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Ion Cutoff Whistlers

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A new phenomenon associated with the propagation of whistlers in the ionosphere has been found in recordings of the Stanford University/Stanford Research Institute VLF experiment aboard satellites OGO 2 and OGO 4. On a frequency-time spectrogram, this phenomenon manifests itself by the turning up of a whistler trace at a frequency below 1 kHz, as illustrated in Figure 1. The four groups of events shown in the figure were taken from two orbits of OGO 4 in August 1967. Events (*a*) and (*b*) were detected over Australia at a geomagnetic latitude of 40°S. Events (*c*) and (*d*) were detected near Santiago, Chile, at a geomagnetic latitude of 21°S. In all four cases the satellite height was in the vicinity of 900 km. Panel (*c*) shows two examples of the new kind of whistler separated by about 1.5 sec, and in panels (*a*) and (*b*) there are several examples more closely spaced. The measured frequency-time curve of this new whistler fits a constant dispersion, or Eckersley, curve over most of the range 0–10 kHz. However, at the low-frequency end, the trace deviates toward increasing time delays, reaches a turning point or minimum frequency, and then rises in frequency to some extent. (In panels (*c*) and (*d*) there is a companion whistler seen in a narrower frequency range, with a greater time delay, which probably followed a different path and is not relevant to this study.)

Comparison of the observed whistlers with the Eckersley curves indicates a dispersion of the order of 30 $\sec^{1/2}$, which suggests an origin in the conjugate hemisphere [*Helliwell*, 1965]. The peculiar behavior of the low-frequency end of the whistlers is attributed to reflection of the downcoming energy near the point where the wave frequency approaches the first cutoff below the proton gyrofrequency, as discussed below. For this reason the name 'ion cutoff whistler' is suggested.

It is well known that the presence of multiple ions in the ionosphere causes the dispersion relation for propagating waves to exhibit a number of cutoffs and resonances [*Stix*, 1962; *Smith and Brice*, 1964]. Figure 2 shows a sketch of wave phase velocity versus frequency for a cold, collisionless ionosphere composed of electrons and the ions of hydrogen, helium, and oxygen, in the frequency range most affected by the proton concentration. The branches labeled *R* and *L* represent the right- and left-hand circularly polarized modes, respectively, for wave normal along the static magnetic field. The branch labeled *e*, for extraordinary, is linearly polarized, and the wave normal is perpendicular to the static magnetic field. For an intermediate wave normal angle the modes are shown dashed. Characteristic frequencies indicated are the proton gyrofrequency F_H, the crossover frequency f_x, and the cutoff frequency f_c. The crossover and cutoff frequencies depend on the relative abundances of the three ions, but are particularly sensitive to that of H^+. When the proton relative concentration decreases, both f_c and f_x move toward F_H, their limiting value. For the fast mode indicated in the figure, the cutoff frequency represents a reflection condition.

The altitude variations of the cutoff frequency f_c and proton gyrofrequency F_H are sketched in Figure 3, assuming a proton relative concentration increasing monotonically with height. This diagram and the accompanying sketch of an ion cutoff whistler can be used to explain the shape of the spectrograms in Figure 1. In the absence of coupling, a wave of frequency f_1 propagating downward in the fast mode will pass the satellite at h_s, be reflected very near the height h_1 at which $f_1 = f_c$, and then be detected a second time as the energy moves upward. The time interval Δt between the reception of the downcoming and upgoing signals depends on the distance to the level of reflection below the satellite Δh and also on the ionosphere model. The mini-

