THE EFFECTS OF GROUND-BASED
VERY LOW FREQUENCY TRANSMITTERS ON THE
IONOSPHERE AND MAGNETOSPHERE

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF
ELECTRICAL ENGINEERING
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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Speaking frankly and speaking the truth are two different things entirely. Honesty is to truth as prow is to stern. Honesty appears first and truth appears last. The interval between varies in direct proportion to the size of the ship. With anything of size, truth takes a long time in coming. Sometimes it only manifests itself posthumously. Therefore, should I impart you with no truth at this juncture, that is through no fault of mine. Nor yours.

–Haruki Murakami, A Wild Sheep Chase
Abstract

Energetic electrons trapped in the Earth’s radiation belts are a threat to sensitive electronics of manmade spacecraft. Both natural and manmade events can greatly increase trapped energetic electron fluxes, motivating the search for a method of radiation belt remediation to mitigate the danger of such an event. Resonant interaction with electromagnetic whistler mode waves is the dominant mechanism for energetic electron removal from the radiation belts at higher altitudes, and the source of whistler mode waves that humans can most readily construct and control is the ground-based very low frequency (VLF) transmitter. Several powerful VLF transmitters exist for the purposes of naval communications, and studies suggest that they may have a significant effect on the dynamics of the Earth’s radiation belts. In an effort to better quantify these effects, a series of controlled modulation experiments are conducted with the 21.4 kHz, 424 kW transmitter NPM located in Lualualei, Hawaii.

Sub-ionospheric VLF remote sensing is used to detect the effects of the transmitter NPM. When VLF waves scatter energetic electrons from their trapped motion in the radiation belts, those electrons precipitate upon the upper atmosphere, producing secondary ionization which modifies the conductivity of the lower ionosphere. This ionospheric disturbance perturbs a sub-ionospheric VLF probe signal propagating through the region, facilitating remote detection of the precipitation. In addition to the effects of VLF transmitters on the radiation belts, however, such transmitters also directly heat the electron population of the lower ionosphere. This heating represents a confounding ionospheric disturbance in these experiments but is also of direct importance to radio wave propagation both below and through the ionosphere. While initial observations recorded during the NPM experiments suggested the detection
of transmitter-induced electron precipitation, improved signal processing illuminates the lack of onset delay in those observations and eliminates transmitter-induced precipitation as a possible cause. Thorough laboratory testing eliminates the possibility of instrumental cross-modulation influencing the results, which leaves ionospheric heating by NPM as the most probable physical explanation. An observed seasonal variation in the detected probe signal perturbations along with the lack of correlation with geomagnetic activity support this conclusion. Arrival azimuth and theoretical analyses of the probe signal scattering geometry both suggest that the VLF probe signal scatters not from the intense off-path ionospheric heating nearby the transmitter NPM but rather from relatively weak ionospheric heating that extends laterally over the probe signal pathway nearly 2,000 kilometers from NPM.

A large-scale computational modeling framework is assembled to theoretically analyze the extended lateral ionospheric heating generated by NPM and the perturbation this heating would induce upon a VLF probe signal. The fields radiated by NPM within the Earth-ionosphere waveguide are computed to radial distances of 5,000 kilometers in each direction, and the heated electron temperature is computed at each point. Propagation of the VLF probe signal along its pathway through this heated ionosphere estimates the probe signal perturbation for comparison to experiment. The computational model confirms theoretically that this form of ionospheric heating can account for the observed probe signal modulations, establishing that the lateral extent of ionospheric heating due to VLF transmitters is several thousand kilometers, significantly greater than previously recognized.

While sub-ionospheric detection techniques succeeded only in detecting the effects of ionospheric heating, theoretical analysis and satellite-based detection still facilitate study of transmitter-induced precipitation. The trans-ionospheric attenuation of VLF waves is a critical component in the process of inducing precipitation with a ground-based VLF source, and trans-ionospheric propagation has only just recently been accurately estimated with an experimentally-validated model. We provide updated estimates for trans-ionospheric attenuation of VLF waves both for the case of total magnetospheric injection from a ground-based VLF transmitter and for
the case of a VLF whistler-mode plane wave vertically incident upon the lower ionosphere. We also assess the various factors affecting the application and interpretation of such estimates. For the satellite-based detection during the NPM experiments, coordinated observations recorded onboard DEMETER satellite provide evidence of NPM-induced precipitation, but instrumental shortcomings for this application prevent robust quantification of those effects.
Acknowledgments

I first visited Stanford in the Spring of 2007, met Umran, and joined Morris, Praj and Nader for dinner at Illusions. Eager to become part of this eclectic mix of brilliantly energetic personalities and unique research opportunities, I moved to Stanford early that Summer to jump into the VLF Group before beginning the notoriously excessive curriculum of Stanford graduate classes. The first year and a half was astounding: weekly meetings both with Umran and with the group kept me engaged in this new world of research, applicable courses complemented my work and helped me grow, and a bevy of welcoming labmates and a familial atmosphere fostered many friendships and collaborations. I talked at my first conference, assembled my first publication, and passed quals. I also joined in and helped organize group events and sports teams. Oh, and my research took me to, among other places, Alaska for two weeks, Antarctica for five weeks, and the Indian Ocean for four. The origins of that last one exemplify my first year of working with Umran: in one of our weekly meetings, Umran held up a globe and said “what we’d really like is to have a receiver here,” as he pointed to the middle of the Indian Ocean. Not knowing any better, I found two islands near that location, discovered them to be French territories housing only wildlife and small research stations, and reported back to Umran. It turns out Umran knew people in the controlling French organization. A mere month later, simultaneously taking Stanford classes and on my very first solo VLF Group mission, I found myself as the only native English-speaker on a French boat departing from Madagascar for a month-long journey to the islands of Crozet, Kerguelen and Amsterdam. The VLF Group was a crazy place.
The VLF Group experience simmered down a bit after Umran assumed his position as President of Koç University in Turkey. What once was a small army of over 30 steadily decreased to a more traditional size of \( \sim 10 \) and stands to continue its reduction. Some of the hustle and bustle is gone, and it is sad to witness the impending end of an era, but the VLF Group remains a productive leader in its field even in its twilight years. The dedication of the VLF Group’s leadership (not just from Umran, but from every senior researcher and staff member of the VLF Group) has been truly admirable in its effort to remain a world-class organization and provide an uncompromisingly elite experience for each student guided along their path toward graduation. Would we remain today a stronger presence in the fields of space and radio sciences if Umran had not accepted a position elsewhere? There’s no question. Are there occasional feelings of unease or even betrayal that could have been avoided? Undoubtedly. But decades pass and circumstances shift. Ten years is a long time to expect no change to occur in the world around you. The VLF Group (though not always by that name) has remained a stalwart figure for over fifty years, with Umran at its lead for over twenty. Life is a series of choices; no one could chagrin Umran for his decision to lead an entire university in an education system seeking his guidance, and the Stanford EE department seemed long set on heading its research efforts in a new direction. I am thankful for Umran’s undying dedication to his students – that through this transition he has spared no effort in guaranteeing that each of his students still has every opportunity to succeed at Stanford and graduate with their PhD. Aside from my family, there is no group in my life of which I am more proud to have been a member than the VLF Group. Thank you Umran, Maria, Forrest, DanDan, Bob, Morris, Jeff, Ben, Nikolai, Shaolan, Helen, Praj, Erin, Cecile, George, Nick, Nicholas, Dan, Ryan, Marek, Mark, Amanda, Jeremy, Nader, Denys, Naoshin, Rob, Andrew, Martin, Don, Tim, Tim Bell, Dave, David, Ivan, Justin, Can, Vijay, Fadi, Patrick, Drew, and every other member of the VLF Group past and present.
In a more traditional showing of gratitude, I owe huge thanks to Maria, my family, and Feng. Thank you Maria for patiently guiding me through my publications, my PhD and my job search. Thank you even more for your honesty and your friendship. Thank you Mom, Dad, Grandma, Granddad and all members of the Graf and Slein clan for all of your love and support (with a special thanks to Aunt Dorrie for your generous donations to my college funds). Thank you Feng for your love and companionship now and in the future. Last but not least, thank you Juan Valentin Rodriguez, for if you had not written this thesis in 1994, there is no way I could have written it again today.

**Kevin Graf**

*Stanford, California*

*November 2013*

This research has been supported by a William R. and Sara Hart Kimball Stanford Graduate Fellowship, by an Achievement Rewards for College Scientists Scholarship, by the Defense Advanced Projects Research Agency and the High Frequency Active Auroral Research Projects Agency under Office of Naval Research grants N00014-06-1-1036, N00014-03-1-0630 and N00014-05-1-0854 to Stanford University, by the Department of Air Force under award FA9453-11-C-0011 to Stanford University, and by the National Science Foundation under awards ANT-1043442, ANT-1141791 and ANT-0538627 to Stanford University.
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Chapter 1

Introduction

Anthropogenic radio transmissions in the very low frequency band (VLF, 3–30 kHz) are primarily utilized for long distance communication with naval vessels. VLF waves propagate in the waveguide formed by the Earth and the lower portion of the ionized upper atmosphere, known as the ionosphere. As the waves propagate within this waveguide, Joule heating and collisional losses heat ionospheric electrons and attenuate the VLF waves. A small fraction of the wave power propagates through the ionosphere into the near-Earth space environment where the waves can resonantly interact with energetic electrons that are trapped in Earth’s radiation belts as a result of the dipolar configuration of the geomagnetic field. One result of these resonant wave-particle interactions can be the removal of the electrons through precipitation into the upper atmosphere. Ground-based VLF transmitters are believed to be a significant driver of electron losses in certain regions of the radiation belts, but detailed quantification of this process remains incomplete.

A series of naval VLF transmitter keying experiments was performed with the 424 kW, 21.4 kHz transmitter NPM in Lualualei, Hawaii between the dates of 25 August 2005 and 2 April 2008. The goal was to better quantify the effects of ground-based VLF transmitters on the Earth’s radiation belts. Specifically, the questions pursued were: How much energetic electron precipitation is induced by NPM; What are the global effects of ground-based VLF transmitters upon radiation belt dynamics; and Would an enhanced global network of ground-based VLF transmitters prove tenable?
as a method for radiation belt remediation? A combination of sub-ionospheric VLF remote sensing techniques and in situ satellite-based measurements applied throughout the NPM experiments provided insight into the effects of the transmitter on the radiation belts, but separate hindrances with each measurement technique ultimately prevented accurate quantification of those effects. Instead, observations provided a data base with which to illuminate the heretofore unrecognized lateral extent of ionospheric heating caused by VLF transmitters. In combination with work by Cohen and Inan [2012] and Cohen et al. [2012], the understanding and estimation of trans-ionospheric propagation of VLF waves are also greatly improved. This dissertation reports upon the NPM keying experiments, discussing the effects of ground-based VLF transmitters on the ionosphere and the radiation belts within the magnetosphere. This chapter first introduces the key components of the near-Earth space environment and the propagation and effects of VLF waves therein, and then presents the motivation for and scientific contributions of this dissertation.

1.1 The Near-Earth Space Environment

Near-Earth space extends from the upper atmosphere at \( \sim 60 \) km altitude to the outer edge of the magnetosphere at \( \sim 10 \) \( R_E \) on the dayside of Earth and over \( 50 \) \( R_E \) on the night side, where \( R_E = 6,371 \) km is the average radius of the Earth. Both cold (\(<1 \) eV) and hot (\( \sim 1 \) keV up to 10’s of MeV) plasma populations exist within the magnetosphere. As we go to higher altitudes, the gaseous upper atmosphere blends into the weakly-ionized plasma of the ionosphere, which merges into the fully-ionized plasma of the plasmasphere. The ionosphere and plasmasphere comprise the background cold plasma population within the magnetosphere while energetic charged particles trapped within the magnetosphere forming the radiation belts comprise the hot plasma population. Each of these components is illustrated in Figure 1.1. While the magnetosphere extends to great distances and hosts complex interactions with the solar wind and interplanetary magnetic field, this dissertation focuses on the relatively stable inner magnetosphere at altitudes less than \( 2 \) \( R_E \). A closer view of the components of this inner region is provided in Figure 1.2, and those components are
the focus of the discussion below. The texts Kivelson and Russel [1995] and Tascione [1994] provide more complete coverage of the near-Earth space environment.

Figure 1.1: Illustration of the components of the near-Earth space environment, depicting the interaction of the solar wind with the geomagnetic field of the magnetosphere and highlighting the magnetosphere, plasmasphere and radiation belts.

1.1.1 The Magnetosphere

The magnetosphere is defined as the region wherein the motion of charged particles is dominated by the Earth’s magnetic field as opposed to either collisional interactions with atmospheric molecules or the interplanetary magnetic field. Electrical currents flowing deep within the Earth generate an approximately dipolar geomagnetic field. The solar wind, which is high-speed plasma ejected from the Sun, travels towards
Figure 1.2: Illustration of the components of the near-Earth space environment most pertinent to this dissertation. The ionosphere is shaded in blue, the plasmasphere is shaded in yellow, the dipole magnetic field is traced in grey, and the inner radiation belt is shaded in green. Figure is not drawn to scale. Modified from Figure 1.3 of Golden [2011].

The Earth and interacts with the geomagnetic field to produce the general bullet-like shape of the magnetosphere shown in Figure 1.1. The interaction of the variable solar wind and interplanetary magnetic field can result in the injection of energetic particles into the magnetosphere as well as modifications of the geomagnetic field setting off complicated reactions affecting both wave and particle behaviors. This geomagnetic variability is most significant in the outer magnetosphere. While the inner magnetosphere can also be affected by geomagnetic variability, it tends to remain relatively stable [Tascione, 1994, Chapter 5].

Within the inner magnetosphere, the geomagnetic field is often approximated as
1.1. THE NEAR-EARTH SPACE ENVIRONMENT

A tilted dipole. Expressed in spherical coordinates:

\[ B_r = -2B_0 \left( \frac{R_E}{r} \right)^3 \sin \lambda \]

\[ B_\lambda = B_0 \left( \frac{R_E}{r} \right)^3 \cos \lambda \]

\[ B = \sqrt{B_r^2 + B_\lambda^2} = B_0 \left( \frac{R_E}{r} \right)^3 \sqrt{1 + 3 \sin^2 \lambda} \]  \hspace{1cm} (1.1)

where \( r \) is radial distance from the center of the dipole, \( \lambda \) is latitude measured from the magnetic equator, and \( B_0 = 3.12 \times 10^{-5} \) T is the mean geomagnetic field value at the Earth’s surface along the equator [Walt, 1994, p. 30]. The dipole field is symmetric about its axis so that \( B_\phi = 0 \) everywhere. While Equations 1.1 provide a reasonable approximation of the geomagnetic field in the inner magnetosphere for use in studies that benefit from a simplified analytical approach, better models exist which account for the offsets and irregularities of the geomagnetic field. The International Geomagnetic Reference Field (IGRF) [Macmillan and Maus, 2005] is a mathematical model based on averaged magnetic field data from both satellite and ground-based observations recorded throughout the magnetosphere and around the world and can be used to compute geomagnetic field values for any location and time for altitudes between 0 and 40,000 km.

1.1.2 The Ionosphere and Plasmasphere

The background cold plasma population comprises the ionosphere and plasmasphere within the inner magnetosphere. The ionosphere is a charge-neutral region of weakly-ionized plasma between approximately 60 and 1,000 km altitude. Plasma forms at higher altitudes due to the decreasing atmospheric density with altitude combined with the increased availability of incoming ionizing radiation [Ratcliffe, 1972, p. 16]. The density of the atmosphere is largely determined by the total weight of the atmosphere above that altitude, which leads to approximately exponentially decreasing atmospheric density with altitude in the upper atmosphere. Absorption of incoming ultraviolet and x-radiations from the Sun and other astronomical sources by the
upper atmosphere, together with nuclear reactions induced in the upper atmosphere by cosmic rays (particles with energies greater than 1 GeV), ionizes a fraction of the neutral particles therein. The absorption of incoming radiation by the atmosphere decreases the intensity of that radiation as it penetrates to lower altitudes where neutral particle densities are higher. The result is an increasing percentage of particle ionization at higher altitudes.

