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Langmuir Probe Response to Periodic Waveforms

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Abstract. The behavior of a Langmuir probe modulated by an externally applied signal differs from its static behavior owing to the presence of additional steady and alternating components of probe current. These may be interpreted to yield data on such plasma parameters as electron temperature and density and electron velocity distribution. Considerable progress has been made recently in this branch of plasma diagnostics. This work is surveyed here and is extended by an analysis of a new ac method of determining electron density and temperature. The new method depends on the measurement of probe admittance and should have useful applications to ionospheric probing by rockets and satellites. Details of an illustrative example are given. Some of the implications of the theory presented bear on the accuracy of probe measurements of electron velocity distributions made by second derivative methods. These errors are discussed briefly.

1. INTRODUCTION

Determining static Langmuir probe characteristics by varying the probe potential sufficiently slowly for transient phenomena to be negligible, while simultaneously recording the probe current, has been a standard plasma diagnostic technique since it was first developed by *Langmuir and Mott-Smith* [1924] forty years ago. Although considerable interest attaches to the dynamic behavior of the probe under conditions in which its potential contains time-varying components, it is only comparatively recently that this field has been extensively explored. As we shall indicate below, study of dynamic probe characteristics opens up a wide range of additional diagnostic techniques involving data reduction methods rather different from the usual semilogarithmic current/voltage plot associated with the static probe.

To facilitate further discussion, we can conveniently differentiate three ranges of dynamic probe operation: (1) very low frequency (VLF), (2) low frequency (LF), and (3) high frequency (HF). In the first of these, the frequency components of the probe potential are considered to be so low that ions and electrons respond without delay, and there are no displacement currents.[2] As the frequencies increase the second range will be entered, where the ion mobility can be considered to be zero, though the electrons can still respond without delay. Further increases in frequency will bring operation within the third range, where the mobilities of both electrons and ions can be regarded as zero and only displacement current flows. For these frequencies, the current flowing is determined by considering the probe as a structure immersed in a medium whose dielectric constant is dependent on the plasma [*Kane et al.*, 1962].

There will, of course, be two transition regions which might be of interest. So far, attention has been concentrated on the higher of the two, between LF and HF behavior, and several workers have shown that a resonance can occur near the local electron plasma frequency. In

[1] A first draft of this paper, devoted mainly to the admittance probe analysis given in section 2B, was written by Dr. Mlodnosky before his death in June 1963. In view of the considerable volume of work published recently in the field of dynamic probes, it was considered necessary to modify his discussion substantially to bring it up to date. This has been done, with some additions to the analysis, by the other author.

[2] Note that this does not necessarily correspond to the frequency band below 30 kc/s commonly termed VLF.

section 2, we shall discuss this phenomenon and the characteristics of VLF and LF probes in more detail.

2. Dynamic Probe Characteristics

A. *VLF probes.* A number of pulsed probe techniques have been developed in which the pulse length is sufficient for the conditions of this probe category to be met [*Waymouth*, 1959; *Bills et al.*, 1962]. They are of use in transient discharges and where relatively rapid probe contamination may be occurring. We shall omit further discussion of them here, since they give the normal static probe characteristic. Instead we shall consider an arbitrary probe characteristic of the form

$$i = f(V) \tag{1}$$

where both i and V contain dc and small-amplitude time-varying components. These will be denoted by subscripts 0 and 1, respectively. We have, on expanding by Taylor's theorem,

$$i_0 + i_1(t) = f(V_0) + V_1(t)f'(V_0) + \frac{V_1(t)^2}{2} f''(V_0) + \cdots \tag{2}$$

This has been studied for general noise signals elsewhere [*Garscadden and Emeleus*, 1962; *Crawford*, 1963], but in the cases of interest to us the modulating signal will be sinusoidal, so that

$$V_1(t) = V_1 \sin \omega t \tag{3}$$

Substituting (3) into (2) and collecting terms gives

$$i_0 = \left[f(V_0) + \frac{V_1^2}{4} f''(V_0) + \frac{V_1^4}{64} f''''(V_0) + \cdots \right] \tag{4}$$

$$i_1(t) = \left[V_1 f'(V_0) + \frac{V_1^3}{8} f'''(V_0) + \cdots \right] \sin \omega t - \left[\frac{V_1^2}{4} f''(V_0) + \frac{V_1^4}{48} f''''(V_0) + \cdots \right] \cos 2\omega t + \cdots \tag{5}$$