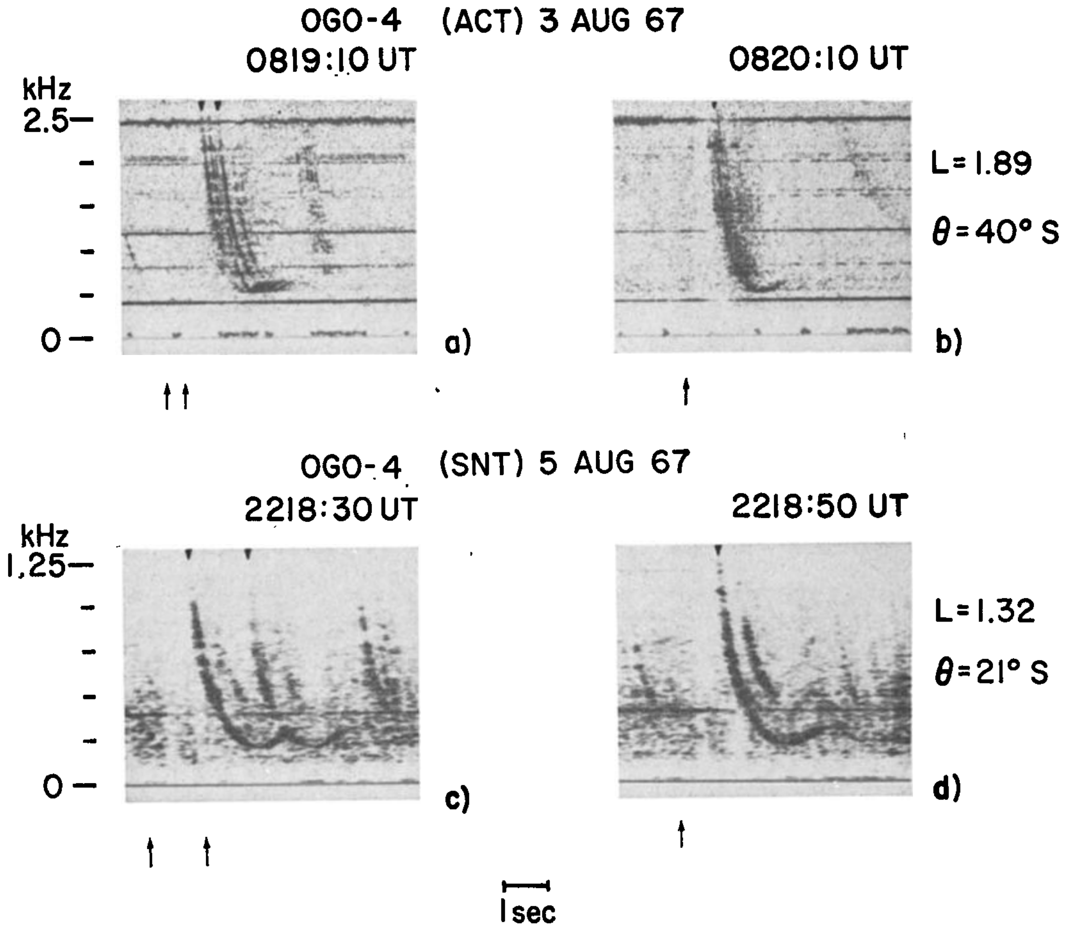

Fig. 1. Spectrograms of ion cutoff whistlers detected by OGO 4. Examples (*a*) and (*b*) were detected at Australia and examples (*c*) and (*d*) at Santiago, Chile. In all four examples the satellite height was about 900 km. Examples (*a*) and (*b*) show multiflash events, in contrast to the isolated ones in examples (*c*) and (*d*). The arrows at the bottom of each spectrogram indicate the probable origin, based on the Eckersley law, of the components indicated by the arrowheads at the top. The great difference in the minimum frequencies in the two passes is due mainly to differences in the magnetic field intensities at the two sites.

mum observed frequency f_m is reflected at the satellite height h_s. Lower frequencies are reflected higher up and are not detected. If coupling occurs, part or all of the energy of a downcoming wave in the fast mode can be coupled to the other mode, around f_x, and continue to propagate downward. In fact, whistlers detected on the ground sometimes show frequencies below the range of local cutoff frequencies.

A possible application of ion cutoff whistlers to the determination of the relative proton concentration may now be considered. For the three-ion model ionosphere considered above, the cutoff frequency f_c is given by [*Gurnett et al.*, 1965]

$$1+\left(\frac{f_N}{f_H}\right)^2 = \frac{f_N^{\,2}}{F_H f_H \Lambda} \cdot \left[1 - \frac{\alpha}{1-\Lambda} - \frac{\beta}{1-4\Lambda} - \frac{\gamma}{1-16\Lambda}\right] \qquad (1)$$

where

$$\Lambda = \frac{f_c}{F_H}, \qquad \alpha = \frac{n[\mathrm{H}^+]}{n_e},$$

$$\beta = \frac{n[\mathrm{H_e}^+]}{n_e}, \qquad \gamma = \frac{n[\mathrm{O}^+]}{n_e}$$

