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Reply

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Before discussing the details of Nunn's comments we wish to restate our approach to the problem as follows. By following the particles instead of the waves we obtain a simple space-time picture of the interaction, as shown in Figure 3, page 7359, of our paper [*Helliwell and Crystal,* 1973]. The laws of motion are used to find the wave-induced nonlinear perturbations in the trajectories of the interacting electrons. We show that these perturbations produce transverse currents that act as sources for stimulated waves. These stimulated components are added linearly to the existing wave to give the total self-consistent wave fields at any point in space and time.

We still believe that our model, though unconventional, contains the essential mechanism required to account for both driven and self-excited narrow band oscillations. Our model differs from Nunn's model in the following respect. Our concept of phase bunching is applicable for all field intensities within the 'slowly varying' approximation and for all times. *Nunn*'s [1974] model, on the other hand, requires a waiting time of 'several trapping periods' before it is effective as a current-stimulating mechanism. This requirement sets a relatively high minimum value on the wave amplitude that can be employed in an inhomogeneous medium in Nunn's model. Our response to Nunn's three comments is as follows:

CALCULATION OF THE RESONANT PARTICLE CURRENT

Nunn questions the validity of our representation of the full distribution function by single values of α and $v_{\parallel}$. At the time of writing (1973), our main justification for this simplification was agreement between the predictions and the observations. More recently we have modeled the full distribution function with 2000 sheets, covering the full range of α and an appropriate range of $v_{\parallel}$. We have calculated the currents without feedback (feedback greatly increases cost), getting results qualitatively similar to those of our simplified model with feedback included. However, with respect to 'phase memory,' we noted on page 7363 of our paper the possibility that phase mixing from off-resonance sheets would reduce the effect. Our 2000-sheet nonfeedback model shows that indeed the current profile resembles the wave form, in agreement with Nunn's comment. However, this result cannot be extrapolated to the feedback case, since such extrapolation would then ignore the self-stimulated wave fields that always accompany these currents. It is this stimulated wave that gives growth in the presence of even a very short (10-ms) triggering pulse (our Figure 10, page 7367).

With regard to the phase of the stimulated current, our own recent calculations show that when we include a slope in the distribution function, we do obtain a small component of current that is colinear with **E**, just as Nunn insists that we must. However, we believe that its contribution to the total stimulated current can be neglected. In referring to the nonlinear treatment of *Abdalla* [1970], Nunn does not substantiate his claim that the stimulated current is colinear with **E**. While Abdalla makes no explicit statement in her paper regarding this phase, we conclude from her Figure 2 (page 1804) that her currents, like ours, are largely colinear with the wave **B**. It should be kept in mind that we make no attempt in our published model to recover the results of linear theory, for we believe that the reported nonlinear stimulated currents dominate the mechanism of generation.

ROLE OF INHOMOGENEITY

We agree that inhomogeneity is important. The parabolic variation of gyrofrequency was invoked by *Helliwell* [1967, 1970] to explain the frequency slope of self-sustained emissions. We think that our criterion for defining the interaction length (the unperturbed phase variation due to inhomogeneity falls within $\pm\pi/2$) is still reasonable. The important point is that in our model, inhomogeneity is not required to produce phase bunching and the corresponding coherent radiation, nor do we expect that it will significantly alter their values within the interaction region. We plan to check this point in the near future by including the inhomogeneity in our model. At this time our justification for assuming a homogeneous interaction region must still rest primarily on agreement between our predictions and the observations.

We are puzzled by Nunn's statement that our treatment 'neglects the effect on resonant particle behavior of changes in wave phase which occur as a result of nonlinear wave-particle interaction.' We believe that we have included the phase change of the stimulated wave as illustrated in our Figure 6 and discussed on page 7364.

AGREEMENT WITH OBSERVATIONS

The point at issue is what wave field intensities are appropriate to use in modeling the interaction process. Inside the plasmapause, satellite-based VLF measurements of both natural emissions [*Gurnett and O'Brien,* 1964] and station NAA signals [*Heyborne,* 1966] suggest a maximum field intensity of 10 mγ, signals as low as 0.5 mγ being reported. This alone would justify the use of either 1-mγ or 10-mγ fields.

In support of his assumption of a $\geq$10-mγ wave, Nunn states that 'in a parabolically varying field a wave with an amplitude of 1 mγ will not be able to trap particles, and the wave-particle interaction process would be purely linear in that case.' We disagree. A large transverse current is formed in one quarter of a trapping period. In this case even though no electron completes a full oscillation in the potential well of the wave, individual electron perturbations are already quite nonlinear, as we show in our Figure 4. Therefore their effects cannot be described with linear theory.

Lastly, it appears to us that Nunn has incorrectly associated the 10-Hz pulsations predicted by our model for CW excitation (see our Figure 7) with the wave's trapping frequency. For the 30° pitch angle electrons used in our model, the trapping frequency is 16 Hz. The 10-Hz pulsations are a fundamental result of the feedback in our model and not a consequence of oscillatory electron perturbation motion.

We realize that we do not yet have a complete theory of narrow band VLF emissions. We find that our present model does account for frequency change, exponential growth, saturation, and bandwidth. However, many phenomena remain to be explained, such as phase locking, wave-wave interactions, triggering, and entrainment, just to mention a few. Thus there should be ample opportunity for further theoretical work.

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