To first order, both the dc and second harmonic components depend on the second derivative of the probe characteristic at V_0. *Druyvesteyn* [1930] showed that electron velocity distributions could be traced out directly by determining the second derivative of a static probe characteristic. Graphical differentiation is highly inaccurate for such an application, and what was probably the first application of a dynamic probe technique was designed to avoid this difficulty by obtaining the second derivative directly [*Sloane and MacGregor*, 1934; *van Gorcum*, 1936]. The principle of the method was to modulate the probe potential with a small low-frequency sinusoidal signal superimposed on the dc potential and to measure the increment in the direct component. The variation of this with V_0 could then be used to determine the velocity distribution.

Experimentally, it is more convenient to have an ac output proportional to the second derivative, and direct measurement of the second harmonic component suggests itself. There are considerable practical difficulties in carrying out this measurement, however, since a very high degree of linearity is required from the modulating and amplifying circuits, and the input signal must be very pure. Details of a satisfactory circuit have been published recently [*Branner et al.*, 1963]. An alternative approach, to amplitude-modulate the applied ac potential, leads to an output signal proportional to the second derivative at the modulating frequency. This technique has been developed by *Malyshev and Fedorov* [1953], who modulated their carrier with a sinusoidal signal, and by *Boyd and Twiddy* [1959], who employed square-wave modulation. Direct differentiation of the probe characteristic can be carried out electronically if the probe potential is swept by a sawtooth waveform [*Crawford and Freeston*, 1963], but considerable difficulties are encountered if the measurement is to be made in the presence of noise. If a second probe is available, this effect can be discriminated against [*Crawford et al.*, 1964].

Considerable simplification of (4) and (5) is possible if the electron velocity distribution is Maxwellian. Then, in the electron-repelling region we have for the probe current

$$i = i_e \exp [V/V_e] - i_i \qquad (6)$$

where i_e and i_i are the electron and ion saturation currents, and V_e is the electron temperature. The probe potential is measured relative to space potential and will be negative in the electron-repelling region. It can be shown that (4) becomes

$$i_0 = i_e I_0(V_1/V_e) - i_i \qquad (7)$$

where I_0 is the modified Bessel function of the first kind [*Takayama et al.*, 1960; *Crawford*, 1963]. This expression is exact provided that the ac signal does not drive the probe into the electron saturation region at any time. If this occurs, a more complicated analysis is required [*Garscadden and Emeleus*, 1962; *Crawford*, 1963; *Boschi and Magistrelli*, 1963].

The fundamental component of (5) can be expressed in terms of a conductance $G(V_0)$ given by

$$G(V_0) = \left(\frac{i_e}{V_e}\right) \cdot \exp \left[\frac{V_0}{V_e}\right]\left\{1 + \frac{1}{8}\left(\frac{V_1}{V_e}\right)^2 + \cdots\right\} \qquad (8)$$

For $V_1 < 0.3V_e$, this conductance can be approximated to within 1 per cent error by

$$G(V_0) = \left(\frac{i_e}{V_e}\right) \exp \left[\frac{V_0}{V_e}\right] = \frac{i + i_i}{V_e} \qquad (9)$$

This expression suggests the possibility of obtaining both the electron temperature and the ion saturation current from a plot of $G(V_0)$ against i. We shall postpone further discussion to section 3.

B. *LF probes.* In this frequency range, it will be necessary to take account of the displacement current i_d that flows while the probe surface charge Q adjusts to follow the time-varying potential. We have

$$Q = CV \qquad (10)$$

where C is the probe/plasma capacitance. Hence

$$i_d = \frac{d(CV)}{dt} = \frac{C\,dV}{dt} + \frac{V\,dC}{dt} \qquad (11)$$

The quantity (dC/dt) will not be zero, since the charge distribution which determines C is changing in response to the potential variations. Since we have postulated that the electrons can follow instantaneously, C is a function of V only, and (11) can be written

$$i_d = \frac{d(CV)}{dV}\frac{dV}{dt} = C_e(V)\frac{dV}{dt} \qquad (12)$$

where we have introduced a voltage-dependent, effective probe/plasma capacitance $C_e(V)$. If we assume, as before, that V consists of a dc term V_0 and a small sinusoidal signal $V_1 \sin \omega t$, we can expand by Taylor's theorem to obtain for the dc component of $C_e(V)$

$$C_{e0} = C_e(V_0) + \frac{V_1^2}{4} C_e''(V_0) + \frac{V_1^4}{64} C''''(V_0) + \cdots \qquad (13)$$