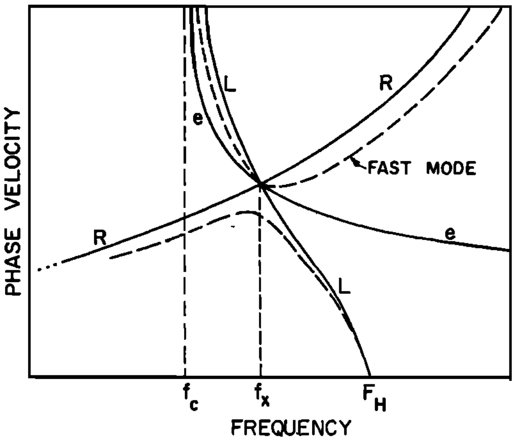

Fig. 2. Phase velocity versus frequency for the principal modes in the frequency region most affected by the H$^+$ concentration. For wave normal along the static magnetic field, *R* and *L* represent the right- and left-hand circularly polarized modes. The mode labeled *e*, for extraordinary, is linearly polarized, and the wave normal is perpendicular to the magnetic field. For an arbitrary wave normal direction the modes are shown dashed. The proton gyrofrequency is F_H, f_x is the crossover frequency, and f_c is the cutoff frequency that sets a reflection condition for the fast mode propagating downward.

f_N is the electron plasma frequency.

f_H is the electron gyrofrequency.

For altitudes above approximately 200 km it can be assumed that $f_N^2 \gg F_H f_H$, so that

$$\frac{f_N^2}{F_H f_H} \gg 1 + \left(\frac{f_N}{f_H}\right)^2$$

With this approximation, equation 1 reduces to

$$1 = \frac{\alpha}{1 - \Lambda} + \frac{\beta}{1 - 4\Lambda} + \frac{\gamma}{1 - 16\Lambda} \qquad (2)$$

where the condition of electrical neutrality requires $\alpha + \beta + \gamma = 1$. If one allows β and γ separately to be zero, the two limiting values of α are given by

$$\tfrac{16}{15}(1 - \Lambda) < \alpha < \tfrac{4}{3}(1 - \Lambda) \qquad (3)$$

which are shown graphically in Figure 4 for Λ ranging from 0.25 to 1. By averaging α between these two limits one obtains

$$\alpha_{av} = \tfrac{6}{5}(1 - \Lambda) \qquad (4)$$

to within ±10% of the correct value. This estimate can be made by using only the ratio of the minimum observed frequency to the proton gyrofrequency at the satellite. A knowledge of another characteristic frequency such as the crossover frequency would permit an exact determination not only of α but also of the other two ratios for a three ion model [*Shawhan and Gurnett*, 1966].

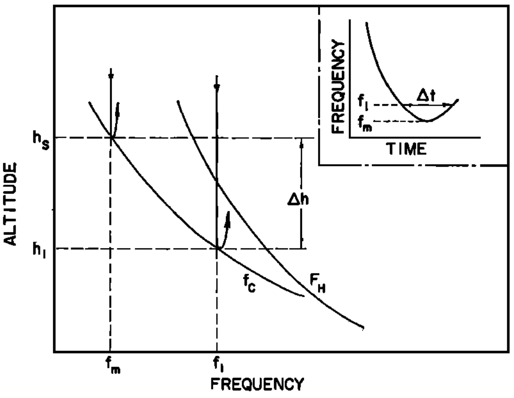

Fig. 3. A schematic representation of the variations of f_c and F_H with altitude and a corresponding ion cutoff whistler spectrogram. The satellite height is represented by h_s. The cutoff frequency f_c approaches the proton gyrofrequency for decreasing altitudes as a result of the decrease in the H$^+$ relative concentration. The frequency f_l is reflected at a depth Δh below the satellite, where $f_l \cong f_c$. The minimum frequency f_m is the one reflected at the satellite height. The time delay Δt is associated with the group velocity of the waves over the path Δh.

