Traveling Wave Amplification of Whistlers

N. M. BRICE

Stanford University
Stanford, California

Traveling wave amplification has been postulated by *Helliwell* [1956] and by *Gallet* [1959] as a possible generation mechanism for audio-frequency ionospheric emissions. The present note discusses the application of the theory of the traveling wave tube [*Chodorow and Susskind*, 1960] to these frequencies with particular reference to whistlers and whistler echo trains.

In a traveling wave tube, the magnitude and direction of the electric field initially seen by the particle depends on the phase of the wave when the particles are injected, so that some of them will be speeded up and some slowed down. Bunching tends to occur in a range around the point where the field is passing through zero from accelerating to decelerating, as seen when moving forward through the wave.

If the electrons have a slightly greater average velocity than the wave, then the particles bunched around the point of zero field will tend to drift forward into the region of retarding potential, so that they are slowed and lose energy. This energy is transferred to the electromagnetic field, increasing the amplitude of the wave. Conversely, if the particles have a slightly smaller average velocity than the wave, then the latter will be decreased in amplitude.

We now consider these principles as they are applied to audio-frequency waves in the ionosphere for particles with constant velocity in the direction of the magnetic field. The wave, in general, will have a component of electric field in the direction of the magnetic field [*Helliwell*, 1956] so that particles moving through the wave will be accelerated and decelerated. On the average, they will neither gain nor lose energy. If, however, the wave phase velocity becomes matched to the particle velocity, then bunching of the particles will occur. The matching condition is that these velocities are sufficiently close for the acceleration or deceleration during one-half cycle of the wave to make them equal.

At the time of trapping the particles have, on the average, the same velocity as the wave. Thus for a fixed frequency wave in a homogeneous medium, no change in amplitude of the wave would occur due to this mechanism. We may expect the wave to be amplified or absorbed according as the phase velocity is decreasing or increasing, respectively, at the point in the wave at which the particles are trapped. This change in velocity may be due to change in the properties of the medium, or, for a wave whose frequency changes with time, to the change in frequency of the wave where the particles are trapped. It will, in general, be a combination of both of these effects. The change in particle velocity may be expressed as

$$dv = \left.\frac{\partial v_p}{\partial s}\right|_f ds + \left.\frac{\partial v_p}{\partial f}\right|_s df \qquad (1)$$

where

v_p = the wave phase velocity.
v = the particle velocity.
f = the wave frequency.
s = distance measured in the direction of particle travel.

The necessary condition for possible amplification, then, is that dv be negative so the particles are decelerated.

The change in frequency df is illustrated in Figure 1 and arises as follows. For whistler mode propagation, the wave phase and group velocities will, in general, be different.

$$v_g = 2v_p(1 - f/f_H) \qquad (2)$$

where

v_g = the wave group velocity.
f_H = the electron gyrofrequency

The particles traveling at the phase velocity are thus continually moving through the wave (except where $f = \frac{1}{2}f_H$), so that if the fre-

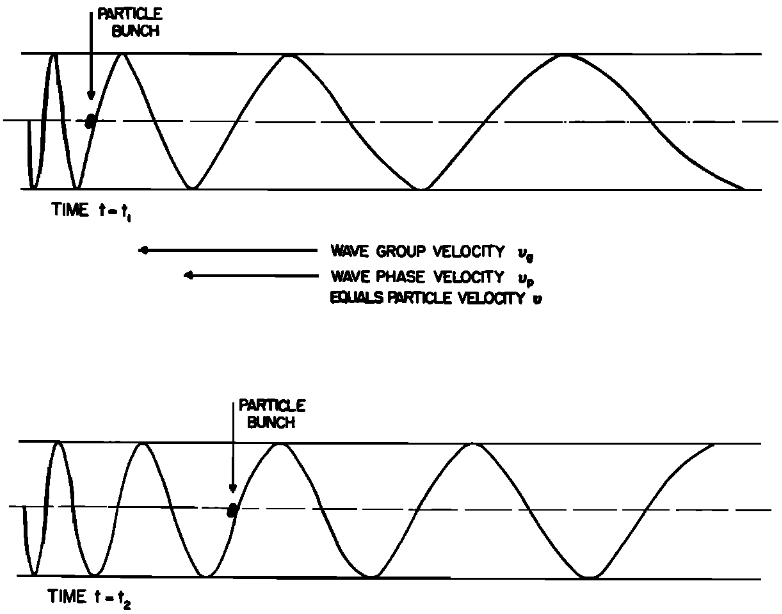

Fig. 1. A schematic diagram of the electric field of a whistler and a trapped bunch of particles at two different times, illustrating that the particles move with respect to the wave group while remaining bunched at a zero crossing of the field.

quency of the wave is changing, so also is the frequency where the particles are trapped.