Ionospheric regions form in altitude due to the presence of several distinct neutral particle populations and incoming types of radiation. Each particle population interacts as described above with each incoming type of radiation. The lowest layer of the ionosphere, for example, occurs in the altitude range 60–90 km and is produced primarily by the most penetrating incoming radiation. This radiation, which is mostly short wavelength ultraviolet and x-radiation, penetrates the farthest into the upper atmosphere, thus forming the lowest region of the ionosphere [Tascione, 1994, Chapter 7]. The daytime ionosphere is generally divided into three regions: D (50–90 km), E (90–140 km), and F (above 140 km). The D region is often said to disappear at nighttime due to its relatively low ionization levels and subsequent lack of impact on >1 MHz radio waves, but it remains of critical importance to the propagation of VLF waves. These ionospheric regions are labeled in Figure 1.3 together with typical electron and neutral particle density profiles for the nighttime ionosphere. For comparison, the density of the air near the surface of the Earth is approximately $10^{19}$ cm$^{-3}$. Due to incoming solar radiation on the day side of the Earth, the electron density of the daytime ionosphere is approximately one to two orders of magnitude greater than the nighttime ionospheric electron density and has a modified altitude structure. As is discussed in Section 1.2, the increased electron density during the daytime leads to a significantly higher rate of attenuation of VLF electromagnetic waves and additionally diminishes the relative effects of small ionospheric disturbances. The effects of VLF transmitters on the ionosphere and inner magnetosphere are far more pronounced and detectable during the nighttime, and thus all experimental results presented in this dissertation are recorded under nighttime conditions.

The International Reference Ionosphere (IRI) [Bilitza and Reinisch, 2008] sponsored by the Committee on Space Research and the International Union of Radio
Science provides an empirical standard model of ionospheric charged particle densities and temperatures based on averaged data from ionosondes, incoherent scatter radars, topside sounders, and in situ instruments. The Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar Extended (NRLMSISE) empirical model [Picone et al., 2002] calculates composition, temperature and total mass density of the neutral atmosphere. While these reference models provide very good estimates for typical ionospheric profiles across date, time, location and geomagnetic conditions, the models are based on averaged observations and thus cannot fully capture day-to-day ionospheric variability. A statistical analysis of data from several in situ rocket studies showed the lower ionosphere nighttime electron density profile to often vary by a factor of five from the typical profile [Tao et al., 2010]. Ionospheric variability must be considered in any scientific study involving the ionosphere.

![Figure 1.3: Nighttime ionosphere electron and neutral particle density profiles. The approximate altitudes of the D, E, and F-regions of the ionosphere are labeled along the electron density profile.](image)

While the ionosphere is a collisional, weakly-ionized plasma with neutral particle densities often much larger than electron densities, the plasma above several thousand kilometers altitude is fully ionized and of sufficiently low density that the medium
can be considered collisionless. Cold plasma densities in the plasmasphere typically range from 10 to \(10^4\) cm\(^{-3}\), and the plasmasphere extends to an outer edge located anywhere between 3 and 7 \(R_E\) [Carpenter, 1963]. Plasma density drops sharply by several orders of magnitude beyond this altitude. The location of this sharp drop can influence very low frequency wave propagation, but the waves injected by ground-based VLF transmitters generally remain confined to the inner magnetosphere within 3 \(R_E\).

### 1.1.3 The Radiation Belts

The configuration of the Earth’s magnetic field is conducive to the trapping of energetic charged particles, forming a type of magnetic bottle capable of confining their motion. Energetic electrons become trapped in regions of the magnetosphere, constituting the radiation belts. An illustrative depiction of these belts is highlighted in Figure 1.1.

**Trapped Particle Motion**

Geomagnetically-trapped energetic charged particles undergo three basic types of motion: gyration, bounce and drift [Inan and Golkowski, 2011, Chapter 2]. The motion of a charged particle in a background magnetic field is governed by the Lorentz force equation \(\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})\), where \(q\) is the particle charge, \(\vec{v}\) is the particle velocity, \(\vec{B}\) is the background magnetic field, and \(\vec{E}\) is the background electric field. \(\vec{E}\) can be considered zero for the purposes of this discussion. A charged particle moving only across a background magnetic field gyrates around that magnetic field line due to the Lorentz force, remaining trapped circling in place. Any motion along the magnetic field line remains unaffected when the magnetic field is completely uniform in that direction. If the magnetic field intensifies so that field lines converge, however, the charged particle experiences a force away from the converging field lines. Within the dipole magnetic field of the Earth, a charged particle starting in the equatorial plane and moving along a field line encounters converging field lines when approaching either the north or south poles. Thus, a charged particle with motion both along and
across the geomagnetic field lines can remain trapped, gyrating around the field line while also bouncing back-and-forth along the line between the northern and southern hemispheres. Background magnetic field gradients and the curved path of the particle within the dipole magnetic field lead to the third form of motion: longitudinal drift of the charged particle to the west or east around the Earth. Since the Earth’s field currently points from South to North, positively charged particles drift west while negatively charged particles drift east. An energetic electron trapped in the Earth’s magnetic field gyrates around a magnetic field line as it bounces back-and-forth between the northern and southern hemispheres and drifts eastward around the Earth [Walt, 1994, Chapter 2]. These three basic motions are illustrated in Figure 1.4a. Typical time scales for each of these periodic motions for a \( \sim 100 \text{ keV} \) electron in the inner radiation belt are \( \sim 10^{-3} \text{ sec} \) for gyration, \( \sim 0.1 \text{ sec} \) for bounce, and \( \sim 10^3 \text{ sec} \) for drift.

![Figure 1.4: (a) The three periodic motions of geomagnetically trapped radiation: gyration, bounce and drift. Electrons drift to the east while protons drift to the west. (b) The helical electron trajectory along a magnetic field line with pitch angle \( \alpha \) labeled. Modified from Figure 2.7 of Bortnik [2004].](image)

While the Lorentz force equation could be used to completely describe the motion of a charged particle within the magnetosphere, applying it to determine the long-term future location of any particle would require numerical integration over many gyration and bounce periods of the particle. To avoid the numerical errors inherent in
such an integration and to provide more insightful analysis, adiabatic invariants are typically derived to describe the particle motion. An adiabatic invariant of a periodic motion remains effectively constant so long as any changes to the system occur slowly relative to the timescale of that periodic motion. The first two adiabatic invariants describe the basic motion of geomagnetically trapped energetic charged particles and are introduced below. The third adiabatic invariant describes particle drift paths during slow changes in the geomagnetic field. It is not discussed here but can be found in Walt [1994, p. 50].

The first adiabatic invariant is obtained by integrating the canonical momentum \( \vec{P} = \vec{p} + q\vec{A} \) around the gyration orbit, where \( \vec{p} \) is the relativistic particle momentum and \( \vec{A} \) is the vector potential of the background magnetic field [Walt, 1994, p. 39]. The first adiabatic invariant is often written as:

\[
\mu = \frac{p^2}{2m_0B}
\]  

(1.2)

where \( p_\perp \) is the component of the relativistic particle momentum orthogonal to the background magnetic field, \( m_0 \) is the rest mass of the particle and \( B \) is the strength of the background magnetic field. Adiabatic invariant \( \mu \) describes the particle bounce motion. Figure 1.4b illustrates the helical trajectory of an electron along a magnetic field line. In general, the electron velocity vector \( \vec{v} \) has components both parallel (\( v_\parallel \)) and perpendicular (\( v_\perp \)) to the magnetic field. The local pitch angle \( \alpha \) of the particle is defined as:

\[
\alpha = \arctan \left( \frac{v_\perp}{v_\parallel} \right).
\]  

(1.3)

Noting that \( p_\perp = p \sin \alpha \) and assuming \( p \) remains constant, the first adiabatic invariant implies:

\[
\frac{\sin^2 \alpha_1}{B_1} = \frac{\sin^2 \alpha_2}{B_2}.
\]  

(1.4)

Under these conditions, if the equatorial pitch angle (\( \alpha_{eq} \)) and magnetic field value (\( B_{eq} \)) are both known, then the magnetic field value of the mirror point (where \( \alpha = 90^\circ \)) can be computed:

\[
B_m = \frac{B_{eq}}{\sin^2 \alpha_{eq}}
\]  

(1.5)
The location of this mirror point can then be determined for a given magnetic field model. The most important conclusion from this derivation is that the value of $B_m$ remains constant throughout the particle motion. Even as the particle drifts around the Earth, $B_m$ remains constant so long as no changes violate the first adiabatic invariant or alter the magnitude of the relativistic particle momentum.

The second adiabatic invariant is obtained by integrating the canonical momentum over the particle bounce motion [Walt, 1994, p. 44] and can be written as:

$$I = \int_{s_m}^{s'_m} \sqrt{1 - \frac{B(s)}{B_m}} \, ds$$  \hspace{1cm} (1.6)

where $s$ is distance along the bounce path and $s_m$ and $s'_m$ are the locations of the mirroring points along a field line. Integral invariant $I$ helps describe drift paths of geomagnetically trapped particles, and its invariance ensures that a particle will return to its original field line following a complete drift period around the Earth. Even in an asymmetric or irregular geomagnetic field, as the particle drifts around the Earth it always mirrors at the same magnetic field value $B_m$ to conserve the first adiabatic invariant, and the equatorial altitude of its motion may vary to conserve the second adiabatic invariant, but it always drifts around the Earth back to its original field line to continue its periodic motions assuming that these invariants are not violated by external effects.

The second adiabatic invariant can be used to define the McIlwain $L$-shell parameter [McIlwain, 1961], which is very useful in discussing the geomagnetic coordinate system. Noting that the lack of symmetry in the irregular geomagnetic field limits the use of a standard spherical coordinate system, McIlwain developed the $L$-shell parameter relating the irregular geomagnetic field to a dipole field in a physically intuitive manner. The $L$ value of a location is based on the values of $B$ and $I$ at that location for the true geomagnetic field and functionally relates them to the equatorial crossing distance for the same $B$ and $I$ within a dipole magnetic field. A particle drifting in an irregular geomagnetic field may change its equatorial altitude, but its $L$-shell value remains invariant just as $I$ and $B_m$ remained invariant. The simple intuitive meaning of $L$ is that a magnetic field line in a dipole field crosses the magnetic
CHAPTER 1. INTRODUCTION

equator at a distance of $L$ Earth radii from the center of the dipole. In the inner magnetosphere, where the geomagnetic field is approximately dipolar, this physical intuition for $L$-shell is often not far from the truth.

**Radiation Belt Structure**

Energetic charged particles can theoretically remain trapped at any $L$-shell within the stable magnetosphere. Due to a combination of source, loss and diffusion mechanisms, however, the radiation is most intense across certain $L$, and the structure is different for different particles and energies. Energetic (100 keV to 10 MeV) electrons exist within an inner belt ($1.4 < L < 2$) and an outer belt ($3 < L < 8$) with a slot region relatively devoid of energetic electrons lying in between ($2 < L < 3$). The outermost boundary, mentioned here as $L = 8$, is highly variable. Lower energy (10–100 keV) electrons exist throughout the region and do not exhibit as substantial of a depletion in the slot region. The trapped particle fluxes vary significantly with energy and $L$-shell, as can be seen in Figure 1.5. While our focus is on the energetic electrons as those are the particles which VLF waves can most readily influence, energetic (100 keV to >50 MeV) protons also exist between $L$ of 1.4 and 8, with the >100 keV flux peaking at $2.8 < L < 3.8$ and the >10 MeV flux peaking at $1.4 < L < 2$ [Walt, 1994, p. 74].

**Energetic Electron Precipitation**

The primary loss mechanism of energetic electrons from the inner magnetosphere is precipitation upon the upper atmosphere. Certain conditions must be met for an energetic electron to be trapped within the Earth’s magnetosphere. The motion of both a trapped and a precipitating (not trapped) electron are illustrated in Figure 1.6. Each subfigure depicts the helical trajectory of an electron along a magnetic field line between the northern and southern hemispheres and highlights the value of the the equatorial pitch angle $\alpha_{eq}$ relative to the loss-cone angle $\alpha_{lc}$ for each case. The loss-cone angle is defined as the equatorial pitch angle below which a particle impacts the upper atmosphere on its very next bounce. If the equatorial pitch angle is close to
0°, then \(v_{\parallel} \gg v_{\perp}\), and the electron travels almost directly along the magnetic field line. If the equatorial pitch angle is close to 90°, then \(v_{\parallel} \ll v_{\perp}\), and the electron primarily gyrates around the magnetic field line, moving very little along it. For a smaller equatorial pitch angle, the electron travels farther along the magnetic field line before mirroring and bouncing back, corresponding to a larger value of \(B_{m}\) and a lower mirroring altitude for a given L-shell. If \(\alpha_{eq} > \alpha_{lc}\), then the electron mirrors and bounces back before encountering the Earth’s upper atmosphere, thus remaining stably trapped. If \(\alpha_{eq} < \alpha_{lc}\), however, then the electron impacts the upper atmosphere before mirroring, likely colliding with neutral gas particles therein and exiting its trapped motion, at which time it is said to be ‘precipitated’.

**Figure 1.5:** Equatorial values of electron flux above various energy thresholds. The structure of the inner and outer radiation belts is most evident for higher energy electrons. Figure courtesy of Walt [1994, Figure 5.15] and based on data supplied by the National Space Science Data Center.
Stably trapped electron

Precipitating electron

$\alpha_{eq} > \alpha_{lc}$
$\alpha_{lc}$
$B_0$

Precipitating electron

$\alpha_{eq} < \alpha_{lc}$
$\alpha_{lc}$
$B_0$

$\alpha_{eq} = \tan^{-1}\left(\frac{v_\parallel}{v_\perp}\right)$

**Figure 1.6:** An illustration of stably trapped and precipitating energetic electrons. The bounce and gyro motions of the electron are depicted for each. A stably trapped electron with $\alpha_{eq} > \alpha_{lc}$ mirrors before reaching the upper atmosphere. A precipitating electron with $\alpha_{eq} < \alpha_{lc}$ collides with the upper atmosphere before mirroring and likely exits its trapped motion. Modified from Figure 2.8 of Bortnik [2004].

It is useful to define both a bounce loss-cone and a drift loss-cone. Due to azimuthal irregularities in the geomagnetic field, a value of $B_m$ may reside above the atmosphere at the local longitude but within the atmosphere at another longitude. A particle in the bounce loss-cone precipitates on its very next bounce, impacting the upper atmosphere at its local longitude. A particle in the drift loss-cone is still locally trapped, but precipitates once it drifts in longitude to a region of weaker geomagnetic field. The most significant irregularity in the geomagnetic field is the South Atlantic Anomaly (SAA), a region of particularly weak geomagnetic field off the east coast of Brazil. The weaker magnetic field means that a given value of $B_m$ occurs at a lower altitude, and thus a geomagnetically trapped particle is more likely to impact the atmosphere before mirroring. A bounce loss-cone particle precipitates locally on its very next bounce while a drift loss-cone particle precipitates only once it drifts in longitude to a region of weaker geomagnetic field. A stably trapped particle continues its periodic gyration, bounce and drift motions indefinitely unless its motion is altered by an external force. In general, studies refer to both the bounce loss-cone and the drift loss-cone as the ‘loss-cone’, only mentioning the specific type of loss-cone when
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it is critical to the discussion at hand.

If an interaction due to external forces decreases the equatorial pitch angle of a stably trapped electron, then $B_m$ increases and mirroring altitude decreases. If the mirror altitude is lowered into the atmosphere, then the particle precipitates. If there exists a population of stably trapped particles, then an interaction which scatters the pitch angle distribution of those particles could push a fraction of those particles into the loss-cone. *Abel and Thorne* [1998a] highlights Coulomb collisions and resonant interactions with whistler mode waves as the dominant factors in these scattering losses.

The whistler mode is a right-hand circularly polarized (RHCP) mode of VLF propagation within a magnetized plasma (see Section 1.2.2), and Coulomb collisions are the elastic collisions between two charged particles. Coulomb collisions occur more frequently at low altitudes where background cold plasma densities are higher. A whistler mode wave is a circularly polarized electromagnetic wave which travels roughly along the geomagnetic field lines once injected into the magnetosphere. The fact that this wave is circularly polarized means its rotating fields form a helical trace capable of synchronizing with the helical motion of a trapped energetic electron. If the frequencies and motions of the particle and wave align so as to resonate, then the particle pitch angle can be altered. If a population of trapped energetic electrons is present, with a distribution in energy, pitch angle, and gyrophase, then resonant interaction with a whistler mode wave can scatter the pitch angle distribution of the trapped particles, potentially scattering some into the loss-cone. The details of this wave-particle interaction are discussed in Section 2.2.

