Comment on 'Fast Time Resolved Spectral Analysis of VLF Banded Emissions' by F. V. Coroniti, R. W. Fredricks, C. F. Kennel, and F. L. Scarf

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This comment proposes an alternate interpretation of the data presented by *Coroniti et al.* [1971]. Those authors postulated the generation of narrow band constant frequency tones to explain the presence of narrow lines ('single modes') in their spectra of band-limited chorus. It is suggested that the apparent single modes could be more easily explained by the relatively simple assumption of closely spaced (~30 ms) continuously rising (or falling) tones. (This explanation is like that presented by *Shaw and Gurnett* [1971] to explain harmonically banded whistlers.)

It is extremely important to choose the correct interpretation, since the implied physical processes differ greatly. The explanation of Coroniti et al. is that the magnetosphere generates a series of constant frequency tones of successively higher (or lower) frequency. *Nunn* [1974] has recently presented theoretical support for this explanation. The interpretation of the present comment, on the other hand, is that single tones of continuously rising (or falling) frequency are generated and that the apparent single modes result when several such tones occur in a period shorter than the analyzer response time. This proposal will be more fully developed after a brief review of the data and the interpretation of Coroniti et al.

Coroniti et al. present spectra of banded chorus (both rising and falling) and two hisslike events. Spectra of rising test tones from an audio sweep generator are also shown and provide a reference for their interpretation of the data. Their analysis was performed on a Federal Scientific hybrid spectrum analyzer, model UA-6.

The first examples presented by Coroniti et al. are of rising chorus. The risers begin as narrow lines and then expand into fairly broad (~150 Hz) features that resemble closely the spectra of the test tones and are thus referred to as 'FM bursts.' Toward the upper end of these risers the bursts appear to decay into a group of narrow band peaks, but the peaks are neither sufficiently well separated in frequency nor of sufficient duration to be classified as individual constant frequency tones (single modes).

The remaining examples of chorus, however, do display well-defined single modes in addition to the FM bursts. The principal features of these modes are (1) very narrow (25–50 Hz) bandwidths, (2) usually constant center frequency, (3) duration greater than the time response (~60 ms) of the analyzer, and (4) deep and often broad minima between adjacent peaks. These modes are seen in one of the examples of hiss as well as in both rising and falling chorus. They at times appear to merge into or to emerge from FM bursts.

There are two possible explanations for the presence of single modes in the spectra. The first is that narrow band constant frequency tones are actually generated in the magnetosphere and the observed single modes truly represent an individual element of chorus. The second is that each chorus element is in fact a continuously rising tone and that the apparent single modes are a result of the particular analysis procedure. (This situation is analogous to that presented by Shaw and Gurnett: the harmonically spaced bands in the whistlers may be assumed to represent actual features of individual events or to be products of the analysis. Shaw and Gurnett show the latter to be the case by clearly resolving two whistlers whose spacing in seconds is the reciprocal of the band spacing in hertz. The bands seen in the original analysis resulted because the spacing in time was less than the response time of the analyzer.)

Coroniti et al. assume that narrow band tones are actually generated in the magnetosphere. As possible mechanisms they suggest (1) a linear instability having a growth rate that is narrowly peaked at the frequencies of the modes, (2) trapping of electrons in the effective potential well of a whistler mode wave, and (3) trapping of electrons by plasma oscillation. It is the present author's contention that the single modes are products of the analysis procedure and are due to the existence of overlapping rising tones spaced in time by the reciprocal of the frequency separation between the modes; the time spacing is again less than the analyzer response time. This assumption is based upon the well-established phenomenon of discrete rising tones and does not require an appeal to theories whose applicability is still a matter of serious discussion.

A digital spectral analysis of four overlapping mathematical FM bursts spaced by 0.03 s is shown in Figure 1*b*. The slope (1025 Hz/s) and the duration (0.32 s) of the individual rising elements were chosen to approximate the data presented in Figure 5 of *Coroniti et al.* [1971]. The scale is linear in amplitude. The spectra in Figures 1*a* and 1*b* were calculated by using the fast Fourier transform [*Bergland*, 1969]. The parameters of the transform were adjusted to match as well as possible the test results of Coroniti et al. This adjustment yielded an effective filter width of about 16 Hz and a corresponding response time of about $\frac{1}{16}$ Hz = 62 ms. The result of applying this transformation to a mathematical test tone identical to the Coroniti et al. signal is shown in Figure 1*a*. The similarity of these spectra to those at the bottom of those authors' Figure 3 indicates that the two procedures produce essentially identical outputs for identical inputs.

A single frequency component of a rising tone would be expected to persist in the spectra for a time equal to the sum of the response time (~62 ms) and the time taken for the tone to sweep through the 16-Hz filter (~7 ms for a 2.4-kHz/s tone). The result of ~70 ms for the test tone agrees fairly well with both our Figure 1*a* and the bottom part of Figure 3 of *Coroniti et al.* [1971].

A comparison of the data with our Figure 1*b* shows that the essential features of the single modes (as listed above) are reproduced by the assumption of overlapping rising tones. The same features are produced by overlapping falling tones.

