements of f_{uco} can in principle be used to distinguish between the two types of guidance. A search of available whistler data is presently underway at the Radioscience Laboratory to attempt to identify cases of plasmapause guidance of whistlers.

It is an interesting fact that for normal ducted whistlers, $f_{uco} \simeq f_{Ho}/2$ throughout the magnetosphere except for whistlers which propagate on the outer surface of, or just outside, the plasmapause [Carpenter, 1968a]. For those exceptional whistlers, f_{uco} may approach $0.9f_{Ho}$, a finding not in keeping with the theory of duct guidance.

Our calculations show that VLF waves can be guided by the outer edge of the plasmapause, and an example of this behavior is shown in Figure 2. Our calculations also show that for this mode of propagation, guidance can take place for all $f < f_{Ho}$, and thus $f_{uco} \to f_{Ho}$. On the basis of our results it is tempting to speculate that whistlers observed to be propagating near the outer surface of the plasmapause with $f_{uco} > f_{Ho}/2$ are actually being guided not by a duct but by the plasmapause outer surface in the manner shown in Figure 2. The major problem with this interpretation is that according to our model the ray of Figure 2 cannot be excited from the ground, and even if it were, the ray would reach the lower ionospheric boundary in the conjugate hemisphere with a wave normal far outside the transmission cone and would not be transmitted to the ground. Thus our model does not predict that whistlers can be guided in this mode and also be observed on the ground. On the other hand, it is possible to suggest at least one circumstance under which the ray of Figure 2 could be excited from the ground and could be transmitted to the ground in the conjugate hemisphere. This situation could come about if the appropriate strong horizontal gradients in plasma density were present in the F region at both the input and output points of the ray. However, we know of no experimental evidence that indicates such specialized conditions occur (or do not occur) during observations of high upper cutoff whistlers. Therefore we conclude that it appears

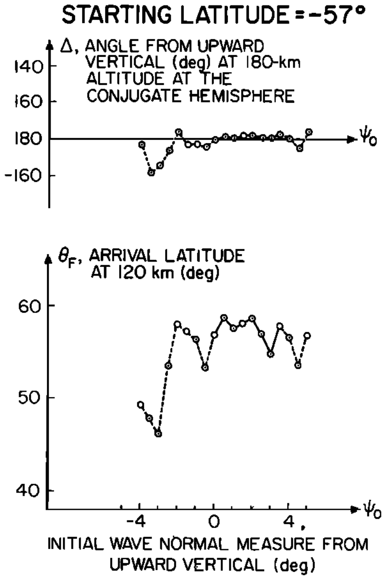

Fig. 8. Ray path parameters for rays injected at 500-km altitude with different wave normal angles. The format is very similar to the formats of Figures 6 and 7 except that wave normal at the equator is not shown in Figure 8.

questionable that high upper cutoff whistlers are plasmapause guided in the manner shown in Figure 2.

For simplicity our model assumes that the plasmapause density gradients extend down to the lower ionosphere in both hemispheres. Experiment shows that this is not always the case and that the plasmapause gradients are sometimes smoothed out below 1000 km [Brace and Theis, 1974]. This fact should have only a minor effect upon our calculations, however, since in our model the majority of rays guided by the plasmapause do not propagate near the plasmapause at altitudes below 4000 km.

Conclusions

We have shown that plasmapause density gradients can guide VLF waves. This kind of guiding is a case of gradient trapping. Using realistic and commonly observed plasmapause density variations, we have shown that the guiding by the plasmapause can be very effective. We have found that for strong density gradients, waves guided by the plasmapause exhibit all the properties of signals guided by normal magnetospheric ducts; namely, these waves can be observed on the ground and can echo between hemispheres. We have shown that plasmapause guiding is possible for wide ranges of initial latitude (~4°) and frequency (1-4 kHz) for excitation from the ground. For excitation by low-altitude satellite-based transmitters, waves having a relatively wide range of initial wave normal angle (~4°) will be similarly guided.

The important practical implications of this mode of propagation are the following.

1. Rays coming from a wide latitude range are focused tightly around the plasmapause field line. For example, rays covering an input latitude range of 8°, or 1000 km, are confined to 0.1L (600 km) as they cross the equator. Thus these rays are focused, resulting in an increase in field intensity at the equator of about 3 dB. This focusing effect causes the vicinity of the plasmapause to be a region where the probability is enhanced for reception by satellites of signals from ground VLF transmitters. This property and the fact that the plasmapause is a natural and readily accessible region enhance the feasibility of in situ satellite measurements during VLF wave injection experiments.