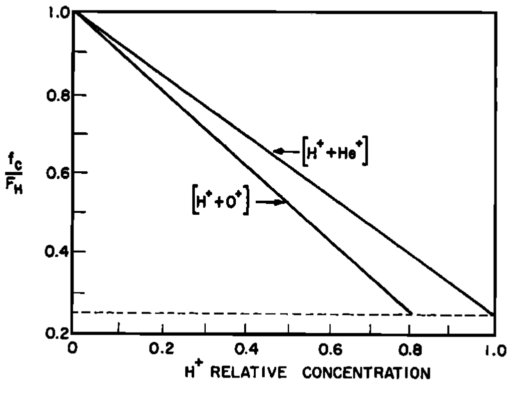

Fig. 4. Dependence of the ratio f_c/F_H on the relative concentration of H$^+$ ions for two extreme ionosphere models. The upper line corresponds to a model made of electrons, H$^+$ and He$^+$ ions, without O$^+$. The lower line corresponds to a model made of electrons, H$^+$ and O$^+$ ions, without He$^+$. Any actual curve must lie between these two lines.

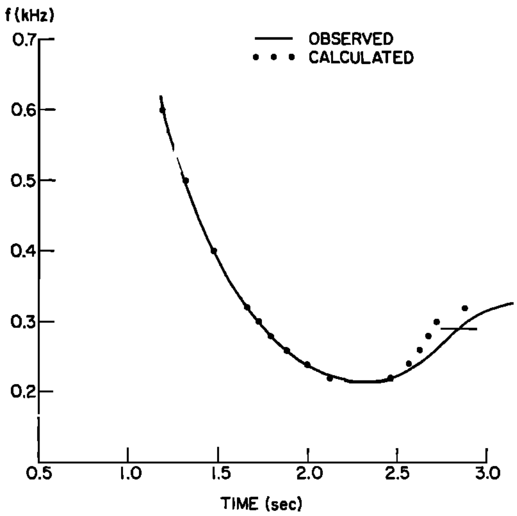

Fig. 5. Comparison of observed and calculated ion cutoff whistler spectra. The continuous line represents the first whistler of Figure 1c. The dots were calculated by ray tracing in a model ionosphere. For convenience in the computations, the medium below the satellite was represented by an isothermal diffusive equilibrium model at 800°K. At the satellite height (900 km) the model parameters are: $F_H = 296$ Hz. $\alpha = 0.333$, $\beta = 0.664$, $\gamma = 0.003$, $F_N = 1.01$ MHz. The apparent mismatch and the low value of T indicate the need for a model that better represents the altitude variations of α. The horizontal bar indicates the average thickness of the whistler trace in the original spectrogram.

Additional information about the value of α and its variation can be obtained by studying the shape of the whistler trace close to the minimum frequency. In Figure 3 one sees that, for a frequency f_1 in the vicinity of the minimum frequency, the depth of penetration below the satellite Δh and thus the delay time Δt, depends on the slope of the curve of f_c versus height, or, what is the same, on the variation of α below the satellite. Ray-tracing techniques become particularly useful at this point. Starting with an ionosphere model with the value of α at the satellite given by equation 4, the model parameters can be subsequently adjusted to achieve the best possible match with the observed trace. In this way a better approximation to the value of α can be attained and also an estimate of its variation with height below the satellite. Figure 5 shows the result of such a procedure. The continuous trace represents the first whistler component of Figure 1c, and the dots were obtained by ray tracing in a model ionosphere. The medium below the satellite was represented by a diffusive equilibrium model whose parameters are indicated in the figure. The value of the electron plasma frequency f_N was adjusted to give a best fit to the dispersion of the whistler for frequencies above 400 Hz. The temperature used is too low for daytime hours, but gave the best approximation to the reflected portion of the trace. This indicates the need for a more realistic model to represent the variation of proton concentration (α) at the lower altitudes.

The expected ±10% accuracy in the determination of α by equation 4 is not as good as the ±3% obtained by the use of proton whistlers and reported by *Shawhan and Gurnett* [1966]. However, the figure obtained by these authors depends on the accurate measurement of the crossover frequency, which is very often not possible. Consequently, the use of ion cutoff whistlers for the determination of proton relative concentration may be regarded as equivalent and complementary to the use of proton whistlers.

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