The three primary sources of whistler mode waves within the inner magnetosphere are plasmaspheric hiss, lightning, and ground-based VLF transmitters. Plasmaspheric hiss consists of incoherent 0.1 to 3 kHz emissions naturally generated within the plasmasphere [*Hayakawa and Sazhin*, 1992]. Lightning and ground-based VLF transmitters both emit significant amounts of electromagnetic energy in the VLF range (and also ELF range for lightning), some of which penetrates through the ionosphere and into the magnetosphere where it propagates in the whistler mode [*Storey*, 1953; *Helliwell*, 1965; *Cohen and Inan*, 2012]. According to *Abel and Thorne* [1998a,b],
Coulomb collisions dominate energetic electron losses at low latitudes ($L < \sim 1.3$), VLF transmitters dominate at the outer edge of the inner zone ($\sim 1.3 < L < 2.4$), lightning-generated whistlers and plasmaspheric hiss dominate in the slot region ($\sim 2.4 < L < 3.5$), and plasmaspheric hiss dominates in the outer zone ($L > \sim 3.5$). Figure 1.7 shows the impact of each of the dominant factors on the precipitation lifetime of 500 keV electrons as a function of $L$, along with observations following the 1962 Starfish high altitude nuclear detonation for comparison. The Starfish detonation was a test conducted by the USA in which a thermonuclear device was detonated at 400 km altitude above a point in the Pacific Ocean. Aspects of the model developed by Abel and Thorne [1998a,b] have been reevaluated by later studies (e.g., trans-ionospheric propagation [Starks et al., 2008; Cohen and Inan, 2012]) and require update, but the suggestion that VLF transmitters may play a significant role in remediating the radiation belts following a significant influx of energetic particles...
has motivated substantial research on the topic, including this dissertation.

1.2 Propagation of VLF Transmitter Signals

Powerful VLF transmitter facilities have been strategically installed around the globe by various nations for the purposes of long distance naval communications. VLF waves offer two notable benefits for their use in this application: 1) VLF waves reflect efficiently from both the Earth and the ionosphere, allowing them to propagate to great distances around the globe with minimal attenuation, especially under a nighttime ionosphere, and 2) VLF waves penetrate some distance into seawater, allowing a submerged vessel to receive communications without the need to completely surface. The distribution of known powerful VLF transmitters is presented in Figure 1.8, where each transmitter is labeled with its unique three letter call sign. The United States of America, possessing the largest naval force in the world, operates many of these transmitters, but several are controlled by foreign nations.

Figure 1.9 presents a conceptual overview and aerial photograph of a typical ground-based VLF transmitter. The constellation of suspended cables operate as a large capacitor which is electrically driven to create an oscillating vertical electric field between the ground plane and the suspended cables. Transmitter NPM consists of two umbrella top-loaded monopoles instead of two star-shaped constellations, but the principle remains the same. In the case of the VLF transmitter NPM, the cables are suspended up to nearly 0.5 km above the ground, but this is still a short distance compared to the 14.0 km wavelength of the transmitted signal. Despite the complicated physical construction, the antenna effectively radiates as a short, vertical monopole above a ground plane. The antenna directivity above the surface of the Earth is therefore that of a short, vertical dipole:

\[ D(\theta, \phi) = \frac{3}{2} \sin^2 \theta \]  

where \( \theta \) is the angle from vertical in spherical coordinates. The radiation pattern shows no azimuthal variation, and antenna directivity \( D \) is defined as the ratio of
Figure 1.8: Map of VLF transmitter locations around the world. Each transmitter is in nearly continuous operation. While their primary purpose is typically for naval communications, each transmitter can also influence the ionosphere and magnetosphere, and provide probe signals for sub-ionospheric VLF remote sensing. The transmitters NPM and NLK are utilized in our experiments.

far-field power density to that of an isotropic antenna:

\[
D(\theta, \phi) = \frac{\langle \vec{S}_{ff} \cdot \hat{r} \rangle}{P_{\text{rad}}/4\pi r^2}
\]  

(1.8)

where \( \vec{S}_{ff} \) is the far-field Poynting vector and \( P_{\text{rad}} \) is the total radiated power [Staelin et al., 1998, p. 410].

The electromagnetic waves emitted by a ground-based VLF transmitter propagate through and interact with each of the elements of the near-Earth space environment introduced in Section 1.1. Much of the wave energy reflects from both the Earth and the ionosphere, propagating sub-ionospherically within the Earth-ionosphere waveguide. Only a fraction of the wave energy couples into the magnetized plasma of
1.2. PROPAGATION OF VLF TRANSMITTER SIGNALS

Figure 1.9: A conceptual overview and aerial photograph of a typical ground-based naval VLF transmitter. The suspended constellation of cables are electrically driven to create a capacitive antenna with the ground plane. This is 24.0 kHz transmitter NAA located in Cutler, ME.

the ionosphere, propagating in the whistler mode after attenuating during its trans-ionospheric propagation up into the magnetosphere. The whistler mode wave energy that reaches the magnetosphere propagates approximately along the geomagnetic field lines, with its propagation influenced by both the background geomagnetic field and cold plasma density.

1.2.1 Sub-Ionospheric Propagation

At very low frequencies, both the Earth and the ionosphere behave as good conductors. The conductivity of the Earth’s surface varies from about 4 S/m for seawater to $10^{-7}$ S/m for certain rocky regions [Rycroft et al., 2008]. Aside for seawater and certain rocky or icy regions, a typical ground conductivity is $10^{-2}$ S/m [Morgan, 1968]. The Earth’s surface efficiently reflects incoming VLF waves, contributing only minimal losses and distortions to the wave. The nighttime ionosphere reaches a conductivity of $10^{-7}$ S/m at about 80 km altitude, but its conductivity and permittivity are both frequency dependent and anisotropic. The ionosphere can often be modeled as a cold, magnetized plasma with complex refractive index described by the
Appleton-Hartree Equation [e.g., Budden, 1985, p. 74]:

\[ n^2 = 1 - \frac{X}{1 - jZ - \frac{Y^2 \sin^2 \theta}{2(1 - X - jZ)} \pm \sqrt{\frac{Y^4 \sin^4 \theta}{4(1 - X - jZ)^2} + Y^2 \cos^2 \theta}} \]  

(1.9)

where

\[ X = \frac{\omega_p^2}{\omega^2} \quad \text{and} \quad \omega_p^2 = \frac{q_e^2 N_e}{m_e c_0} \]  

(1.10)

\[ Y = \frac{\omega_c}{\omega} \quad \text{and} \quad \omega_c = \frac{q_e B_0}{m_e} \]  

(1.11)

\[ Z = \frac{\nu_e}{\omega} \]  

(1.12)

In these equations, \( \omega_p \) is the plasma frequency, \( \omega_c \) is the electron gyrofrequency, and \( \omega \) is the wave frequency. \( N_e, m_e, q_e \) and \( \nu_e \) are the electron density, mass, charge and collision frequency, respectively. \( B_0 \) is the magnitude of the background magnetic field, and \( \theta \) is the angle between the wave vector \( \vec{k} \) and the background magnetic field vector \( \vec{B}_0 \). For an unmagnetized, collisionless plasma, plasma oscillations would facilitate reflection for \( X > 1 \). In a collisional plasma with \( Z > X \), however, collisions prevent the plasma oscillations from reflecting the wave and instead facilitate propagating modes with attenuation. Reflection can occur either when \( n \) changes rapidly with altitude or when \( X > 1 \) and \( X \geq Z \) [Ratcliffe, 1959, Ch. 12]. For the lower nighttime ionosphere, VLF waves mostly reflect near 85 km altitude, above which point \( X \geq Z \).

Since both the Earth and the ionosphere reflect most of the incident wave energy at very low frequencies, the Earth-ionosphere structure effectively acts as a guide for VLF waves. VLF waves can propagate to great distances within the Earth-ionosphere waveguide, suffering only \( \sim 2 \) dB/Mm attenuation far from the transmitter under nighttime conditions [Davies, 1990, p. 387]. While many methods exist for the computation of VLF propagation within the Earth-ionosphere waveguide, discussions of the topic typically apply either ray theory or mode theory. The ray theory approach treats propagation as a sum of plane waves radiated by the transmitter, each with a
1.2. PROPAGATION OF VLF TRANSMITTER SIGNALS

The wave vector traveling along ray paths that may repeatedly reflect and/or refract from the waveguide boundaries before ultimately reaching the destination point. A ray theory representation of VLF propagation within the Earth-ionosphere waveguide is depicted in Figure 1.10. To compute the radiated field at any point, all ray paths reaching that point must be considered, and each ray is scaled by the appropriate antenna directivity and propagation and reflection losses for that ray. The ray theory approach can be insightful for analysis near the transmitter where only a handful of ray paths need be considered for a given point. At distances greater than \( \sim 500 \) km, however, the large number of ray paths that must be considered makes the ray theory approach untenable, or, at the very least, limits the insight it can provide.

![Figure 1.10: A ray theory representation of VLF propagation in the Earth-ionosphere waveguide. The VLF transmitter is marked on the left, and sample ray paths to the receiver are traced in red.](image)

The mode theory approach treats propagation as a superposition of discrete waveguide modes excited by the transmitter. The received signal at any point is the vector sum of signals propagating in each of these modes, and the discrete modes must each satisfy Maxwell’s equations for the given waveguide configuration and boundary conditions. For the Earth-ionosphere waveguide, modes can be determined by the allowed complex incident angles \( \theta_m \) which satisfy the fundamental equation of mode theory [Budden, 1961, p. 115]:

\[
R_I(\theta)R_G(\theta)e^{-2jkh\sin\theta} = I
\]  

(1.13)

where \( k \) is the wavenumber, \( h \) is the height of the base of the ionosphere, \( I \) is the
identity matrix, and $\mathbf{R}_I$ and $\mathbf{R}_G$ are the reflection matrices for the ionosphere and ground:

$$
\mathbf{R}_I(\theta) = \begin{bmatrix}
\parallel R_{\parallel}(\theta) & \parallel R_{\perp}(\theta) \\
\perp R_{\parallel}(\theta) & \perp R_{\perp}(\theta)
\end{bmatrix}
$$

(1.14)

$$
\mathbf{R}_G(\theta) = \begin{bmatrix}
\parallel R'_{\parallel}(\theta) & 0 \\
0 & \parallel R'_{\perp}(\theta)
\end{bmatrix}
$$

(1.15)

For each matrix element above, the left subscript denotes the polarization of the incident wave while the right subscript denotes the polarization of the reflected wave. The diagonal terms are for waves polarized with electric fields parallel to and perpendicular to the plane of incidence, respectively. The off-diagonal terms represent mode conversion between parallel and perpendicular polarizations and appear in $\mathbf{R}_I$ due to the anisotropy of the ionosphere resulting from the geomagnetic field. The ground is taken to be isotropic, so the off-diagonal terms of $\mathbf{R}_G$ are zero. One method for computing the ionospheric reflection matrix is outlined by Budden [1955]. In this method, the ionosphere is divided into horizontal layers with reflection and transmission coefficients computed at each boundary. Two initial solutions at high altitude are integrated downward through the ionosphere and separated into upward and downward propagating waves to compute the total ionospheric reflection coefficients at the base of the ionosphere.

The mode theory approach becomes very useful at greater distances in the Earth-ionosphere waveguide where higher order modes have attenuated and field values are dominated by only a handful of lower order modes. For sub-ionospheric propagation from a VLF transmitter, field values at distances greater than 2000 km are typically dominated by the second and third quasi-transverse magnetic (QTM) modes [Tolstoy et al., 1982; Foley et al., 1973]. Ground-based VLF transmitters preferentially excite the QTM modes, and the lowest order mode is poorly excited due to its Earth-detached nature [Ferguson and Snyder, 1980; Ramo et al., 1965]. Scattering from waveguide disturbances can re-excite higher order modes and also scatter energy into quasi-transverse electric (QTE) modes. Scattering of VLF waves from ionospheric
disturbances is discussed in Section 2.3.1.

1.2.2 Trans-Ionospheric Propagation

While most of the VLF wave energy emitted by a ground-based transmitter remains confined within the Earth-ionosphere waveguide, the complex nature of the refractive index of the ionosphere (Equation 1.9) due to the presence of a background magnetic field allows some energy to couple into and propagate upward through the ionosphere. The only propagating mode at very low frequencies throughout the ionosphere is the whistler mode. While the refractive index cannot in general be simplified beyond that of Equation 1.9, adopting the quasi-longitudinal (QL) approximation [Ratcliffe, 1959], which assumes that the direction of propagation is close to the direction of the Earth’s magnetic field, provides insightful analysis in certain situations. Following the approach of Helliwell [1965, p. 27], the QL condition can be expressed as:

\[
\frac{Y^2 \sin^4 \theta}{4 \cos^2 \theta} < |(1 - X - jZ)^2|.
\]

Introducing this condition to Equation 1.9 results in the simplified refractive index expression:

\[
n^2 = 1 - \frac{X}{1 - jZ \pm |Y \cos \theta|}
\]

where the minus sign corresponds to the RHCP whistler mode and the positive sign corresponds to the evanescent (for \(\sim 20 \) kHz waves at higher altitudes in the ionosphere) left-hand circularly polarized (LHCP) mode.

The amount of energy entering the ionosphere in the whistler mode depends on the boundary conditions at the lower edge of the ionosphere and the properties of the incident wave. Ionospheric reflection and transmission coefficients can be computed for the base of the ionosphere as described in the previous section. Propagation within the ionosphere is then described by Equation 1.17 for the case of a QL whistler. Integration of the imaginary part of the refractive index through the ionosphere provides an estimate of trans-ionospheric absorption and facilitates the computation of VLF wave energy injected into the magnetosphere. Helliwell [1965, p. 62] performed this
integration and provided representative curves for the trans-ionospheric absorption of
a whistler mode plane wave vertically incident upon the base of a specified ionosphere
in his Figure 3.27. The reality of trans-ionospheric propagation of VLF waves from a
ground-based transmitter is far more complicated, however, as multiple incidence an-
gles, wave polarizations, bearing angles and ionospheric profiles must be considered,
and in many cases the QL approximation is invalid. Chapter 5 covers this topic in
detail.

1.2.3 Magnetospheric Propagation

Aside for quasi-electrostatic modes excited by the scattering of whistler mode waves
from magnetic-field-aligned plasma density irregularities \cite{bell1990}, only
whistler mode waves penetrate through the smooth ionosphere into the magneto-
sphere from a ground-based VLF transmitter. Propagation of these whistler mode
waves is dictated by the background magnetic field together with the background cold
(<1 eV) and suprathermal (100 eV – 1 keV) plasma densities. In the relatively sparse,
fully-ionized plasma of the plasmasphere, the effect of collisions becomes insignificant
and Equation 1.17 can be further simplified to:

\begin{equation}
 n^2 = 1 - \frac{X}{1 \pm |Y \cos \theta|}. \tag{1.18}
\end{equation}

Within the plasmasphere, both \(X\) and \(Y\) are greater than 1 for \(~20\) kHz waves.
When \(\theta\) is close to 0\(^\circ\), the RHCP whistler mode (minus sign) is always a propagating
mode within the plasmasphere. The LHCP mode (plus sign) results in an imaginary
refractive index unless \((1 + Y \cos \theta) > X\), which rarely occurs for \(~20\) kHz waves
in the inner magnetosphere. Whistler mode waves in the inner magnetosphere tend
to propagate approximately along geomagnetic field lines and have high \(n\) (10–100)
which produces low group and phase velocities (0.01–0.1\(c\)). The inclusion of ions
facilitates the magnetospheric reflection of whistler mode waves for frequencies less
than \(~2\) kHz \cite{edgar1976} but not for the \(~20\) kHz waves emitted by naval VLF
transmitters \cite{kulkarni2008a,kulkarni2008b}.

The propagation and attenuation of whistler mode waves in the magnetosphere is
typically estimated via numerical ray tracing together with Landau damping. General ray tracing through inhomogeneous, anisotropic media is complex and requires careful assumptions [e.g., Budden, 1985, Ch. 14]. For VLF propagation in the magnetosphere, the medium is slowly varying and can be subdivided into uniform slabs with the solution to Snell’s law at each stratification used for tracing ray paths through the magnetosphere. Wave attenuation due to Landau damping is not captured by the cold plasma approach taken in Equations 1.9–1.18 and instead requires the kinetic approach provided by the more general Vlasov equation together with Maxwell’s equations [Kennel, 1966; Bittencourt, 2005, p. 500]. The presence of suprathermal particles traveling at approximately the same phase velocity as the wave facilitates an exchange of energy between the wave and the particle population. Particles traveling slightly faster than the wave tend to amplify the wave while particles traveling slightly slower than the wave tend to attenuate the wave. Since particle density decreases with increasing particle velocity in the plasmasphere under typical conditions, there exists a greater number of particles traveling slower than the wave as opposed to faster, resulting in a net attenuation of the wave. The Stanford VLF ray tracing code is one such code used to compute ray paths and Landau damping of VLF waves propagating in the magnetosphere [Inan and Bell, 1977; Golden et al., 2010] and is applied in the Whistler-Induced Particle Precipitation (WIPP) model discussed in Section 2.4.4.