In traveling a distance Δs, the particles will be delayed with respect to the wave by an amount

$$\Delta t = \frac{\Delta s}{v_p} - \frac{\Delta s}{v_g} \tag{3}$$

so that the corresponding change in frequency is given by

$$\Delta f = \left(\frac{1}{v_p} - \frac{1}{v_g}\right)\frac{\partial f}{\partial t}\,\Delta s = \frac{(1 - 2f/f_H)}{2v_p(1 - f/f_H)}\frac{\partial f}{\partial t}\,\Delta s \tag{4}$$

Now v_p may be expressed as

$$v_p = cf^{\frac{1}{2}}(f_H - f)^{\frac{1}{2}}/f_0 \tag{5}$$

so that

$$\frac{\partial v_p}{\partial f} = \frac{v_p(1 - 2f/f_H)}{2f(1 - f/f_H)} \tag{6}$$

From (4) and (6) we obtain

$$\frac{\partial v_p}{\partial f}\,df = \frac{(1 - 2f/f_H)^2}{4f(1 - f/f_H)^2}\frac{\partial f}{\partial t}\,ds \tag{7}$$

Note that the sign of this expression is dependent only on the sign of $\partial f/\partial t$ so that if the wave frequency decreases with increasing time, then in a homogeneous medium any trapped particles may amplify the wave via the traveling wave mechanism.

For a whistler, the trapping conditions depend on the magnitude of the wave frequency compared with the instantaneous nose frequency [*Helliwell, Crary, Pope, and Smith,* 1956] and half the electron gyrofrequency, both of which vary during the propagation. It is not proposed to consider all possible situations here. In the simplest and most common case, the wave frequency is below both the nose and half the gyrofrequency. Then the wave group velocity exceeds the phase velocity, the latter decreasing with frequency. Particles with velocities in the range of interest move back with respect to the wave until they reach a frequency where they are trapped. The bunching process then begins. For a medium that is symmetrical about the top of the path, those frequencies with $\partial f/\partial t$ negative will have a bias in favor of amplification. Further, in a whistler train, $|\partial f/\partial t|$ decreases with increasing number of hops; the asymmetry in dv, or bias in favor of amplification, decreases accordingly. This feature offers an explanation for the increase of amplitude of the first few echoes of a train [*Storey*, 1953].

Detailed calculations have been made for a model electron-density distribution in which the electron density is proportional to the magnetic field. These results and evidence of amplification in a whistler train will be published at a later date.

Acknowledgment. I wish to express my thanks to Dr. R. A. Helliwell of this department for helpful criticism of this work, which was sponsored by U. S. Air Force research contract AF 18(603)-126.

References

Chodorow, M., and C. Susskind, *Fundamentals of Microwave Tubes,* McGraw-Hill, New York (in press).

Helliwell, R. A., Low frequency propagation studies, part I: Whistlers and related phenomena: Final report, Contract AF 19(604)795, June 15, 1953, to September 30, 1956 (revised, May 28, 1958), AFCRC-TR-56-189, ASTIA Document AD 110184, 1956.

Helliwell, R. A., J. H. Crary, J. H. Pope, and R. L. Smith, The 'nose' whistler—a new high latitude phenomenon, *J. Geophys. Research, 61,* 139-142, 1956.

Gallet, R. M., The very low frequency emissions generated in the ionosphere, *Proc. IRE, 47,* 211-231, 1959.

Storey, L. R. O., An investigation of whistling atmospherics, *Phil. Trans. Roy. Soc. London, A, 246,* 113-141, 1953.

(Received August 1, 1960.)