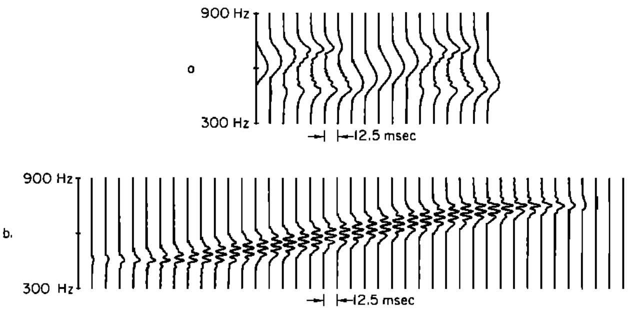

Fig. 1. (*a*) Fast Fourier spectra of a test tone identical to that used by Coroniti et al. Each rising portion lasts 0.125 s and has a slope of 2.4 kHz/s. A single frequency component lasts for approximately 65 ms. (*b*) Fast Fourier spectra of four overlapping rising tones. Each tone lasts 0.32 s and has a slope of 1.025 kHz/s. Successive tones are spaced by 0.03 s. Note the similarity between the well-defined constant frequency peaks and the single modes seen in Figure 5 of *Coroniti et al.* [1971].

The mathematical basis of this interpretation is outlined in Figure 2. We will derive four overlapping risers by convolving a single riser with a sequence of four impulse functions spaced by τ s. The resulting spectrum may most easily be obtained by using the relationship that the transform of the convolution of two signals is equal to the product of the transforms of the individual signals (and vice versa) [*Bracewell,* 1965].

The sequence of four impulses (Figure 2*c*) may be obtained by multiplying an infinite train of impulses (Figure 2*a*) by a square time window (Figure 2*b*). The transform of the four impulses is found by convolving the transform of the impulse train (which is another impulse train of spacing $1/\tau$ Hz) with the transform of the square window (which is of the form ($\sin f)/f$). This convolution simply smooths the pulse train, as is seen in Figure 2*c*.

The sequence of four rising tones is now derived by convolving the single FM burst (Figure 2*d*) with the four impulses (Figure 2*c*). The transform of the result (Figure 2*e*) is quickly found by multiplying the spectra of Figures 2*c* and 2*d*. We have finally a sequence of smooth peaks spaced by $1/\tau$ Hz; the envelope of the peaks is the envelope of the FM burst. It is important to note also that the frequency of each peak is an integral multiple of $1/\tau$ Hz. The location of the peaks thus does not depend upon the center frequency of the riser, as might be expected from the similarity to a frequency-modulated signal. This fact further insures that the peaks in successive traces will occur at the same frequencies.

The spectra of Figure 2 have been derived by assuming that the time functions were Fourier-integrated from $t = -\infty$ to $t = +\infty$; this corresponds to using an analyzer with infinite response time. To derive the spectra produced by an analyzer with a finite response time (assumed greater than τ for our purposes), the time function of Figure 2*e* may be multiplied by a weighting function corresponding to the analyzer characteristics. The result, as exemplified in Figure 1*b*, is simply a selection and further smoothing of those peaks corresponding to the frequencies of the signals at the time that the analyzer sampled the data.

We might also mention that the individual risers cannot be resolved unless two conditions are satisfied:

1. The separation in time, at a given frequency, between risers must be greater than the analyzer response time.

2. The separation in frequency, at a given time, must be greater than the analyzer filter bandwidth.

If we let f' denote the slope of the risers, Δt the response time, $\Delta f = 1/\Delta t$ the filter bandwidth, and τ the spacing between risers, then requiring $\tau \gtrsim 1.5\Delta t$ and $f'\tau \gtrsim 1.5\Delta f$ leads to the criterion that $f' \gtrsim 2.25\tau^{-2}$ for the individual elements to be resolved. If this condition is not met, no adjustment of the analyzer parameters will permit resolution.

To summarize, any spectral analysis technique that is unable to resolve multiple closely spaced events temporally responds by producing frequency bands, or modes, separated by the reciprocal of the time between events.

We should emphasize that the duration of a single mode is not limited to the analyzer response time. A single frequency component from multiple rising tones will persist for a time equal to the sum of the persistence time for one rising tone plus the time between the first and the last tones. From Figure 1*b* we can see that the individual peaks (modes) are well defined for a time roughly equal to the time between the first and the last tones (90 ms in the present case). The fact that some of the modes in the data are longer than those in our example may simply suggest that there were more than four rising tones present. We might add that it would be useful to know if the spectra of Coroniti et al. were plotted logarithmically; a logarithmic presentation could enhance the discrete appearance of the modes.