1.3 Effects of VLF Transmitters on the Near-Earth Space Environment

As VLF waves interact with the lower ionosphere, the wave electric field accelerates lightweight electrons, raising the temperature of the electron population through Joule heating. The increased electron temperature affects the collision frequency and (over time) the electron density of the lower ionosphere, thus affecting the propagation of additional radio waves through the medium. VLF waves penetrating into the magnetosphere can resonantly interact with geomagnetically-trapped energetic
electrons near the equatorial plane, thereby inducing electron precipitation. Precipitating electrons deposit their energy in the upper atmosphere, producing secondary ionization. Transmitter-induced precipitation of electron radiation is illustrated in Figure 1.11. Both ionospheric heating by VLF waves and transmitter-induced precipitation of electron radiation is covered in detail in Chapter 2.

\[ \Delta N_e \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{A cartoon illustration of transmitter-induced precipitation of electron radiation. Magnetospheric injection of VLF waves from a ground-based transmitter is traced in red. These waves interact with trapped radiation near the equatorial plane, inducing electron precipitation traced in green. Precipitating energetic electrons create secondary ionization in the ionosphere. Modified from Figure 1.3 of Golden [2011].}
\end{figure}
1.4 Motivation and Scope

The energetic particles trapped in the Earth’s magnetosphere threaten the functionality and longevity of manmade spacecraft. Relativistic (>100 keV) electrons can readily penetrate spacecraft shielding and be deposited in dielectric materials, building up until internal electric discharge occurs with potentially damaging effects. Lower energy (10 to 100 keV) electrons can accumulate on satellite surfaces, also with the potential to create damaging discharges or generate disorienting electronic signals. Energetic ions impacting micro-miniaturized electronics leave ionization tracks which can disrupt memory storage and other sensitive electronics, creating single-event upsets [Baker, 2002]. Specific episodes of satellite failure have been linked to the increased energetic particle fluxes of geomagnetic storms [e.g., Baker et al., 1998; Webb and Allen, 2004], and Pilipenko et al. [2006] present statistical analysis showing the patterns of upset types through the different phases of the solar cycle. The occurrence of operational anomalies on spacecraft due to space weather effects is a trend which further increases with the miniaturization of spacecraft electronics [Baker, 2000]. Both natural and manmade events represent threats to spacecraft electronics: coronal mass ejections and solar wind fluctuations affect the Earth’s so-called ‘space weather’ and can greatly increase trapped energetic particle fluxes, and high altitude nuclear detonations release large amounts of radiation which can remain trapped in the magnetosphere for several months to years [Barth, 2003].

As was discussed in Section 1.1.3, resonant wave-particle interactions involving electromagnetic whistler mode waves are the primary mechanism for the removal of stably trapped energetic particles from the radiation belts at altitudes above 2,000 km [Abel and Thorne, 1998a]. The three primary sources of whistler mode waves within the magnetosphere are plasmaspheric hiss, lightning, and ground-based VLF transmitters. While man has little control over lightning and plasmaspheric hiss, powerful ground-based VLF transmitters have been in operation for many years for the purposes of naval communications. It is potentially possible to build and operate as many ground-based VLF transmitters as necessary, but much work remains to properly quantify the effects of such transmitters both on the radiation belts and on
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the ionosphere. The focus of this dissertation is to study these effects.

The region of stably trapped energetic electrons within the magnetosphere extends from $\sim 0.2$–40.0 Mm altitude at the equator [Walt, 1994, p. 80], existing most intensely in the inner radiation belt ($\sim 2.5$–6.4 Mm) and the outer radiation belt ($\sim 12.8$–40.0 Mm) with the slot region in between. Proton radiation belts also exist, but this dissertation focuses on the electron radiation belts as only the trapped electron population can be influenced efficiently through gyroresonance with the $\sim 20 $ kHz electromagnetic whistler mode waves emitted by a ground-based VLF transmitter. While gyrofrequencies of trapped energetic electrons are on the order of 5 kHz to 1 MHz, gyrofrequencies of the more massive protons are 3 to 300 Hz [Walt, 1994, p. 118]. The lower gyrofrequency prevents efficient resonance. Gyroresonance and the details of this wave-particle interaction are discussed in Section 2.2. The scattering of $\sim 20 $ kHz electromagnetic waves into quasi-electrostatic whistler modes [Bell and Ngo, 1990; Foust et al., 2010] could potentially influence the proton population, but analysis of that interaction is beyond the scope of this dissertation.

Most manmade satellites orbit the Earth at altitudes ranging from 0.2 to 36.0 Mm, overlapping entirely with the typical extent of the radiation belts ($\sim 0.2$ to 40.0 Mm). Manned space stations and many communication and remote sensing satellites reside in lower Earth orbit (LEO; 0.2–2.0 Mm) staying closer to the ground and moving more rapidly around the Earth. Global positioning satellites (GPS) and many additional navigation, communication and remote sensing satellites reside in a medium Earth orbit (MEO; 2.0–35.0 Mm). GPS satellites are commonly placed near 20.0 Mm to achieve a 12 hour orbital period. Broadcast and weather satellites typically use a geostationary orbit (GEO; 35.786 Mm) to move in sync with the Earth’s rotation and thus remain over a fixed point on the ground. Finally, any higher altitude orbit is classified as high Earth orbit (HEO; $> 35.786 $ Mm). A graveyard orbit exists several hundred kilometers above GEO to which many GEO satellites are pushed following decommission. Many additional orbit types and classification systems exist, but this altitude classification system illustrates that the common orbits of manmade satellites covers nearly the full range of the radiation belts. In regards to geomagnetically trapped radiation, this dissertation focuses primarily on the inner
1.5. REVIEW OF PAST WORK

Due to the broad scope of this dissertation, the review of past work is divided into two sections and only the most relevant works are mentioned. The two sections cover the two main effects of VLF transmitters to be discussed: 1) ionospheric heating, and 2) transmitter-induced precipitation of electron radiation.

1.5.1 Ionospheric Heating

The first recognized occurrence of ionospheric heating by radio waves was the so-called “Luxembourg” or “ionospheric cross-modulation” effect reported by Tellegen [1933], where the 252.1 kHz Luxembourg broadcast was detected by radio receivers in Eindhoven, Holland cross-modulated upon the 652.2 kHz Beromünster, Switzerland broadcast. Luxembourg is located nearly along the line joining Beromünster and Eindhoven, and it was suggested that the Luxembourg effect was due to interaction between the two waves as they passed through the ionosphere [Bailey and Martin, 1934; Bailey, 1938]. Huxley and Ratcliffe [1949] provided a survey of the early theoretical and experimental knowledge concerning ionospheric cross-modulation, and Gurevich [1978] extended the theory while summarizing nonlinear phenomena in the ionosphere. Fejer [1970] analyzed the potential of applying this ionospheric cross-modulation effect to probe the lower ionosphere. Galejs [1972] outlined the ionospheric heating and cross-modulation theory specifically for VLF radio waves, with additional work on this topic performed by Ginzburg and Gurevich [1960] and Maslin [1975, 1976].

VLF waves typically reflect from the D-region of the nighttime ionosphere, and since radio waves tend to be most sensitive to ionospheric changes near their reflection height, VLF waves are uniquely well-suited for probing the lower ionosphere. Inan
[1990] provided the first experimental evidence of ionospheric cross-modulation with VLF transmitters, while Taranenko et al. [1992, 1993] and Barr and Stubbe [1992] provided theoretical analysis and a comparison between VLF and HF heating of the lower ionosphere. Inan et al. [1992], Rodriguez and Inan [1994] and Rodriguez et al. [1994] provided additional observations of VLF transmitter heating effects on sub-ionospherically-propagating VLF probe signals (a technique to be discussed in Section 2.3.1) and showed the observations to be consistent with 3-D modeling of the VLF heating and probe signal perturbation.

Rodriguez et al. [1994] showed that powerful, ground-based VLF transmitters in continual operation enhance the nearby ionospheric temperature by as much as a factor of 3 for a 1 MW transmitter, with the enhancement most intense in an annular ring in the lower ionosphere within 150 km radius of the transmitter. Rodriguez and Inan [1994] showed that the heating leads to an electron density depletion of up to 30% due to an increase in the effective three-body electron attachment rate. While the heating and cooling rates are very rapid (on the order of msec or less), the electron density changes occur far more slowly (tens of sec) [Glukhov et al., 1992; Rodriguez and Inan, 1994]. Much of the additional attention given to ionospheric heating by radio waves concerns the use of HF waves in ELF/VLF generation [e.g., James, 1985; Rietveld et al., 1986; Moore et al., 2007; Cohen et al., 2010a; Jin et al., 2013]. Critical to the more advanced heating models are the electron cooling rates and studies of ionospheric chemistry [e.g., Schunk and Nagy, 1978; Tomko et al., 1980].

1.5.2 Transmitter-Induced Precipitation of Electron Radiation

The capability of ground-based VLF transmitters to induce the precipitation of radiation belt electrons has been well-established by satellite-based observations [Vampola, 1977; Imhof et al., 1983; Koons et al., 1981]. Theoretical analysis attributed the phenomenon to gyroresonant pitch-angle scattering of the trapped energetic electron population by whistler mode VLF waves [Inan, 1987, and references therein], and computational modeling efforts have served to theoretically quantify the effects
using the best parameter estimates available at the time [Abel and Thorne, 1998a,b; Kulkarni et al., 2008b].

Sub-ionospheric VLF remote sensing is the only known method potentially capable of measuring transmitter-induced precipitation from the ground [Marshall et al., 2010]. Sub-ionospheric VLF remote sensing has successfully detected electron precipitation due to lightning [e.g., Helliwell et al., 1973; Johnson et al., 1999; Peter and Inan, 2007; Cotts et al., 2011], but the perturbations associated with VLF transmitter-induced precipitation have been considerably less distinct [Inan et al., 2007b; Graf et al., 2011]. A key factor affecting the efficiency with which ground-based VLF transmitters can induce energetic electron precipitation is the rate of trans-ionospheric attenuation of VLF signals, a factor that has only recently been accurately ascertained and validated [Helliwell, 1965; Starks et al., 2008; Tao et al., 2010; Cohen and Inan, 2012; Cohen et al., 2012; Graf et al., 2013a]. Previous efforts to quantify the role played by ground-based VLF transmitters in shaping the Earth’s radiation belts [e.g., Abel and Thorne, 1998a,b; Kim et al., 2011] thus warrant revisitation.

1.6 Contributions of this Research

The focus of this dissertation is to discuss the effects of ground-based VLF transmitters on the ionosphere and magnetosphere. We will recount a series of VLF transmitter keying experiments which were initially performed with the goal of better quantifying the effect of ground-based VLF transmitters upon the Earth’s radiation belts. These experiments were not directly successful toward that goal but have instead illuminated the confounding effect of extended lateral ionospheric heating by VLF transmitters. This ionospheric heating is of potential importance to both radio wave propagation and any future attempts to detect transmitter-induced electron precipitation via sub-ionospheric techniques. In the following chapters, we report observations from our multi-year VLF transmitter keying experiments and thoroughly assess the results via a combination of data analysis and computational modeling. Following the introductory and background material of Chapters 1 and 2, we categorically assess the experimental observations and their physical cause in Chapter 3.
This assessment puts forth extended lateral heating of the nighttime ionosphere by the keyed VLF transmitter as the physical cause, an effect which is covered with much greater detail and computational modeling in Chapter 4. We then shift our focus back to the topic of inducing electron precipitation from the radiation belts using a ground-based source in Chapter 5. There, we provide an update to and analysis of trans-ionospheric attenuation estimates of VLF waves, which is a feature absolutely critical to inducing radiation belt losses using a ground-based source. Then we return to experimental results in Chapter 6, analyzing satellite-based measurements from transmitter keying experiments to report on the topic of inducing energetic electron losses from the radiation belts. While we are unable to directly quantify the radiation belt losses induced by a ground-based VLF transmitter, we do detect evidence of its occurrence. Chapter 7 concludes with a discussion of these results and suggestions for future work on the topic.

The scientific contributions provided by this research are:

1. Determined that heating of the ionosphere by VLF transmitters is the cause of modulation observed on probe signals during controlled experiments.

2. Established experimentally that the lateral extent of ionospheric heating due to VLF transmitters is several thousand kilometers, significantly greater than previously recognized.

3. Developed a large-scale modeling framework to confirm theoretically that ionospheric heating can account for the observed probe signal modulations.

4. Identified the causes for discrepancy between observations and theoretical estimates of trans-ionospheric attenuation of VLF waves and provided an updated set of estimates based on full-wave modeling.
Chapter 2

VLF Transmitter Effects and Experiments

While the primary use of currently existing powerful ground-based VLF transmitters is to transmit communications to naval vessels, they also induce a pair of secondary effects on the near-Earth space environment: 1) interaction of the electromagnetic wave with the lower ionosphere produces collisional heating, and 2) waves penetrating into the magnetosphere induce the precipitation of energetic electrons from the radiation belts. Most VLF transmitters are in continuous operation, transmitting MSK (Minimum Shift Keying) modulated communications while also having a continuous effect on the ionosphere and magnetosphere. To better study the secondary effects, the US Space and Naval Warfare Systems Command (SPAWAR), Air Force Research Laboratories (AFRL) and Office of Naval Research (ONR) assisted in orchestrating a series of coordinated experiments with particular VLF transmitters. Most notably, in a specific effort to better quantify the effects of ground-based VLF transmitters on the radiation belts, a series of keying experiments were performed with the 424 kW, 21.4 kHz naval VLF transmitter NPM in Lualualei, Hawaii. This chapter introduces those NPM keying experiments, covering the effects such a transmitter could theoretically have on the ionosphere and magnetosphere, as well as detailing the observation and modeling tools used to quantify and assess experimental results.
2.1 Ionospheric Heating

As a VLF wave propagates through the weakly-ionized, collisional plasma of the lower ionosphere, the wave electric field accelerates lightweight electrons, thus heating the electron population. Collisions between the accelerated electrons and ambient neutral particles transfer energy away from the electrons thus cooling the electron population back towards their ambient temperature. Unless the temperature of the electron population is increased by several orders of magnitude, the neutral population acts as an infinite heat sink and remains unaffected due to the much greater density of neutral particles in the lower ionosphere (see Figure 1.3). When balancing electron heating, $U$, with electron cooling, $L_e$, it follows that [Huxley and Ratcliffe, 1949]:

$$\frac{3}{2} k_B N_e \frac{dT_e}{dt} = U - L_e,$$

where $k_B$ is Boltzmann’s constant, and $N_e$ and $T_e$ are electron density and temperature, respectively. Once $U$ and $L_e$ are defined, this differential equation can be solved for $T_e$ either as a function of time or in the steady-state. Multiple methods exist for estimating $U$ and $L_e$, and we introduce two common methods for each below. An illustration of ionospheric heating induced by a sub-ionospherically propagating VLF wave is provided in Figure 2.1.

![Figure 2.1: A cartoon illustration of VLF propagation in the Earth-ionosphere waveguide and ionospheric heating. The VLF transmitter is marked on the left, and a sample ray path is traced in red. The ionosphere is shaded in blue, with heating shaded in red.](image-url)
2.1. IONOSPHERIC HEATING

2.1.1 Electron Heating $U$

Electron heating due to a wave electric field accelerating lightweight electrons can be computed either from the wave attenuation as estimated with the imaginary part of the refractive index of the medium or from the work performed on the electron population by the wave electric field. Both approaches are physically sound and should provide equivalent results. In the first approach, we express the electric field of a wave traveling in the $z$-direction as:

$$E = E_0 e^{-j k_0 z e^{-k_0 \chi z}}$$

(2.2)

where $k_0$ is the free space wavenumber, and $\beta$ and $\chi$ are the real and imaginary parts of the refractive index such that $n = \beta - j \chi$. The refractive index $n$ is defined by the Appleton-Hartree equation (Equation 1.9). The wave attenuates with distance at a rate determined by $\chi$, and the energy lost from the wave goes directly into heating the electron population. The heating rate can therefore be expressed as:

$$U = 2k_0 \chi S$$

(2.3)

where $S$ is the wave power density. Equation 2.3 provides a convenient method for computing $U$ as long as the refractive index $n$ is known at each point. The refractive index in the anisotropic ionosphere depends on the angle $\theta$ between the background magnetic field and the wave vector $\vec{k}$. Many computational models of sub-ionospheric and trans-ionospheric VLF wave propagation, such as those discussed in Sections 2.4.1–2.4.3, do not readily provide sufficient information to compute $\vec{k}$ or $\theta$, thus complicating use of Equation 2.3.