In the small-signal limit, (13) becomes

$$i_d = C_e(V_0)\, dV/dt \qquad (14)$$

and we see that the effective admittance of the probe is

$$Y(V_0) = G(V_0) + j\omega C_e(V_0) \qquad (15)$$

For measurement of the admittance to be useful as a diagnostic technique, it will be necessary for $C_e(V_0)$ to be easily expressible as a function of the electron temperature and charge density. An illustrative example is given in section 3, showing that this may be so under some conditions.

We have supposed that the electronic conduction current is the same in this region as in the VLF range so that measurements of the incremental dc component should still be valid for obtaining second derivatives of the probe characteristic. This will not be true of the second harmonic method unless attention is paid to the phase of the output signal. From consideration of the expansions of (5) and (13), we see that the second harmonic component is given to first order by

$$i_2(t) = -\frac{V_1^2}{4}[f''(V_0) \cos 2\omega t - 2\omega C_e'(V_0) \sin 2\omega t] \qquad (16)$$

and that, if only the amplitude is to be measured, it is required that

$$50[2\omega C_e'(V_0)/f''(V_0)]^2 < 1 \qquad (17)$$

for the error to be brought within 1 per cent.

C. *The resonance probe.* We have already shown that the incremental dc component of the electronic conduction current component will remain unchanged as frequency is varied through the VLF and LF ranges. It was first noted by *Takayama et al.* [1960] that as the frequency of the modulating signal is raised the resonance effect shown in Figure 1 occurs. Preliminary measurements of the static probe characteristic suggested that the resonance occurred at the local electron plasma frequency ω_{pe} given by

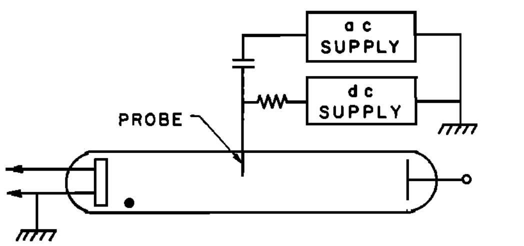
$$\omega_{pe}^2 = n_e q^2/\epsilon_0 m \qquad (18)$$

where n_e is the electronic number density, q and m are the electronic charge and mass, and ϵ_0 is the permittivity of free space.

A first attempt at an analysis of the resonance effect for an infinite plane probe was made by *Ichikawa and Ikegami* [1962] and was extended later by *Ichikawa* [1963]. The grossly simplified model used led to the conclusions that resonance should occur at ω_{pe} and that the sharpness of the resonance depends on the combined effects of Landau damping and electron-neutral collisions.

It is not immediately obvious that the resonance should occur at ω_{pe}, nor does it in fact generally do so. The phenomenon is best understood physically as a series resonance effect in the plasma. At resonance the RF electric field in the sheath becomes high and causes an increased rectified component to flow. To describe it completely, however, would require an electron trajectory analysis considering the combined influences of the dc and ac electric fields and taking into account finite probe geometry. This is an extremely complicated problem. It has, however, been attacked with considerable success by *Harp* [1963] by direct measurement, and also numerically using some computed results of *Pavkovich* and *Kino* [1964]. Pavkovich and Kino analyzed the distribution of a RF electric field in a semi-infinite plasma having a wall sheath in which the dc potential distribution was taken to be parabolic. Measurements by *Harp and Kino* [1964], using an electron beam probing technique in a low-density mercury-vapor discharge to determine dc and RF electric field strengths, showed that the assumption about the dc potential profile is good and indicated close agreement between measured and computed values of RF electric field components both in phase and in quadrature with the injected RF current.

Figure 2 shows a typical result for the quadrature component computed by *Pavkovich and Kino* [1964]. The loss component is relatively small and is not shown. The field at large distances is constant and is inversely proportional to the



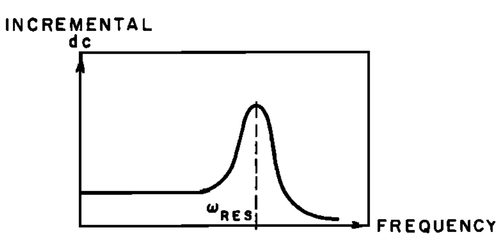


Fig. 1. Resonance probe characteristics.