Further justification for this interpretation may be found in published data. Overlapping chorus bursts are quite common. *Burtis and Helliwell* [1969] show examples with spacings of 100–300 ms. Overlapping elements also appear to be present in the *f-t* diagrams of the paper under discussion, although the scale is such that it is difficult to be sure. (The overlap in the Coroniti et al. amplitude spectra might be due to the fairly long time response of the analyzer; see, for example, their test tones.) Both the Burtis and the Coroniti et al. data also show that the spacing between elements may vary considerably over a time scale as short as a few seconds. In light of these facts it does not appear unreasonable to assume that chorus bursts

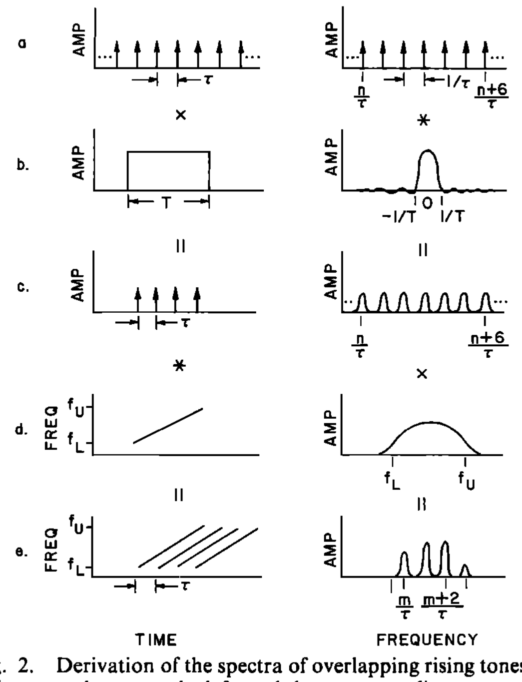

Fig. 2. Derivation of the spectra of overlapping rising tones. Time functions are shown on the left, and the corresponding spectra on the right. The crosses denote multiplication, and the asterisks convolution. The peaks in *e* will always be located at multiples of $1/\tau$, where τ is the spacing between events.

may occur as close together as 20–30 ms and that what Coroniti et al. considered to be individual elements of chorus was actually composed of several closely spaced bursts.

It has also been reported [*Burtis and Helliwell*, 1969; *Burtis*, 1969] that banded chorus often occurs simultaneously with banded hiss. The presence of closely spaced risers within the hiss would thus account for the single modes seen in Figure 8 of *Coroniti et al.* [1971]. (It is possible that the hiss band may actually be composed entirely of very dense chorus; it is not uncommon for banded chorus to appear to merge into a hiss-like structure (W. J. Burtis, private communication, 1973).) It should be noted that this example of hiss occurs within 1 min of one of the chorus examples and that Coroniti et al. recognize two choruslike features within the hiss.

We may further employ Burtis' data to help account for an additional feature of the Coroniti et al. spectra. As was mentioned above, the single modes typically appear with the broader spectral features that are referred to as FM bursts. In our model spectra of Figure 1, however, only single modes are present. A possible explanation of this discrepancy may be found in work of *Burtis and Helliwell* [1969], where we see that adjacent chorus elements may vary significantly in both their starting frequency and their duration; the range of overlap thus may not cover the entire frequency range of the individual elements. We would expect to see single modes in the region where the risers overlap and the broader 'FM bursts' where they do not. The appearance of both modes and FM bursts in a single event might also be due to a large change in the relative amplitudes of two overlapping signals; *Burtis* [1969] indicates that the amplitude of a single riser may change quite rapidly.

One comment might be made on the spacing between the modes. As Figure 1*b* shows, uniform overlapping tones produce uniformly spaced peaks. Although the uniformity of the spacing in the Coroniti et al. data is difficult to determine accurately due to the few occurrences of three or more well-defined adjacent modes, there do appear to be regions of fairly regular spacing.

We should finally note that the solution employed by Shaw and Gurnett, that of improving time resolution to separate closely spaced signals, is not available in the present case. The postulated overlapping tones are spaced by about 30 ms. To assure that a single analyzer filter sees only one tone, we thus require a filter response time of less than 30 ms. This value implies a filter bandwidth of more than 33 Hz. However, due to the slope (1025 Hz/s) of the risers, two tones are spaced by only $1025 \times 0.03 \cong 31$ Hz. The separate tones would thus not be clearly resolved in frequency; application of the previously derived criterion leads to the same conclusion. If additional single-mode examples with greater slopes are found, it may be possible to resolve the individual elements by improving the time resolution.

CONCLUSION

This comment gives an alternate interpretation of the data presented by Coroniti et al. It is shown here that constant frequency single modes in the spectra of banded chorus can in fact be produced by closely spaced rising or falling tones of the type observed in nature. This interpretation is considerably simpler physically than the Coroniti et al. postulate that constant frequency tones are generated in the magnetosphere. It is concluded here that it has not been established that single modes may constitute a part of the structure of chorus.

Acknowledgments. The author would like to thank R. A. Helliwell and W. J. Burtis for their comments and suggestions. This work was supported in part by the Office of Naval Research under contract NONR N00014-67-A-0112-0012 and in part by the Air Force Office of Scientific Research under grant F4462-72-C-0058.

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(Received March 6, 1974;
revised June 6, 1974;
accepted April 25, 1975.)