Alternatively, electron heating can be computed from the differential form of the Joule heating formula:

$$U = \frac{1}{2} \text{Re}(\vec{J}^* \cdot \vec{E}),$$

(2.4)

where $\vec{E}$ is the wave electric field vector and $\vec{J} = \vec{\sigma} \cdot \vec{E}$ is current density. The conductivity matrix $\vec{\sigma}$ can be derived from the momentum transport equation for a plasma and is a function of electron density, collision frequency, geomagnetic field, and
wave frequency [Inan and Golkowski, 2011, p. 163]. Using Equation 2.4 to compute $U$ requires only knowledge of the wave electric field vector and the background media parameters at each point.

### 2.1.2 Electron Cooling $L_e$

Electron cooling must account for the collisional transfer of energy from accelerated electrons to the ambient neutral population. Earlier studies used a single expression for this energy transfer [e.g., Inan et al., 1992; Rodriguez and Inan, 1992], facilitating an explicit analytical expression for the steady-state heated electron temperature. Estimating the fraction of energy lost per collision as $\delta$, the cooling rate can be expressed as:

$$L_e = \frac{3}{2}k_B(T_e - T_0)\delta \nu_e N_e$$

(2.5)

where $T_0$ is the ambient temperature and $\nu_e$ is the electron-neutral collision frequency. Plugging this expression for $L_e$ into the energy balance Equation 2.1 and taking the steady-state solution gives:

$$T_e = T_0 + \frac{2U}{3\delta \nu_e N_e k_B}$$

(2.6)

Note that Equation 2.6 is still an implicit expression for $T_e$ because $\nu_e$ is a function of electron temperature. While kinetic theory would suggest $\nu \propto v \propto T^{1/2}$, empirical evidence shows that $\nu \propto v^2 \propto T$ for low energy electrons [Budden, 1985, p. 58]. Inserting $\nu_e = (T_e/T_0)\nu_0$ into Equation 2.6 and solving the resultant quadratic provides an explicit expression for steady-state heated electron temperature:

$$T_e = \frac{T_0}{2} + \sqrt{\left(\frac{T_0}{2}\right)^2 + \frac{2T_0U}{3\delta N_e k_B \nu_0}}$$

(2.7)

This explicit expression for heated electron temperature facilitates analytical analysis of ionospheric heating, but it relies on an accurate estimate of fractional energy loss per collision $\delta$. In this context, $\delta$ is a function of electron and ambient temperature, electron density, and neutral composition. Fejer [1970] suggests $\delta = 6 \times 10^{-3}$ as a reasonable estimate for 85 km altitude in the nighttime ionosphere with values
ranging from $2.5 \times 10^{-3}$ to $12.5 \times 10^{-3}$. Studies by Inan et al. [1992] and Rodriguez and Inan [1992] used a value of $\delta = 1.3 \times 10^{-3}$ based on previous work by Maslin [1974]. This approach provides a reasonable estimate of electron heating if an appropriate value of $\delta$ is known for each point in the ionosphere, but $\delta$ varies with altitude and accurate altitude profiles of $\delta$ have not been tabulated.

![Typical Cooling Rates at 85 km Altitude, Nighttime](image)

**Figure 2.2:** Typical values of the cooling rate terms of Equation 2.8 for 85 km altitude in the nighttime ionosphere. The range of $\Delta T_e$ shown is applicable for ionospheric heating by a ground-based VLF transmitter.

A more rigorous approach is to apply the empirically- and theoretically-derived expressions for electron cooling in a weakly-ionized plasma due to the elastic, rotational and vibrational transfer of energy to the dominant neutral particle populations. This approach does not facilitate an explicit analytical expression for heated electron temperature, but it allows for more accurate computation of electron cooling rate as a function of altitude accounting for variations in the electron and neutral particle densities and temperatures. In the lower ionosphere, electron cooling is dominated
by collisional energy transfer to the diatomic N\textsubscript{2} and O\textsubscript{2} neutral particle populations and is a function of electron density \(N_e\), neutral densities \(N_{\text{N}_2}\) and \(N_{\text{O}_2}\), ambient temperature \(T_0\), and electron temperature \(T_e\). Defining \(L_{\text{rot}}\), \(L_{\text{elast}}\) and \(L_{\text{vib}}\) as the rotational, elastic and vibrational cooling rates, respectively, the following expression for \(L_e\) was derived through a combination of empirical and theoretical analyses by Mentzoni and Row [1963] and Dalgarno et al. [1968] (rotational), Banks [1966] (elastic), Stubbe and Varnum [1972] (vibrational), and Schunk and Nagy [1978] (total) and was used previously by Rodriguez et al. [1994]:

\[
L_e = (L_{\text{rot}}(e^-, N_2) + L_{\text{elast}}(e^-, N_2) + L_{\text{vib}}(e^-, N_2)) + (L_{\text{rot}}(e^-, O_2) + L_{\text{elast}}(e^-, O_2) + L_{\text{vib}}(e^-, O_2))
\]

\[
L_{\text{rot}}(e^-, N_2) = 4.65 \times 10^{-39} N_e N_{\text{N}_2} \frac{T_e - T_0}{\sqrt{T_e}}
\]

\[
L_{\text{rot}}(e^-, O_2) = 1.11 \times 10^{-38} N_e N_{\text{O}_2} \frac{T_e - T_0}{\sqrt{T_e}}
\]

\[
L_{\text{elast}}(e^-, N_2) = 1.89 \times 10^{-44} N_e N_{\text{N}_2} (1 - 1.21 \times 10^{-4} T_e) T_e (T_e - T_0)
\]

\[
L_{\text{elast}}(e^-, O_2) = 1.29 \times 10^{-43} N_e N_{\text{O}_2} (1 + 3.6 \times 10^{-2} \sqrt{T_e}) \sqrt{T_e} (T_e - T_0)
\]

\[
L_{\text{vib}}(e^-, N_2) = 4.79 \times 10^{-37} N_e N_{\text{N}_2} \exp \left[ f_{\text{N}_2} \frac{(T_e - 2000)}{2000 T_e} \right] \left( 1 - \exp \left[ -g \frac{T_e - T_0}{T_e T_0} \right] \right)
\]

\[
L_{\text{vib}}(e^-, O_2) = 8.32 \times 10^{-38} N_e N_{\text{O}_2} \exp \left[ f_{\text{O}_2} \frac{(T_e - 700)}{700 T_e} \right] \left( 1 - \exp \left[ -2700 \frac{T_e - T_0}{T_e T_0} \right] \right)
\]

\[
f_{\text{N}_2} = (1.06 \times 10^4) + (7.51 \times 10^3) \tanh(0.0011 (T_e - 1800))
\]

\[
f_{\text{O}_2} = 3300 - 839 \sin(0.000191 (T_e - 2700))
\]

\[
g = 3300 + 1.233(T_e - 1000) - (2.056 \times 10^{-4}) (T_e - 1000)(T_e - 4000)
\]

Typical values for the cooling rate terms as applicable to VLF heating of the lower nighttime ionosphere are shown in Figure 2.2. For this regime, electron cooling is dominated by the rotational transfer of energy. Vibrational energy transfer becomes dominant at much higher electron temperatures (\(\Delta T_e > 1000\) K), but ionospheric
heating by ground-based VLF transmitters is not expected to exceed \( \Delta T_e \approx 500 \text{ K} \). Combining Equation 2.8 for \( L_e \) with the energy balance equation (Equation 2.1) and either of the expressions for \( U \) (Equations 2.3 or 2.4) describes ionospheric heating by a VLF wave.

### 2.1.3 Single Cell Heating Example

It is instructive to consider the pulsed heating of a single uniform cell of ionospheric plasma over time. Figure 2.3 presents computed electron temperature values for the pulsed heating of a typical nighttime ionosphere at 85 km altitude by a 1 MW, 20 kHz ground-based transmitter in the region of maximum heating. For comparison, electron temperatures are computed using both the \( \chi S \) wave attenuation approach of Equation 2.3 and the \( \vec{J} \cdot \vec{E} \) approach of Equation 2.4. The two methods should give identical results. The small discrepancy between the methods apparent in Figure 2.3c is attributed to approximations made in computing an equivalent electric field vector for a given wave power density. Wave parameters in this example are meant to mimic the maximum heating found at 85 km altitude overhead a 1 MW VLF transmitter at nighttime. Wave intensity is pulsed on-off at 100 Hz with a 50% duty cycle in this example. While this pulsing frequency is far higher than an actual naval VLF transmitter could achieve, it helps illustrate the heating and cooling rates typical of ionospheric heating by a VLF transmitter. Both heating and cooling processes effectively reach saturation level in less than 2 msec. The time constants for the heating and cooling rates in this example are approximately 0.1 msec and 0.2 msec, respectively. If the time scale of interest for a study is greater than several msec, as is the case for experiments discussed in this dissertation, then the heating and cooling processes can be treated as instantaneous and the steady-state solution to Equation 2.1 can be assumed.
Figure 2.3: An example of the pulsed heating of a single cell of ionosphere at 85 km altitude by a 20 kHz wave. (a) Wave power density $S$ as a function of time. (b) The equivalent wave electric field magnitude $E$ for this point in the ionosphere. (c) Electron temperature as a function of time, computed using both approaches to electron heating for comparison.

2.2 Transmitter-Induced Precipitation of Electron Radiation (TIPER)

Whistler mode VLF waves which penetrate into the magnetosphere from ground-based transmitters can induce the precipitation of energetic electrons from the radiation belts. Recall from Section 1.1.3 that energetic electrons trapped in the Earth’s radiation belts move along the geomagnetic field lines in helical trajectories with equatorial pitch angle $\alpha_{eq} = \arctan(v_{\perp}/v_{\parallel})$. The helical trajectory of the electron can
resonate with the helical trajectory traced by the wave electric and magnetic field vectors of a propagating circularly polarized whistler mode wave. Resonance occurs when the Doppler-shifted frequency of the whistler wave is equal to an integer multiple of the electron gyrofrequency in the electron frame of reference. For a wave and electron propagating in opposite directions along a geomagnetic field line:

$$\omega + k_\parallel v_\parallel = l\omega_c/\gamma$$

(2.9)

where $\gamma$ is the relativistic Lorentz factor and $l=(0, \pm 1, \pm 2, \ldots)$ is the harmonic resonance number. Non-zero $l$ indicates the cyclotron resonances which are the primary contributions to energetic electron precipitation while $l=0$ represents the Landau resonance. For as long as wave and particle remain in resonance, the Lorentz force enacted by the wave magnetic field can redirect the electron momentum, thereby altering the electron pitch angle. If the electron pitch angle is altered such that $\alpha_{eq} < \alpha_{lc}$, then the electron is destined to precipitate.

The resonance condition specifies the electron energies and conditions under which a given wave can effectively interact with a particle. For any harmonic resonance number, the resonance condition is satisfied along an ellipse in electron velocity space [Walker, 1993], and the lowest resonant kinetic energy along this ellipse is found for particles near the loss cone boundary (where $v_\parallel$ is largest relative to $v_\perp$) and occurs near the equatorial plane (where weaker magnetic field produces smaller $\omega_c$) [Bolton and Thorne, 1995; Abel and Thorne, 1998a]. The lowest resonant energy is of particular interest primarily because it produces the most effective wave-particle interaction. Resonant energy decreases with $L$-shell and wave frequency and increases with harmonic resonance number $l$. The lowest resonant energy of $\sim 20$ kHz waves ranges from $\sim 500$ keV at $L=2.4$ to $\sim 50$ keV at $L=1.6$.

Wave-particle interactions are most effective near the geomagnetic equatorial plane. Varying magnetic field intensity along a field line alters the electron gyrofrequency $\omega_c$ and disrupts the resonance condition, but the magnetic field changes more slowly near the equatorial plane, remaining nearly constant over a larger range of latitude and facilitating a longer duration of resonance. The fact that the wave-particle
interaction occurs dominantly near the equatorial plane influences the location and onset delay of the induced precipitation of energetic electrons. Consider Figure 1.11. The whistler mode VLF waves penetrating into the magnetosphere do not propagate strictly along geomagnetic field lines unless they enter into a duct [e.g., Angerami, 1970]. Instead, obliquely propagating VLF waves drift slightly outward in the magnetosphere to higher $L$-shell as they propagate toward the equatorial plane. The electrons scattered into the loss cone by this wave then travel directly along the local geomagnetic field line as they precipitate at the higher $L$-shell. Due to the decreased velocity of whistler mode waves ($\sim 0.01–0.1 c$) and the bounce period of energetic electrons ($\sim 0.25$ sec for a 100 keV electron at $L = 2$), the onset delay between a VLF transmitted pulse at mid-latitudes and the resultant induced electron precipitation impinging upon the upper atmosphere is at least 0.2 sec [Inan et al., 1985]. Practically, the onset delay and rise time is more likely to be between 0.7 and 2.0 sec considering additional propagation paths, multiple bounce interactions in the equatorial plane, and multiple atmospheric backscatter interactions in both hemispheres [Inan et al., 1985; Cotts et al., 2011]. As a result of the wave-particle interaction occurring dominantly near the equatorial plane, the region of induced electron precipitation is shifted poleward from a low- or mid-latitude ground-based VLF transmitter, and there exists a significant onset delay between the transmission of a VLF wave and the induced precipitation of energetic electrons.

While the resonance condition (Equation 2.9) helps to explain the wave-particle interaction in physical terms, the change of the electron pitch angle $\alpha$ with time requires a complete solution to the resonant interaction between an obliquely-propagating whistler mode wave and an energetic electron. Inan et al. [1978, 1982] provided the first numerical solutions and estimates of precipitation induced by monochromatic parallel-propagating waves while Chang and Inan [1983] and Chang et al. [1983] provided estimates for waves with frequencies varying in time. Bell [1984] provided the first estimates for obliquely-propagating waves by solving the Langevin equation of motion in conjunction with Maxwell’s equations and gyro-averaging the result. Bortnik et al. [2006] applied this result to develop a Whistler-Induced Particle Precipitation model (to be discussed in Section 2.4.4) for estimating the energetic electron
2.2. TRANSMITTER-INDUCED PRECIPITATION

precipitation induced by whistler mode waves. An example of a predicted energetic electron precipitation region, as computed for the transmitter NPM, is presented in Figure 2.4. The precipitation region is shifted poleward from the NPM transmitter and aligns along $L$-shell and magnetic longitude as opposed to along geographic latitude and longitude. The precipitation of $>100$ keV electrons peaks near $L=2$. Only the local bounce loss-cone precipitation region of the northern hemisphere is shown in Figure 2.4. Precipitation also occurs in the magnetically conjugate region of the southern hemisphere, and particles scattered into the drift loss-cone precipitate to the east as they approach the South Atlantic Anomaly.

The precipitating energetic electron flux subsequently collides with neutral particles of the upper atmosphere producing ionization which enhances the electron density of the ionosphere. The altitude of energy deposition is highly dependent on the energy and pitch angle of the precipitating particle. Higher energy and lower pitch angle electrons both deposit more energy and deposit relatively more of that energy at lower altitudes [Banks et al., 1974]. Electrons in the 100–300 keV range deposit the majority of their energy near 85 km altitude in the lower ionosphere, making them

![Figure 2.4: Location and magnitude of the predicted electron precipitation region for VLF transmitter NPM. The magnetic longitude of NPM and select $L$-shell lines along the surface of the Earth are labeled.](image-url)
of critical importance to studies of the lower ionosphere. The energy deposition into
the ionosphere as a function of $L$-shell and altitude for a given precipitation flux can
be computed with a Monte Carlo simulation of the penetration of energetic electrons
into the ionosphere [Lehtinen et al., 2001; Peter and Inan, 2007]. The electron density
enhancement is then computed assuming the production of one ion-electron pair per
35 eV deposited [Rees, 1963].