2. As shown in Figures 1 and 5, the ray paths deviate from the plasmapause field line at 4000 to 6000-km altitude. They therefore reach the lower ionosphere as much as 5° (5° in latitude near L = 4 represents about one unit in L value and 500-600 km in horizontal distance) away from the end of the plasmapause field line along which the ray has propagated for most of its path. We have shown that for strong plasmapause gradients the waves can penetrate the ionosphere and be observed on the ground exactly like whistler mode waves propagating in normal ducts. The fact that the rays come out at a point 500-600 km away from the field line of propagation could cause errors in the determination of the plasmapause location using VLF direction finding (DF) methods. A practical problem related to this may occasionally arise in connection with balloon and rocket measurements of Xray flux and particle precipitation produced during VLF wave injection experiments. For such measurements it is crucial to launch a rocket (or balloon) into (or under) that ionospheric region where field-aligned particle precipitation due to wave-particle interactions is expected to occur. If on a particular day whistler ducts are absent and only plasmapause guiding is taking place and if one relies on VLF DF techniques to attempt to identify the field lines on which the particle scattering takes place, then it is possible that the region of interest will be missed by as much as 600 km. Thus on occasion the DF method may need to be complemented by other methods (e.g., nose frequency analysis) to ensure the accuracy necessary for the application. In our calculations we assumed that low-altitude,horizontal density gradients can be neglected in mid-latitude transionospheric propagation. Although there are times when this assumption is valid [Chan and Colin, 1969], there are also times when this assumption is incorrect [Cairo and Cerisier, 1976]. Thus one of the next steps that should be taken is to investigate the effects of these gradients upon the wave guiding properties of the plasmapause.

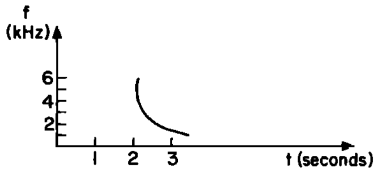

Fig. 9. Computed frequency-time spectrum of a plasmapause-guided whistler as it would be observed on the ground.

Appendix

The ray paths were found by using the Stanford VLF ray tracing program on an IBM 370/168 computer. This program was written by Walter [1969] and later modified by Burtis [1974]; it is similar to that developed by Kimura [1966].

The geomagnetic field model used was a centered dipole with electron gyrofrequency of 880 kHz at the ground at the equator.

The electron and ion densities above 1000 km were represented by a field-aligned isothermal diffusive equilibrium model at 3200 K, with appropriate factors included for the plasmapause variation, an r^{-a} type of density variation outside the plasmapause, and the lower ionosphere density variation. The electron density is given by

$$N = N_B \cdot N_{DE} \cdot N_{LI} \cdot N_{PL} \qquad \text{(A1)}$$

where N_B is the density at the base of the diffusive equilibrium (DE) model [Angerami and Thomas, 1964].

$$N_{DE}(r) = \left[\sum_{i=1}^{n} \delta_i \, e^{G/S_i} \right]^{1/2} \qquad \text{(A2)}$$

where the δ_i are the relative concentrations of the ionic species at the base of the diffusive equilibrium model, n is the number of species, $G = r_b[1-(r_b/r)]$, r_b being the geocentric distance (in kilometers) to the base of the DE model, $S_i = 1.506T(r_b/7370)^2(1/4^{i-1})$, where T is the temperature at the base of the DE model, and r is the geocentric distance (in kilometers) to the point where the density is evaluated. N_{LI} is the factor due to the lower ionosphere:

$$N_{LI}(r) = 1 - e^{-[(r-r_o)/H]^2} \tag{A3}$$

where r_o is the geocentric distance (in kilometers) to the level at the bottom of the lower ionosphere where the density goes to zero and H is the scale height (in kilometers) of the bottomside of the lower ionosphere. N_{PL} is the factor due to the plasmapause. If the plasmapause is placed at $L = L_P$, then this factor is unity for $L < L_P$. For $L \geq L_P$,

$$N_{PL}(r, L) = e^{-[(L-L_P)/W]} + \left[1 - e^{-[(L-L_P)/W]}\right]\left(\frac{r_c}{r}\right)^a + \left[1 - \left(\frac{r_c}{r}\right)^a\right]e^{-[(r-r_c)/H_S]^2} \tag{A4}$$

where L is the L value defining the particular field line, W is the half width (in L) of the plasmapause boundary, r_c is the geocentric distance (in kilometers) to the level at which the density outside the plasmapause field line is equal to the density inside, a is the exponent of the exponential component of the density decrease outside the plasmapause, r^{-a}, and H_S is the scale height of the radial density decrease for $r \geq r_c$ and outside the plasmapause.

Although the evaluation of the density profile is rather complex, the model allows much flexibility in obtaining desired densities at different points along the field line. Typical values of the parameters used for our computations are tabulated below:

Parameter	Value
N_B	6443 el/cm^3
Ionic species (1000 km)	
H^+	8%
He^+	2%
O^+	90%
r_b	7370 km
T	1600°K
r_o	6460 km
H	140 km
L_P	4.1
W	0.24
a	-2.9
r_c	6400 km
H_S	2500 km

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