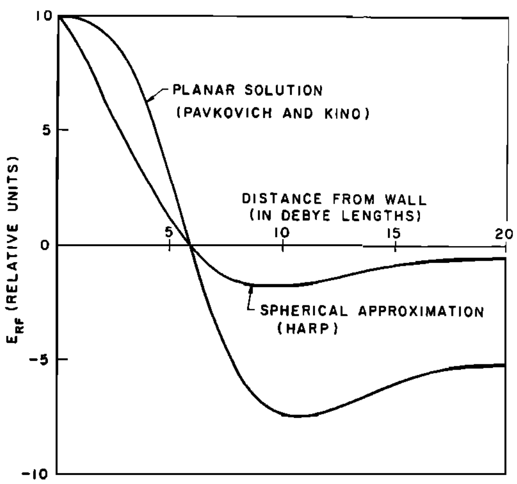


Fig. 2. RF sheath fields.

zero-temperature dielectric constant of the plasma. We note that the quadrature component of potential could be obtained by integration of the electric field. This would, however, lead to an infinite result in the case of the semi-infinite plane plasma. *Harp* [1963] pointed out that, for the finite probes used experimentally, the electric fields will fall off approximately inversely as the square of distance at distances comparable to the probe dimensions, and that the infinite potential result will not occur in practice. It will also disappear in the case of bounded plasmas.

As an illustrative example, consider a spherical probe of radius a, equal to $(9\lambda_e)$, where λ_e is the electronic Debye length. This implies approximate equality between the radius and the sheath thickness. *Harp* [1963] modifies the computed curve for planar geometry by multiplying it by (a^2/r^2) to give the approximate curve applying to spherical geometry shown in Figure 2. Integration of this yields a finite value for the potential. If this is carried out for a range of variation of the parameter (ω/ω_{pe}), the resonant point, at which the potential is zero, can be determined. In the case under consideration, this occurs at $(0.6\omega_{pe})$, and experimental measurements under appropriate conditions have confirmed the result to within an accuracy of 10 per cent. The following empirical formula is deduced by *Harp* [1963] to generalize this example to different probe radii:

$$\omega_{\mathrm{res}} = \frac{\omega_{pe}}{\{1 + (a/5\lambda_e)\}^{1/2}} \qquad (19)$$

For the frequencies above about $(0.5\omega_{pe})$, the results of *Pavkovich and Kino* [1963] show an increasing component of RF electric field in phase with the current. This is a collisionless effect equivalent to Landau damping and would probably preclude observation of the resonance for probes of radius smaller than about $(2\lambda_e)$.

The theory has been further extended by *Harp* [1964] to show that where Landau damping is negligible the half-power width of the resonance gives the electron-neutral collision frequency and to describe the behavior of the probe at potentials other than floating potential. His conclusions can be used to explain the corresponding experimental results obtained by *Cairns* [1963] and by *Peter et al.* [1964]. It has also been possible to extend the treatment to cover the behavior of the probe in a static magnetic field and to explain the resonant frequency shifts observed under these conditions by *Uramoto et al* [1963].

3. A Space Application of the LF Probe

A. *Explicit forms of the parameters.* It was pointed out in section 2B that, for the admittance measurement to be useful, a relatively simple analytic form should be available for the effective probe/plasma capacitance $C_e(V_0)$. In this section, we shall derive a very much simplified expression for a plane satellite- or rocket-borne probe of area A, flush-mounted normal to the direction of motion. Conditions will be such that the vehicle velocity u greatly exceeds the ion thermal velocity but is much below that of the electrons [*Jastrow and Pearse*, 1957]. This means that the ion current swept up by the probe is unchanging with potential and is given by

$$i_i = An_i qu \qquad (20)$$

By assuming a Maxwellian electron velocity distribution and ignoring secondary emission effects we can derive the floating potential of the probe V_f from

$$i_i = i_e \exp(V_f/V_e) \qquad (21)$$

where

$$i_e = An_e q(qV_e/2\pi m)^{1/2} \qquad (22)$$

and we can assume charge neutrality ($n_e = n_i \equiv n_0$) in the undisturbed plasma.