2.3 Observation Techniques

One of the most challenging aspects of studying the secondary effects of VLF trans-
mitters is detecting the ionospheric heating and induced precipitation phenomena.
Induced precipitation can potentially be detected directly by satellite-based measure-
ments, but this requires a satellite with a specifically-aligned and narrow field of view
energetic particle detector to orbit over the precipitation region at the proper time
and observe the energetic electrons as they precipitate. Optical instruments have been
successfully used in studying aurora [e.g., Maggs and Davis, 1968; Mende and Eather,
1976] and transient luminous events [e.g., Franz et al., 1990], but Marshall et al. [2010]
showed that the optical signature of transmitter-induced precipitation would be unde-
tectable by modern instruments. Detection of the localized ionospheric conductivity
changes induced by precipitating electrons provides an additional non-satellite-based
method for detection, but such a measurement of a precipitation-induced ionospheric
disturbance presents its own complications. The ionospheric disturbances induced by
both electron precipitation and ionospheric heating occur primarily at altitudes too
high for airplanes or balloons but too low for orbiting satellites, thus precluding any
attempts at consistent in situ observation. Rocket studies can directly measure the
ionospheric profile [e.g., Friedrich and Torchar, 2001] but only for the duration and
the particular location of the rocket’s flight. These challenges have led to the develop-
ment of sub-ionospheric VLF remote sensing techniques. If an ionospheric disturbance
occurs along the path of a sub-ionospherically propagating VLF probe signal, then
the amplitude and/or phase of the received probe signal may be perturbed, providing
information about the ionospheric disturbance. Once an ionospheric disturbance is
detected, however, interpreting and assessing the cause of that disturbance presents additional challenges.

2.3.1 Sub-Ionospheric VLF Remote Sensing

VLF electromagnetic waves are effective in detection of conductivity changes in the nighttime D-region ionosphere. Several sources of D-region conductivity disturbances have been detected and analyzed via perturbations on sub-ionospherically propagating VLF probe signals [e.g., Barr et al., 1985; Inan and Carpenter, 1987; Rodriguez et al., 1994; Inan et al., 2010]. The basic process of sub-ionospheric VLF remote sensing is illustrated in Figure 2.5. The cartoon illustration in panel (a) depicts the propagation of a VLF probe signal in the Earth-ionosphere waveguide in the presence of an ionospheric disturbance. The ionospheric disturbance in this case is a change in collision frequency \( \nu \), but any change to the refractive index of the medium would constitute an ionospheric disturbance. The VLF probe signal emitted by the transmitter on the left reflects from both the Earth and the ionosphere as it propagates toward the receiver on the right. The introduction of the ionospheric disturbance, illustrated in pink, perturbs the propagation of the VLF probe signal. A perturbation to the VLF signal detected by the VLF receiver on the right provides information about that ionospheric disturbance. The signal could be perturbed by a disturbance introduced at any altitude in the waveguide, but it is particularly sensitive to disturbances near its reflection height in the D-region of the ionosphere [Inan et al., 2010].

Panels (b) and (c) of Figure 2.5 present computational modeling results for the perturbation of a VLF probe signal by a very large hypothetical ionospheric disturbance. In this simulation, a Gaussian-shaped ionospheric electron density disturbance is placed in the D-region half way between the transmitter and the receiver. The maximum electron density change is set to 100% at 85 km altitude, and the two-dimensional Gaussian shape has a full width at half maximum of approximately 100 km in the along-path direction and 10 km in the vertical direction. The probe signal perturbation is computed as the signal amplitude in the presence of the disturbance minus the signal amplitude without the disturbance present. This computed
Figure 2.5: An illustration and example modeling result depicting sub-ionospheric VLF remote sensing. (a) Cartoon illustration of VLF probe signal propagation in the Earth-ionosphere waveguide and perturbation of that signal by an ionospheric disturbance. (b) Example computational modeling result for propagation of a VLF probe signal in the Earth-ionosphere waveguide from a transmitter on the left to a receiver located some distance to the right. (c) Computed perturbation to the probe signal of (b) induced by the ionospheric disturbance illustrated in pink. The received perturbation to this probe signal provides information about the ionospheric disturbance.

perturbation is plotted in panel (c) with red indicating an increase in amplitude, blue indicating a decrease in amplitude, and white indicating no change in amplitude. Once the probe signal encounters the ionospheric disturbance, its amplitude thereafter is noticeably perturbed. Some ionospheric disturbances can create wide-angle scattering or even back-scatter of the VLF probe signal, but the gradually spatially
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varying disturbance modeled here generates very little back-scatter and thus only produces probe signal perturbations at distances at or beyond the disturbance itself. A VLF receiver located some distance beyond this ionospheric disturbance is likely to detect a perturbation to the received probe signal. Nulls exist in the perturbation pattern, at which points a receiver would observe no amplitude change to the received signal, but at most other locations past the disturbance some perturbation to the probe signal would be expected.

Three facts make sub-ionospheric VLF remote sensing a particularly appealing tool for probing the lower ionosphere at nighttime: 1) VLF waves are particularly sensitive to ionospheric disturbances in the D-region while other measurement techniques have trouble probing this region of space [Inan et al., 2010], 2) VLF waves propagate to great distances (many megameters) with low losses (∼2 dB/Mm) within the Earth-ionosphere waveguide at nighttime [Davies, 1990, p. 387], and 3) the existence of numerous continually-operating naval VLF transmitters (see Figure 1.8) provides a web of VLF probe signals covering nearly the entire globe. Both the ionospheric heating and induced precipitation phenomena produce ionospheric disturbances which can be studied with sub-ionospheric VLF remote sensing techniques [Inan and Carpenter, 1987; Rodriguez et al., 1994].

2.3.2 Satellite-Based Measurements

Satellite-based observations are capable of providing the most direct measurement of the effect of VLF transmitters on the radiation belts. As a satellite orbits over the precipitation region, a properly-aligned energetic particle detector can observe energetic electrons as they travel down the geomagnetic field line. Ideally, the detector would be aligned with a narrow field of view in pitch angle so as to only observe precipitating electrons while excluding trapped electrons, but useful information can still be gathered even when detector alignment is not ideal. Alternately, a pair of detectors could be utilized, one facing up the geomagnetic field line and one facing down, to measure both the energetic particles that travel down the field line and those that mirror and travel back. The difference in the observed flux between the two
detectors would estimate the precipitating particle flux. As one example of satellite-based detection, the Stimulated Emission of Energetic Particles (SEEP) experiment conducted by Lockheed Palo Alto Research Laboratories and Stanford University provided satellite-based observation of bursts of transmitter-induced precipitation induced by the NAA transmitter (44.7°N, 67.3°E, L = 2.85) in Cutler, ME [Imhof et al., 1983; Inan et al., 1985].

Figure 2.6: A computer-generated image of DEMETER Satellite. Image courtesy of Space Scientific Missions of the French National Space Agency (SMSC-CNES).

During the course of the NPM keying experiments to be analyzed in this dissertation, many transmission periods were selected to correspond to traverses of the DEMETER satellite (Figure 2.6) above the region of expected electron precipitation in the northern hemisphere or its corresponding conjugate in the southern hemisphere. DEMETER is a microsatellite developed by the French National Center for Space Studies (CNES) with a ~700 km altitude, 98.3° inclination orbit [Parrot,
An onboard electric field instrument (ICE) measures electric field fluctuations of up to 20 kHz in burst mode, and an instrument for particle detection (IDP) measures 72.9 keV–2.35 MeV electrons with 8.9 keV resolution in burst mode at one sample per second [Sauvaud et al., 2006]. The 21.4 kHz transmission frequency of NPM places it above the cutoff of the ICE, but a powerful aliased signal is still received at 18.6 kHz. The in situ measurements of both electromagnetic field and energetic particle fluxes provided by DEMETER are analyzed for correlations between NPM transmission bursts and particle flux bursts to identify cases of NPM-induced precipitation. The DEMETER IDP is not aligned ideally for the detection of the immediately-precipitating bounce loss-cone electrons, but detected bursts of energetic particle flux are still indicative of induced particle precipitation. This IDP alignment issue is discussed in more detail in Chapter 6.

The DEMETER satellite has previously been used to detect both lightning-induced electron precipitation events [Inan et al., 2007a] and transmitter-induced scattering of radiation belt electrons [Selesnick et al., 2013]. Its low Earth orbit and multi-year lifetime have made DEMETER especially well-suited for long-term studies of the scattering and precipitation of particles from the radiation belts. For example, DEMETER particle data were used by Gemelos et al. [2009] to study the seasonal dependence of lightning-induced electron precipitation, and by Sauvaud et al. [2008] to show a persistent enhancement of drift loss-cone electron fluxes associated with the VLF transmitter NWC.

## 2.4 Modeling Tools

Computational modeling is useful both for predicting the effects of physical phenomena and for comparing theoretical hypotheses to experimental observations. Many models have been developed for the simulation of specific aspects of the near-Earth space environment and of the behavior of waves and particles therein. Some general-purpose models exist, but often times a model is custom-designed for a specific application. Here we introduce the computational models that are directly applied in this dissertation.
2.4.1 Stanford Full Wave Method (FWM)

The full wave method (FWM) described in Lehtinen and Inan [2008, 2009] is a versatile and computationally-efficient modeling approach for finding a three-dimensional full wave solution to Maxwell’s equations in a horizontally-stratified medium. For user-specified source currents and media parameters, this model computes the electromagnetic field vectors throughout the domain. Maxwell’s equations are solved in each layer after computing the reflection coefficients at the boundary between each layer. The FWM model solves for the reflection coefficients at each boundary in a manner that avoids the numerical “swamping” instability that is often a concern for full wave method efforts [Nygrén, 1982]. The method was inspired by Wait [1970], and a detailed introduction to full wave methods can be found in Budden [1985, Ch. 15-19].

For VLF propagation, the FWM model developed at Stanford can compute field values within the Earth-ionosphere waveguide, within the ionosphere, and even up through the ionosphere for lateral distances within $\sim 500$ km of the source. Lehtinen and Inan [2008] and Piddyachiy et al. [2008] applied the FWM to compute ELF/VLF radiation excited by a high frequency ionospheric heater, comparing model results to both ground- and satellite-based observations. Lehtinen et al. [2010] used the FWM to calculate the scattered VLF field in the near zone of ionospheric disturbances created by lightning electromagnetic pulses. Lehtinen and Inan [2009] provided an initial comparison between model and observation for estimating the trans-ionospheric propagation of VLF waves, while Cohen and Inan [2012] and Cohen et al. [2012] would later experimentally-validate use of the FWM model for this application. In Chapter 4 of this dissertation, the FWM model is used to compute VLF field values radiated by a ground-based VLF transmitter up to an altitude of 100 km within 500 km lateral distance of the source. In Chapter 5, the FWM model is applied to trans-ionospheric propagation to estimate magnetospheric injection from a ground-based VLF transmitter and to assess the factors affecting that process.
2.4.2 Discontinuous Galerkin (DG) Method

The DG method described by Foust et al. [2011a,b] is based on the nodal discontinuous Galerkin formulation of Hesthaven and Warburton [2002]. The method is a hybrid between finite volume and finite element techniques, solving conservation law problems with a local basis expansion in each element. The high order, high accuracy, and amenability of the DG method to nonuniform meshes make it appealing to a variety of wave propagation problems. Foust et al. [2011b], for example, applied the DG method to model VLF scattering from lightning-induced ionospheric disturbances, and in Chapter 4 we apply it to compute wave propagation to great distances within the Earth-ionosphere waveguide from a ground-based VLF transmitter. The adaptable nature of the DG method allows for the accurate and efficient accommodation of wave spreading, Earth curvature, inhomogeneous background magnetic field and ground conductivity, and it can accurately compute wave electric field penetrating into the ionosphere.

2.4.3 Finite Difference Frequency Domain (FDFD) Model

The segmented long path (SLP) arrangement of the FDFD code developed by Chevalier and Inan [2006] and Chevalier et al. [2008] allows efficient modeling of VLF propagation over long paths within the Earth-ionosphere waveguide. The path is broken into segments, with Maxwell’s equations solved in the frequency domain in each segment via matrix inversion for a monochromatic wave and the solution sequentially sources the next segment to reach great distances. The ability to solve Maxwell’s equations efficiently over great distances for a single-frequency source makes SLP an appealing method for modeling propagation in the Earth-ionosphere waveguide from a VLF transmitter. Modeling the propagation with different ionospheric profiles allows the estimation of the effects of those ionospheric changes on the propagating VLF signal. This FDFD model has been used in multiple previous studies [e.g., Chevalier et al., 2007; Marshall et al., 2008; Marshall and Inan, 2010] to estimate the perturbation to a sub-ionospherically propagating VLF probe signal induced by an ionospheric disturbance, and that is how we apply it in Chapter 4.
2.4.4 Whistler-Induced Particle Precipitation (WIPP) Model

The WIPP model developed over the years at Stanford [Inan et al., 1982; Lauben et al., 2001; Bortnik et al., 2006] estimates the energetic particle precipitation induced by either a ground- [Kulkarni et al., 2008b] or space- [Kulkarni et al., 2008a] based source of VLF waves. Whistler wave propagation in the magnetosphere is simulated using the Stanford ray tracing code [Inan and Bell, 1977; Golden et al., 2010] including Landau damping effects in accordance with theoretical formulation of Brinca [1972]. Pitch angle scattering of energetic particles into the loss-cone by the whistler wave is calculated according to the work of Bortnik et al. [2006] and yields precipitated flux as a function of energy, L-shell, longitude, and time. The wave-particle interaction model assumes oblique propagation of a weak whistler mode wave and has been built upon a wealth of past work on the topic [e.g., Inan et al., 1978, 1982; Chang and Inan, 1983; Lauben et al., 2001]. We apply the WIPP code in Chapter 6 to estimate particle precipitation induced by the NPM transmitter for comparison to satellite-based observations recorded onboard DEMETER.

2.5 NPM Keying Experiments

A series of naval VLF transmitter keying experiments was conducted in recent years with the goal of better quantifying the effect of ground-based VLF transmitters on the Earth’s radiation belts. In these experiments, the 424 kW, 21.4 kHz transmitter NPM in Lualualei, Hawaii (21.4°N, 158.2°W; \(L = 1.17\)) was modulated in periodic keying formats (e.g., 5-sec on/5-sec off) while the 200 kW, 24.8 kHz transmitter NLK in Jim Creek, Washington (48.20°N, 121.92°W) provided a continuous probe signal for the detection of induced ionospheric disturbances. The VLF receiver MI was installed on Midway Atoll (28.21°N, 177.38°W) to detect the NLK probe signal propagating through the theoretical NPM precipitation region, and the received NLK-MI probe signal was analyzed for the presence of periodic perturbations matching those of the modulated NPM signal. The geographic locations of NPM, NLK and MI are marked in Figure 2.7, together with the great circle paths (GCPs) of the NPM and NLK.
signals received at MI. Many of the NPM keying sessions were also coordinated with passes of the DEMETER satellite to facilitate in situ detection of energetic electron precipitation.

The transmitter NPM was keyed on/off in periodic formats for two thirty minute periods on most days from 25 August 2005 through 2 April 2008. On some days, the two thirty minute periods were placed back-to-back to create a single sixty minute keying session. The majority of the transmissions utilized a 0.1 Hz (5-sec on/5-sec off) periodic keying format. For a 5-sec on/5-sec off format, NPM transmits its 21.4 kHz signal at nearly full power for five seconds, is then turned off for five seconds, and repeats this cycle for the duration of the thirty minute keying period. This controlled modulation is the key advantage of these experiments over the previous study of Rodriguez et al. [1994], where only the effects of single, fortuitous on/off transitions of VLF transmitters were studied as opposed to their periodic modulation.