To determine the probe/plasma capacitance, only changes in electron surface charge need be calculated: the vehicle motion ensures that the ion current is unaffected by potential. Hence, changes in the sheath and total surface charge with potential can result only from electron displacements.

Poisson's equation in one dimension is

$$d^2V_0/dx^2 = -\rho/\epsilon_0 \qquad (23)$$

where ρ is the net positive charge density, and x is measured from the probe surface. Now the ion density in the space-charge sheath is constant at the ambient value, so that (23) becomes

$$\frac{d^2V_0}{dx^2} = \frac{n_0 q}{\epsilon_0}\left[\exp\left(\frac{V_0}{V_e}\right) - 1\right] \qquad (24)$$

which can be integrated to give

$$\left(\frac{dV_0}{dx}\right)^2 = \frac{2n_0 qV_e}{\epsilon_0}\left[\exp\left(\frac{V_0}{V_e}\right) - 1 - \frac{V_0}{V_e}\right] \qquad (25)$$

Consider now the electric displacement D in the plasma. We have

$$\frac{dD}{dV_0} = \frac{dD}{dx}\frac{dx}{dV_0} = \frac{\rho}{(dV_0/dx)} \tag{26}$$

If this is evaluated at the probe surface ($x = 0$), we obtain

$$\frac{C_e(V_0)}{A} = \left[\frac{\rho}{(dV_0/dx)}\right]_{x=0} \tag{27}$$

Substitution in (26) yields

$$C_e(V_0) = \left(\frac{A\epsilon_0}{2^{1/2}\lambda_e}\right) \cdot \frac{\{1 - \exp(V_0/V_e)\}}{\{(-V_0/V_e) - [1 - \exp(V_0/V_e)]\}^{1/2}} \tag{28}$$

where λ_e has been written for the Debye length $(n_0q/\epsilon_0V_e)^{1/2}$.

The expression of (28) can be approximated by

$$C_e(V_0) = \left(\frac{A_0\epsilon_0}{2^{1/2}\lambda_e}\right)\left[\left(-\frac{V_0}{V_e}\right) - 1\right]^{1/2} \tag{29}$$

The comparison with the expression of (28) is shown in Figure 3. For $(-V_0/V_e)$ greater than 3.25, the error will be less than 5 per cent. For values greater than 5, the error is below 1 per cent.

Strictly, Boltzmann's law for the electron density distribution can only be used in (24) and (25) for large negative values of (V_0/V_e). If the exact expressions are employed, the exponential factors in (28) are both multiplied by 0.5. Expression 29 is unaffected, however.

At floating potential we have, from (20)-(22),

$$-\frac{V_f}{V_e} = \ln\left[\frac{1}{u}\left(\frac{qV_e}{2\pi m}\right)^{1/2}\right] \tag{30}$$

and (29) becomes, finally,

$$C_e(V_f) = \left(\frac{A\epsilon_0}{2^{1/2}\lambda_e}\right)\left\{\ln\left[\frac{1}{ue}\left(\frac{qV_e}{2\pi m}\right)^{1/2}\right]\right\}^{-1/2} \tag{31}$$

For the conductance under similar conditions, (9) and (20) yield

$$G(V_f) = \frac{An_0qu}{V_e} = \frac{A\epsilon_0u}{\lambda_e^2} \tag{32}$$

These can be combined to give an expression for the electron temperature:

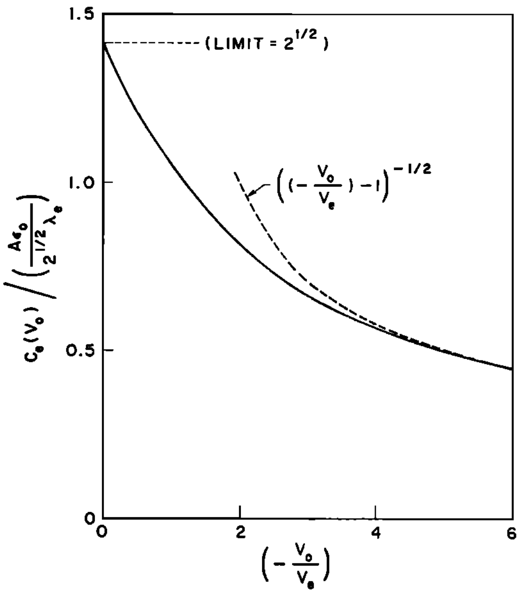


Fig. 3. Approximation to probe capacitance variation.

$$V_e = \frac{2\pi mu^2}{q}\exp\left[\frac{A\epsilon_0G(V_f)}{uC_e(V_f)^2}\right] \tag{33}$$

After we have determined the electron temperature, substitution in (32) will yield the charge density.

B. *Errors.* It is important to see that the simplification of (13) which we have used is in order. By using (29), we have

$$C_e''(V_0) = \frac{3C_e(V_0)}{4V_e^2[(-V_0/V_e) - 1]^2} \tag{34}$$

If V_1 is put equal to $(0.3V_e)$, the value for which errors in the conductance measurement reach 1 per cent, then the first-order correction in (13) is less than 1 per cent for $(-V_0/V_e)$ greater than 2.3. The susceptance measurement is, therefore, much less sensitive than the conductance measurement to finite signal amplitude effects.

We can also use the expression of (29) to assess the errors in the Druyvesteyn analysis described by (17). Upon substituting the appropriate expressions, assuming for illustration that the electron velocity distribution is Maxwellian, we obtain

$$\frac{\omega}{\omega_p} < \frac{[(-V_0/V_e) - 1]^{3/2}\exp(V_0/V_e)}{(50\pi)^{1/2}} \tag{35}$$

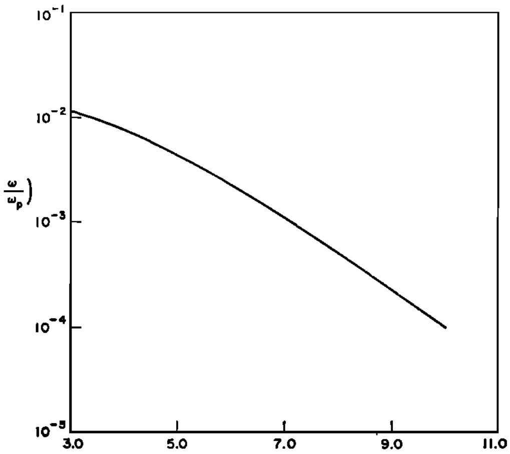


Fig. 4. Limits of accuracy of the Druyvesteyn method.

The error increases, then, with increasing $(-V_0/V_e)$, for a given applied frequency. Figure 4 shows the variation of (ω/ω_p) with maximum $(-V_0/V_e)$ measurable for the 1 per cent level to be maintained.

C. *Choice of working frequency.* Before leaving the low-frequency probe, we can calculate an appropriate working frequency. The admittance measurements will probably be most accurate when real and imaginary parts are equal. This gives

$$1 = \frac{G(V_0)}{\omega C_e(V_0)} = \left(\frac{\omega_{pe}}{\omega\pi^{1/2}}\right) \frac{\exp(V_0/V_e)[(-V_0/V_e) - (1 - \exp(V_0/V_e))]^{1/2}}{1 - \exp(V_0/V_e)} \qquad (36)$$

where (9) and (28) have been used. If measurements are to be made at space potential, the expression yields a working frequency of $(0.4\omega_{pe})$ and measurements could probably be made at this frequency of densities up to 100 times higher, when $[G(V_0)/\omega C_e(V_0)]$ could reach 10.

The range of measurement is widened considerably if the probe is at floating potential, or below. For a typical satellite orbit, $u \approx 8.10^5$ cm/sec and $V_e \approx 0.15$ ev. Use of (20)–(22) gives equal real and imaginary components at a working frequency of about $(0.1\omega_{pe})$ for a probe at floating potential. Measurements could be made under these circumstances for densities up to 100 times higher and 10 times lower.

If the range of electron densities expected is greater than 3 orders of magnitude, a possible solution would be to measure the conductance at one frequency, and the susceptive component at another, rather higher, frequency. The choice of frequencies and amplitudes would then have to be made carefully for cross-modulation components to be negligible.

It is important to note from (36) that VLF conductance measurements will be complicated by the presence of an admittance component for large values of $(-V_0/V_e)$. Simplifying, and expressing (36) in terms of the ion plasma frequency, gives

$$\frac{\omega}{\omega_{pi}} = \left(\frac{M}{\pi m}\right)^{1/2} \frac{[-(V_0/V_e) - 1]}{\exp(-V_0/V_e)} \qquad (37)$$

where M is the ion mass. Assuming that the most likely ion is O^+, and that it would be required to vary $(-V_0/V_e)$ up to 10, yields a working frequency of $(0.013\omega_i)$. Measurements could be made for all higher densities and for densities up to 2 orders lower when $[G(V_0)/\omega C_e(V_0)]$ would reach 0.1.