![Figure 2.7](image.png)

**Figure 2.7:** Map marking the location of VLF transmitters NPM in Lualualei, Hawaii, NLK in Jim Creek, Washington, and VLF receiver MI on Midway Atoll. The blue and brown lines denote the sub-ionospheric great circle paths from the NLK and NPM transmitters to the MI receiver, respectively. Cartoon illustrations of hypothetical ionospheric disturbances due to NPM-induced ionospheric heating and electron precipitation are shaded in red and green, respectively. These are included for illustration purposes only and are not meant to indicate their relative intensity.
The two-channel VLF receiver MI installed at Midway Atoll provided ground-based VLF measurements of the local transverse magnetic field during these experiments. This receiver is similar to the Atmospheric Weather Electromagnetic System for Observation, Modeling, and Education (AWESOME) instrument described by Cohen et al. [2010b]. The receiver features a pair of crossed, air-core, wire-loop antennas for detecting the magnetic field in the north-south and east-west planes. Data are recorded at 100 kHz sampling rate with 16-bits per sample, and the amplitude and phase of narrowband signals at specific frequency channels are demodulated in software and recorded at 50 Hz. One of these narrowband frequency channels records the 24.8 kHz signal from the transmitter NLK while another records that of the keyed NPM transmitter. This provides a continuous probe signal for measuring ionospheric disturbances along the NLK-MI great circle pathway from the transmitter NLK to the receiver MI. Limited data were also recorded during these experiments at additional VLF receiver sites in the Pacific at Waimea, Hawaii (WM, 21.96°N, 159.67°W) and Kwajalein Atoll (KJ, 8.72°N, 167.72°E), and in the Antarctic at Palmer Station (PA, 64.77°S, 64.05°W).

Figure 2.7 also provides cartoon illustrations of theoretical ionospheric disturbances caused by NPM through direct ionospheric heating (in red) and induced electron precipitation (in green). Rodriguez et al. [1994] showed the ionospheric heating by a ground-based VLF transmitter to be strongest in an annular ring within 150 km of the transmitter. The dipole radiation pattern of a typical VLF transmitter antenna produces a null directly overhead so that the most intense heating occurs at approximately 80 km radial distance. Heating is known to decrease at greater distances as the VLF wave energy spreads and attenuates, though Rodriguez et al. [1994] was interested primarily in the most intense portions of the heating region and thus only modeled the ionospheric heating to a radial extent of 250 km. The cartoon heating region illustrated in red in Figure 2.7 is provided for illustrative purposes only, with its scale exaggerated to show the intense ring of heating near NPM and relatively weak heating extending laterally within the Earth-ionosphere waveguide to great distances. A quantitative analysis of the lateral extent and intensity of this heating region is provided in Section 4.1.2.
The location and extent of the ionospheric disturbance due to NPM-induced electron precipitation (in green) is based on the WIPP model of Kulkarni et al. [2008b], where we note that the precipitation region is shifted poleward from NPM due to the non-ducted, whistler mode propagation of the VLF waves within the magnetosphere. The lateral alignment of the region is in the shape of an ellipse with major axis orthogonal to the magnetic meridian since the latitudinal extent is limited in geomagnetic latitude. The precise location and intensity of this ionospheric disturbance is not definitively known due to a lack of experimental measurements for its quantification [Graf et al., 2011] and also due to the recently updated trans-ionospheric attenuation estimates of VLF waves [Cohen and Inan, 2012; Cohen et al., 2012; Graf et al., 2013a], which is discussed in Chapter 5. The existence and parameters of the precipitation region is also heavily dependent upon the pitch angle distribution of the trapped energetic electron population in the radiation belts [Graf et al., 2009], which is discussed in Chapter 6. It is shown in Chapter 3 that the ionospheric effect of this NPM-induced electron precipitation lies below the threshold of detectability for this set of NPM keying experiments. As such, we illustrate the theoretical NPM-induced electron precipitation region in Figure 2.7 only as a point of reference for discussion. In Chapter 3 we present representative observations from the NPM keying experiments and assess the most probable physical cause of the perturbations observed on the NLK-MI probe signal during the keying experiments.
Chapter 3

Ionospheric Heating Rather Than Electron Precipitation

We presented scientific background material and an introduction to the NPM keying experiments in Chapters 1 and 2, and we proceed now to an analysis of those experiments. The NPM keying experiments have been analyzed in previous studies for the secondary effects of ground-based VLF transmitters, but the fact that both ionospheric heating and electron precipitation induced by the keyed transmitter can produce ionospheric disturbances capable of perturbing a sub-ionospherically propagating probe signal has led to some confusion in analyzing experimental results. Inan et al. [2007b] initially reported the detection of induced electron precipitation during the NPM keying experiments, showing that the periodic modulation of the NPM transmitter led to the periodic perturbation of the NLK probe signal. They concluded that the observed perturbations were due to induced precipitation, in part because of the geometry of the detection network: the NLK-MI probe signal pathway propagates directly through the theoretical NPM precipitation region and is never closer than 1750 km to the NPM transmitter. At this large distance from the NPM transmitter, Inan et al. [2007b] discarded off-path scattering from the NPM heating region as a possible link, and they did not consider the possibility of extended lateral heating by NPM affecting the NLK probe signal along its path. Our analysis in this chapter and the next shows that assumption to be premature. The material discussed in this
chapter constituted the subject matter of Graf et al. [2011], and is largely taken from therein.

In order to detect the potentially weak periodic perturbations of the ionosphere induced by the keyed NPM transmitter, Inan et al. [2007b] utilized both superposed epoch and time-integrated Fourier analysis to analyze the received VLF probe signals at each VLF receiver station for the available NPM keying sessions. The key parameters for determining the cause of VLF probe signal perturbations are the onset delay and lag of the periodic perturbations with respect to the keyed transmitter signal. We define “onset delay” to be the time delay, within a keying period, from when the NPM transmitter switches on to when a perturbation begins to appear on the probe signal. The “lag” is the time delay, over many keying periods, from when NPM begins its on/off keying for the session to when a perturbation begins to appear on the probe signal. Since onset delay is critical in determining the cause of the perturbation, we focus our attention only on superposed epoch analysis. The signal-to-noise ratio in the initial superposed epoch analysis of Inan et al. [2007b] prevented the determination of the presence or absence of an onset delay. In this chapter, we remove impulsive noise prior to computing the superposed epoch to greatly increase the signal-to-noise ratio. Any impulsive noise (defined as at least one standard-deviation above the local 3-second median) of less than half a second in duration is replaced by the local mean of the data. Much of the impulsive noise in VLF receiver data is caused by sferics (the electromagnetic impulse from lightning discharges), and the processing to remove sferics should not affect our time resolution or the presence of the small periodic perturbations we aim to detect.

### 3.1 NLK-MI Observations

A representative set of NPM keying session observations is presented in Figure 3.1 for the date of 19 August 2007. On this date, the NPM transmitter was keyed in a 5-sec on/5-sec off periodic format from 9:45 to 10:15 UT (11:45 PM to 12:15 AM local time), and the MI receiver recorded both the NPM and NLK signals. In each subplot of Figure 3.1, we plot the amplitude of the NPM-MI signal in brown, as
3.1. NLK-MI OBSERVATIONS

labeled on the right-side y-axis, and we plot the change in amplitude of the NLK-MI signal in blue, as labeled on the left-side y-axis. The top plot (a) is for the 30 minutes prior to the NPM keying session, the middle plot (b) is for the 30 minutes during the keying session, and the bottom plot (c) is for the 30 minutes after. Each plot shows the superposed epoch of each received signal, meaning we averaged over each 10 second block of data to improve the signal-to-noise ratio in detecting small periodic perturbations on the NLK-MI probe signal. The presence of impulsive noise due to sferics (the electromagnetic impulse from lightning discharges) common in narrowband VLF data would hinder this superposed epoch averaging, so we first replace any impulsive noise (defined as at least one standard-deviation above the local 3-second median) of less than half a second in duration with the local mean of the data. In Figure 3.1b, the NLK-MI signal in blue is clearly perturbed in-step with the NPM on/off keying. The NPM signal in brown is off for the first 5 seconds and on for the final 5 seconds of each 10 second period. When NPM switches from off to on, the NLK-MI signal amplitude immediately decreases by 3 fT, which is approximately a $-0.25\%$ change to its amplitude. A quick look at Figure 3.1a,c shows that during those neighboring times when NPM is not keying on/off (and is instead in continuous operation) the NLK-MI probe signal shows no such periodic perturbation. It is clear that the NPM on/off keying leads to the periodic perturbation of the NLK-MI probe signal.

While we do not analyze signal phase in detail in this study, we provide the superposed epoch of the NLK-MI phase data in Figure 3.2 for this date of 19 August 2007. Just as in the amplitude data, the NLK-MI probe signal is clearly perturbed by the NPM on/off keying. Compared to probe signal amplitude, signal phase is typically more sensitive to small ionospheric disturbances. In addition, an amplitude perturbation does not necessarily coincide with a phase perturbation or vice versa [e.g., Barr et al., 1984]. Detection rates and perturbation analyses improve if both amplitude and phase data are available and reliable for analysis. Unfortunately, it is often not possible to accurately extract phase data, which tend to be noisier, containing occasional phase jumps, and exhibiting much less consistency. In general, the NLK-MI phase data did not respond as well to this form of periodic perturbation
CHAPTER 3. IONOSPHERIC HEATING

Figure 3.1: Experimental observations at MI on 19 August 2007. Plots present 10-second superposed epochs for the 30 minutes prior to (a), during (b), and after (c) the 9:45 to 10:15 UT NPM 5-sec on/5-sec off keying session. Received NPM signal amplitude is plotted in brown in pT as labeled on the right y-axis. Perturbation to the received NLK signal amplitude is plotted in blue in fT as labeled on the left y-axis. Plot (b) clearly shows the NLK perturbation in tandem with the NPM on/off keying. This perturbation and keying are not present in the before (a) or after (c) plots.

One more example of superposed epoch analysis of the NLK-MI probe signal is provided in Figure 3.3 for the date of 26 February 2008 when a different keying periodicity was used. During this forty minute portion of a sixty minute keying session, the NPM transmitter was keyed in a 0.2 Hz (3-sec on/2-sec off) periodic format. The NPM signal in this plot clearly shows the on/off keying format, with NPM being on for approximately the first three seconds of each epoch, and off for the final two seconds. The NLK signal shows a distinct amplitude decrease when NPM is on. To confirm that such a periodicity was not inherent in the NLK signal analysis as did the NLK-MI amplitude data. Therefore, we focus on probe signal amplitude perturbations for the bulk of the analysis.
3.1. NLK-MI OBSERVATIONS

![Figure 3.2: Experimental phase observations at MI on 19 August 2007. Plot presents 10-second superposed epoch (SE) for the 30 minutes during the 9:45 to 10:15 UT NPM 5-sec on/5-sec off keying session. Received NPM signal amplitude is plotted in brown as labeled on the right y-axis. The received NLK phase is plotted in blue as labeled on the left y-axis.](image)

at this time, the superposed epoch was also computed for the thirty minutes prior to and following this keying session, and no such 0.2 Hz periodicity was observed in the signal at those times. Similar results are achieved on this NLK probe signal on a regular basis, from keying sessions of 0.05 Hz, 0.1 Hz, 0.2 Hz, and 0.5 Hz periodicity over the course of the NPM keying experiments.

The removal of impulsive noise prior to averaging is the primary reason why these probe signal perturbations appear significantly more distinct here than in the similar case published in Figure 2 of Inan et al. [2007b]. The perturbation is still very small ($\Delta A \simeq -0.006$ dB compared to $\Delta A \simeq -0.012$ dB reported in Inan et al. [2007b]), but the noise fluctuations superposed on the averaged probe signal have been significantly reduced. With the perturbation now appearing distinctly, it is clear that there is no discernible onset delay from when NPM turns on to when the perturbation appears on the NLK probe signal. Likewise, there is no discernible delay between NPM turning off and the disappearance of the perturbation. This lack of onset delay indicates that the perturbation is not caused by NPM-induced electron precipitation since a delay of at least 200 msec is to be expected based simply on the kinematics of the interaction.
Figure 3.3: Superposed epoch (SE) of the NLK and NPM signals received at Midway (MI), generated from 40 minutes of data recorded during the keying session of 26 February 2008. The NPM signal in brown is keyed in a 0.2 Hz (3-sec on/2-sec off) periodic format. A perturbation of the same periodicity and no relative onset delay is seen in the NLK probe signal plotted in blue.

(i.e., the travel time of the wave to the equatorial interactions region and the travel time of the scattered electrons from there to the ionosphere). The time resolution of the superposed epoch data is 20 msec.

### 3.2 NLK-PA Observations

In addition to the results from MI in the Pacific analyzed for NPM-induced precipitation in the local bounce loss-cone, Inan et al. [2007b] also presented results from Palmer Station (PA) in Antarctica for detection of NPM-induced precipitation in the drift loss-cone. The authors suggested that NPM had scattered energetic electrons into the drift loss-cone. These electrons would precipitate after taking several minutes to drift eastward toward the South Atlantic Anomaly, thus perturbing the NLK-PA probe signal after some lag time. A series of superposed epochs of the NLK probe signal amplitude as received at PA on 14 January 2008 is plotted in Figure 3.4 (comparable to Figure 3 of Inan et al. [2007b]). During the removal of impulsive noise
for this data set, each signal is also processed with a 220 msec running-median filter. This processing is necessary to further reduce the impulsive noise in this case, due to the need to determine the presence of a very small perturbation, but it also lowers the time resolution with which we can specify an onset delay. The six plots presented are all for the same 08:15–08:45 UT NPM keying session, but the averaging window for each superposed epoch is taken to be only fifteen minutes and is gradually shifted through that thirty minute keying session in ten minute steps. The first plot averages fifteen minutes of data entirely before the NPM keying session has begun, the second plot averages fifteen minutes of data for which only ten minutes include NPM keying, the third plot is entirely within the NPM keying session, and so on out through the tail end of the keying session. Outside of the keying session (the first and last plots) the NLK signal has a consistent amplitude of \( \sim 28 \text{ dB} \) with no inherent 0.1 Hz periodicity.

Figure 3.4: Progressive superposed epochs of NLK and NPM signals received at Palmer (PA) on 14 January 2008. The averaging window for each superposed epoch plot is taken to be only fifteen minutes, and that window is gradually shifted through the thirty minute keying session in ten minute steps to create these six sequential plots. The NPM signal in brown is keyed in a 0.1 Hz (5-sec on/5-sec off) periodic format from 08:15–08:45 UT. A perturbation of the same periodicity and no relative lag or onset delay is seen in the NLK probe signal plotted in blue.

The third and fourth plots of Figure 3.4 can be analyzed much like the plot of Figure 3.3. Again, there is a distinct amplitude perturbation on the probe signal.
when NPM is on. Also, while our effective time resolution has been lowered in this case, there is nevertheless no discernible onset delay between NPM turning on and the NLK probe signal being perturbed. The added detail provided by the series of plots is that there is no significant lag between the beginning of the NPM keying session at 08:15 UT and the appearance of the perturbation on the NLK probe signal. The perturbation looks to be present already in the second plot which overlaps with the first ten minutes of the NPM keying session. While it is not presented here, a more thorough analysis involving additional window sizes and smaller steps than ten minutes suggests that there is no discernible lag between the beginning of the NPM keying session and the appearance of the perturbations on the NLK to PA probe signal.

3.3 Additional Observations

In addition to the MI and PA sites discussed here, Inan et al. [2007b] also mentioned observations at Waimea and Kwajalein in the Pacific region. Inan et al. [2007b] stated that Waimea was too close to NPM to be used for the detection of transmitter-induced precipitation, and that Kwajalein data was too inconsistent and with too low signal-to-noise levels to show any evidence of detection. Additionally, due to the limited operation of the Waimea and Kwajalein sites, data is only available from these locations for the first few months of the NPM keying experiments. Analysis of the available Waimea and Kwajalein data sets using the sferic removal technique applied in this chapter does little to change these statements of Inan et al. [2007b]. Waimea is very close to the NPM transmitter and its data consistently shows a strong perturbation similar to that presented in Figure 3.3 for MI. The absence of an onset delay in the Waimea data is very clear. Kwajalein data is less useful for these studies due to weaker probe signal strengths at that location, higher local noise levels, and the frequent prevalence of strong sferics in the vicinity. While some periodic perturbations have now been discovered on the probe signals at Kwajalein, and no onset delay is evident in those observations, the low signal-to-noise levels and the infrequency of the detections limits the conclusions that can be drawn.
3.4 Determining the Physical Cause

The NPM transmitter is keyed periodically for up to thirty minutes to an hour, and that same periodicity is observed on a probe VLF signal. As described above, the two possible physical causes of such an observed periodicity in the probe signal are direct heating of the lower ionosphere by NPM and induced electron precipitation by NPM. A third possible cause could be instrumental, that is, signal cross-modulation due to nonlinearities within the VLF receiver. Inan et al. [2007b] presented results similar to the two cases shown here and suggested that the perturbations were due to transmitter-induced electron precipitation. The effects of ionospheric heating were ruled out at the time due to the >1000 km distance between the NPM transmitter and the signal path of NLK to MI with references to theoretical results of Rodriguez [1994]. Cross-modulation within the receiver was ruled out on grounds that the perturbation did not appear on other VLF signals not passing through the heating or precipitation regions. Unfortunately, due to a lack of the sferic removal analysis technique prior to averaging in Inan et al. [2007b], the specification of onset delays and lag times in their observations was not possible.