4. Discussion

We have tried to give in this paper an indication of the wide variety of ways in which the response of Langmuir probes to periodic waveforms can be interpreted to yield information either on laboratory plasmas or on charged particles in space, and in particular we have shown that it is feasible to obtain electron temperature and charge density data from measurements of probe terminal admittance. The techniques involved can be divided, in general, into those capable of giving continuous measurements of the relevant parameters and those in which some sweeping action (of frequency or probe potential, for example) takes place and a finite time is required for the sampling. Measurements of capacitance at frequencies well above the local electron plasma frequency and admittance measurements at a fixed probe potential come into the former category; the resonance probe and Druyvesteyn determinations of electron velocity distributions fall within the latter.

Of the methods involving sweeping, the resonance probe is particularly attractive. The success of *Harp*'s [1963] analyses suggests that this method can be developed into a powerful tool for plasma studies. There still remain to be explained one or two additional resonances that have sometimes occurred at rather higher frequencies than the main resonance. It seems likely that these are due to standing-wave effects in the finite geometry of the experimental system or are related to the well-known Tonks-Dattner resonances which have been explained recently [*Crawford and Kino*, 1964; *Nickel et al.*, 1963].

As far as the Druyvesteyn method is concerned, the considerations of section 3B apply to such applications as the recent measurements with the Ariel satellite [*Willmore et al.*, 1963]. That is, the working frequency must be limited or the dc component must be measured for errors to be avoided.

The measurement of terminal admittance has some important advantages over the use of a static probe. For direct measurements in space, the structure can be a long thin cylinder with sufficient collecting area to measure very low densities, say 1 to 10 electrons/cm^3. The instrumentation is very simple, and conventional impedance measuring techniques can be used. Some measurements have already been made in the lower ionosphere using a split dipole antenna as the structure [*Mlodnosky and Garriott*, 1963]. This probe could certainly be applied to the study of transient laboratory plasmas, but difficulties may be encountered under certain conditions in space. For example, the presence of a large flux of very energetic particles, such as Van Allen belt electrons, may alter the static probe current and floating potential, thereby changing the measured values of admittance. Also, for some collector structures, a complicated space-charge sheath forms because of effects such as the earth's magnetic field, photoemission, or motion of the vehicle, and it may not always be possible to derive as simple an explicit form for the effective capacitance as that of section 3A. Under these conditions, the conductance may still be calculable with considerable confidence, however, and valuable information can be obtained, as described in section 2A, by measuring the conductance as the probe potential is swept. One special advantage of this technique is that, for all geometries of the probe structure, space potential appears as a maximum in the conductance and is measured much more precisely than by a static probe characteristic.

The most favorable conditions for use of the admittance method are clearly those in which the flux of very energetic particles is small and the expression for the susceptance can be determined in a relatively simple form. This method will be most advantageous when the electron thermal velocity is very much greater than the vehicle velocity. In the example of section 3A the simplified form (equation 29) is only applicable to values of $(-V_0/V_e)$ greater than about 4. The satellite example given in section 3C implies a value of 2.1, and it would be necessary to use an unsimplified form of (29) in the data reduction or to bias the probe.

Our description of the various frequency ranges of probe operation was based on an arbitrary neglect of either or both of the charged particle mobilities. It seems very likely that at all frequencies substantially below the electron plasma frequency the electron mobility can be safely neglected and that the ion mobility is negligible far below the ion plasma frequency. The regions where these assumptions begin to break down have received some attention recently. At high frequencies, the experiments on resonance probes are relevant, while extensive studies have been carried out by *Oskam et al.* [1964] to determine the response of low-pressure discharges to square-wave and step-function inputs. Their first-order analysis confirms the importance of ion mobility and space-charge sheath capacitance. An experimental study giving very good agreement with the calculated probe response to square waves and sinusoidal signals fed to it through a series capacitance has been published by *Butler and Kino* [1963].

Further work in this field is still required. For example, it is clear that except for very special conditions, such as those of section 3, the time-averaged ion current can, in general, vary between the VLF and the LF ranges. Some detailed study of this variation and of any resonances that may occur would be valuable.

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