3.4.1 Eliminating Induced Electron Precipitation

Now that the onset delays and lag times are more discernible, it appears that the observations are not consistent with NPM-induced precipitation as the cause of the perturbation. There is no observable onset delay greater than our time resolution in either case (20 msec in the MI observations of Figure 3.3, ~220 msec in the PA observations of Figure 3.4). It would take at least 200 msec for the VLF radiation from NPM to propagate to the magnetospheric equatorial plane, interact with and scatter trapped radiation, the scattered energetic electrons to subsequently travel down to the ionosphere and precipitate, and for the resultant secondary ionization in the upper atmosphere to perturb the probe signal. Therefore, transmitter-induced precipitation cannot be the cause of the perturbations on the VLF probe signal for the MI observations.

Similarly, the absence of a lag between the beginning of the NPM keying session
and the appearance of perturbations on the NLK-PA probe signal is inconsistent with NPM-induced drift loss-cone precipitation as the cause. The NLK to PA signal pathway passes through the South Atlantic Anomaly (SAA) where drift loss-cone particles precipitate. Inan et al. [2007b] argued that the keyed NPM transmitter periodically scattered electrons into the drift loss-cone. These electrons would drift eastward and precipitate in the SAA several minutes later, perturbing a probe signal in that region with the NPM keying periodicity. The time for 100 keV electrons to drift from the geomagnetic longitude of NPM to that of the NLK-PA path near the SAA is at least 13 minutes. Therefore, one would expect a lag time of several minutes before the perturbations appear on this NLK-PA probe signal if induced drift loss-cone precipitation were the cause. Since no such lag is observed, we conclude that this type of drift loss-cone precipitation cannot be the cause. There is still the possibility that longitudinal spreading of the NPM signal could induce precipitation of particles trapped closer to the SAA so that little to no drifting or lag would be required before effecting a perturbation on the probe signal, but, given the fact that we already expect the effect for induced precipitation to be rather weak at the longitude of NPM, we would not expect the effect at this more distant longitude to be any more detectable. Additionally, while the processing of the PA signal changed its time resolution from 20 ms to \( \sim 220 \) ms, the lack of any observable onset delay still suggests that the cause of these perturbation is likely not induced precipitation.

In summary, since there is no discernible onset delay or lag time, we conclude that the observed perturbations are not due to NPM-induced electron precipitation. It should also be noted that Inan et al. [2007b] found no correlation between the observations and the Kp or DST geomagnetic indices. Since the trapped particle population is dependent on geomagnetic activity, and induced precipitation is dependent on the trapped particle population, this lack of correlation with the geomagnetic indices further suggests that induced precipitation is unlikely to be the cause of the observed perturbations. We elaborate further concerning this correlation analysis in Section 4.2.
3.4.2 Eliminating Instrumental Cross-Modulation

We have eliminated transmitter-induced precipitation as a possible cause of the observed perturbations, thus leaving instrumental cross-modulation and direct heating of the lower ionosphere as the remaining options. Instrumental cross-modulation in the VLF receiver is unlikely to be the cause due to the lack of a perturbation on certain VLF signals not passing near the heating region of the keyed VLF transmitter. In the NPM (21.4 kHz) experiments, for example, many keying sessions exist where the periodic perturbation is observed on the NLK (24.8 kHz) signal at MI receiver, but not on the JJI (22.2 kHz) signal. The JJI signal originates in Japan (32.04°N, 130.81°E), so it is less likely to experience a perturbation from any NPM-induced ionospheric perturbation. The frequency of JJI is closer to that of NPM, and its received power is similar to that of NLK, so cross-modulation would be more likely. The fact that a perturbation is observed on NLK and not JJI means cross-modulation is unlikely to be the cause. A similar argument can be made for data received at PA, just using the 25.2 kHz NLM (46.37°N, 98.33°W) or 24.0 kHz NAA (44.65°N, 67.28°W) in place of JJI. That argument is less definitive, however, because NPM and NLK are the two strongest signals received at PA during the keying experiments, with NLM and NAA being the next strongest at only about half the amplitude of NLK on average.

To definitively verify that cross-modulation within the receiver is not producing the observed perturbations, the VLF receiver that was deployed at MI during the keying experiments was brought to Stanford and thoroughly tested in the laboratory. Test signals were injected into the MI receiver to mimic the observations of Figure 3.3. With the antenna disconnected and a dummy loop installed in its place, test signals of 21.4 kHz (TS1) and 24.8 kHz (TS2) were injected into the dummy loop. Amplitudes were tuned to recreate the signal levels observed in the experiment, and the 21.4 kHz signal was keyed in a 1-sec on/2-sec off periodic format for two hours while the 24.8 kHz signal was left as a constant tone. Since the receiver has two channels (one north-south and one east-west), and the NPM transmitter signal is observed on both channels with differing amplitudes, the larger of these observed signal strengths was simply injected into both channels of the receiver during the test. This should increase the possibility of cross-modulation within the receiver and thus constitutes a
Figure 3.5: Superposed epoch (SE) results of cross-modulation test executed on the MI receiver. Signals TS1 and TS2 are injected from a function generator with their amplitude and frequency set to mimic the NPM and NLK signals received at MI on 26 February 2008. The 21.4 kHz TS1 plotted in brown is keyed in a 1-sec on/2-sec off periodic format for two hours, and no significant perturbation of this periodicity is observed in the 24.8 kHz TS2 plotted in blue.

Through eliminating the other possible causes of the observed VLF probe signal perturbations, we conclude that direct heating of the lower ionosphere by NPM is the most likely cause. The lack of onset delay and/or lag (depending on the experimental setup) between the periodic perturbations on the VLF probe signal and the keyed

3.4.3 Ionospheric Heating

Through eliminating the other possible causes of the observed VLF probe signal perturbations, we conclude that direct heating of the lower ionosphere by NPM is the most likely cause. The lack of onset delay and/or lag (depending on the experimental setup) between the periodic perturbations on the VLF probe signal and the keyed
VLF transmitter eliminates transmitter-induced precipitation as a possible cause, contrary to the suggestions of Inan et al. [2007b]. Extensive testing of the VLF receiver system used in the experiment eliminates instrumental cross-modulation as a possible cause. With transmitter-induced precipitation and instrumental cross-modulation eliminated, we conclude that direct heating of the lower ionosphere by NPM is the most likely cause of the observed VLF probe signal perturbations. This finding suggests that the effects of ionospheric heating by powerful VLF transmitters are detectable over very long distances. Rodriguez et al. [1994] detected such perturbations in cases when the distance from the heating transmitter to the closest approach of the probe signal pathway was up to 770 km, and those observations were taken from single on/off events without the benefit of any periodic keying for superposed epoch analysis. In this connection, observation of the effects of ionospheric heating induced by NPM on the NLK to MI probe signal pathway at 1750 km distance is in retrospect perhaps to be expected considering the fact that averaging over many cycles of on/off keying brings out substantial improvement in signal-to-noise ratio. Perturbations induced at such great distances may seem unlikely at first, but they become more feasible when the extended lateral ionospheric heating of the sub-ionospherically propagating NPM signal is taken into account. Since a 20 kHz signal propagates in the nighttime Earth-ionosphere waveguide with only \( \sim 2 \) dB/Mm of attenuation at great distances [Davies, 1990, p. 387], the NPM signal strength at 2000 km would only be \( \sim 2 \) dB less than its strength at 1000 km.

With both induced precipitation and instrumental cross-modulation eliminated as possible causes for the observed NLK-MI probe signal perturbations, we proceed to investigate direct heating of the ionosphere by NPM as the possible link. We note first that ionospheric heating and cooling rates match the observed time signature of the perturbation. Heating occurs on the order of \( \mu \)sec while cooling occurs in msec. Given the time resolution of the recorded narrowband data, both the perturbation onset delay and turn-off delay are less than 20 msec. Thus, the perturbation time signature is consistent with ionospheric heating as the causative physical process. The computational modeling of the next chapter serves to assess whether ionospheric heating by the NPM transmitter could realistically produce the observed probe signal
perturbations.

While observations have also been presented here for the NLK-PA probe signal, these observations serve primarily as a necessary rebuttal to Figure 3 of Inan et al. [2007b]. PA features an older receiver design which has not been thoroughly tested for cross-modulation under these conditions, and detecting the perturbation in Figure 3.4 requires additional signal processing which hinders analysis compared to study of the NLK-MI probe signal. The NLK-PA signal is also less consistently available than the NLK-MI signal over the course of the NPM keying experiments. The end result is that any analysis of the NLK-PA probe signal is not fruitful in the context of this study. Further analysis thus focuses solely on the NLK-MI probe signal, for which data from over 1000 keying sessions exists for detailed analysis.

3.5 Summary

Following improved processing of the NLK-MI probe signal from the NPM keying experiments, analysis contradicts Inan et al. [2007b] by demonstrating that the observations could not have been caused by induced electron precipitation. Improved signal processing significantly reduces the noise in the probe signal perturbation analysis, clearly highlighting the lack of onset delay in the observed perturbations. For mid-latitude VLF transmitters, the onset delay between a VLF transmitted pulse and the resultant induced electron precipitation impinging upon the upper atmosphere would be at least 0.2 sec, and, practically, the onset delay and rise time is more likely to be between 0.7 and 2.0 sec [Inan et al., 1985; Cotts et al., 2011]. Thus, the $<0.02$ sec onset delay and rise time observed in the NPM keying experiments eliminates induced electron precipitation as a possible cause and contradicts the conclusions of Inan et al. [2007b]. Furthermore, laboratory test results of the MI receiver eliminate the possibility of instrumental cross-modulation, leading to the conclusion that scattering from the NPM heating region is the most likely cause of the observed NLK-MI probe signal perturbations in the NPM keying experiments.
Chapter 4

Extended Lateral Heating of the Nighttime Ionosphere

In the previous chapter, we concluded that scattering from the NPM heating region was the most likely cause of the observed NLK-MI probe signal perturbations in the NPM keying experiments. In this chapter, we extend the analysis by combining additional experimental observations and data analysis with computational modeling to confirm theoretically that ionospheric heating can account for the observed probe signal modulations. We further show that the observed NLK-MI probe signal modulations are likely due to along-path scattering from extended lateral heating by NPM as opposed to off-path scattering from the more intense ionospheric heating near NPM. In the process, we develop a large-scale computational modeling framework for ionospheric heating by a ground-based VLF transmitter, and we show that the lateral extent of ionospheric heating due to VLF transmitters is several thousand kilometers, significantly greater than previously recognized. The material discussed in this chapter constituted the subject matter of Graf et al. [2013b].

The superposed epoch plots in Figures 3.1 and 3.3 of the previous chapter presented NLK-MI probe signal perturbation analysis for two specific NPM keying sessions. Over the course of the experiments, the NLK signal was recorded at MI for 1250 such keying sessions. Limiting analysis to keying sessions of the 5-sec on/5-sec off format and discarding any sessions for which either the NPM or NLK signals were
weak or had signal drop-outs, we are left with 930 keying sessions for analysis. For each of these 930 sessions, we perform the superposed epoch analysis and compute the amplitude perturbation $\Delta A$ of the NLK-MI probe signal as we did in Figure 3.1 of the previous chapter. The value $\Delta A$ is computed for during the NPM keying session as the average of the NLK-MI probe signal during the times when NPM is on minus the average of the NLK-MI signal during the times when NPM is off. The distribution of $\Delta A$ for these 930 keying sessions is presented in Figure 4.1. The result presented in Figure 3.1 was an extreme case at $-3 \text{ fT}$, but the NLK-MI probe signal is consistently perturbed by the NPM keying. Based on the typical noise levels of the received NLK-MI signal, most periodic perturbations greater than 0.5 fT in magnitude would be detectable in the superposed epoch analysis, so the majority of the keying sessions produce a detectable perturbation to the NLK-MI signal. The average perturbation is $-0.8 \text{ fT}$. The average NLK-MI signal amplitude is 1.2 pT, so $-0.8 \text{ fT}$ represents a $-0.07\%$ change. We also note that the perturbation is consistently negative, meaning the received NLK-MI signal is typically lower in amplitude when NPM is on. These observed perturbations can be compared to a computational model for the proposed physical process.

### 4.1 Computational Modeling

In this section we develop a large scale computational model for ionospheric heating by the VLF transmitter NPM and the effect of this heating on the NLK-MI probe signal. The model consists of three components: 1) computation of the wave electric field radiated by the NPM transmitter throughout the region of space, 2) computation of the increase in electron temperature generated by that wave electric field at each point, and 3) propagation of the NLK-MI probe signal through the heated Earth-ionosphere waveguide to estimate the amount it is perturbed by the NPM-induced ionospheric heating. This probe signal perturbation estimate is then compared to that measured during the NPM transmitter keying experiments. Rodriguez et al. [1994] performed similar steps in their study of ionospheric heating by VLF transmitters, but their modeling only considered heating within 250 km radial distance of the
Figure 4.1: Distribution of perturbations to the NLK-MI probe signal for 930 30-minute 5-sec on/5-sec off nighttime NPM keying sessions. The average perturbation is $-0.80 \, \text{fT}$, which is approximately a $-0.07\%$ change.

Note that we first compute the radiated wave electric field at each point in the domain, and then compute the resultant increase in electron temperature separately as a post-processing step. This approach is not self-consistent due to the fact that a change in electron temperature would alter propagation of the heating wave itself, and thus some form of feedback would occur. A change in electron temperature would also affect electron density \cite{Rodriguez1994}, which would in turn influence both wave propagation and heating behavior. While the decision to separate the wave propagation and ionospheric heating into separate steps without feedback is not ideal, the error associated with this lack of self-consistency should be very small, especially at great distances from the transmitter. This issue is discussed in more detail in Section 4.2.3.

Configurable media parameters input to the model are electron, nitrogen, and
oxygen densities, ambient temperature, electron-neutral collision frequency, and background geomagnetic field. Ambient profiles used in this study are presented in Figure 4.2. This electron density profile was used by Taranenko et al. [1992] and as profile II of Inan et al. [1992]. It is based on data from the International Reference Ionosphere model [Rawer et al., 1978; Bilitza and Reinisch, 2008] and represents a typical nighttime ionosphere electron density profile to be used throughout the region for ambient conditions. The neutral particle densities and ambient temperature profiles are from the NRLMSISE-00 model [Picone et al., 2002] for local midnight, and collision frequency is based on Swamy [1992]. Each of these ionospheric profiles varies with altitude but do not vary latitude or longitude within our model. In other words, we use each of the profiles plotted in Figure 4.2 throughout the domain for ambient conditions. We do, however, vary the background geomagnetic field throughout the region by using values from the IGRF10 model [Macmillan and Maus, 2005] for each point in latitude, longitude, and altitude.

![Figure 4.2: Ambient nighttime ionosphere profiles to be used throughout the computational model. Ambient temperature and neutral densities are from the NRLMSISE-00 [Picone et al., 2002]. Electron density is from profile II of Inan et al. [1992], and collision frequency is based on Swamy [1992]. Geomagnetic field (not shown here) is from IGRF10 [Macmillan and Maus, 2005].](image-url)
4.1.1 NPM Radiated Fields

To compute the ionospheric heating generated by the VLF transmitter NPM at great distances, we first compute the radiated wave electric field at each point in the domain. Computation of the wave electric field throughout the entire region of 5000 km radius is problematic because it is simply too large for a full 3D model considering available computational resources. We resolve this issue by computing the wave propagation individually along many 2D paths in the Earth-ionosphere waveguide extending radially from the NPM transmitter. We then combine these paths to fill the 3D space. We compute wave propagation along these 2D paths with a segmented long path (SLP) arrangement of the discontinuous Galerkin (DG) method [Foust et al., 2011b]. In addition to handling basic wave propagation in the Earth-ionosphere waveguide, this model also accurately accounts for spreading of the wave energy, curvature of the Earth, inhomogeneous background magnetic field and ground conductivity, and can accurately compute wave electric field penetrating up into the ionosphere. The model is accurate for great distances from the source, but, due to the source implementation as a ring offset from the pole in the simulation space, it does not accurately represent the fields near the transmitter. For this reason, we only use data from this DG model for distances greater than 500 km from the VLF transmitter NPM, and we use a different model for closer distances. For distances within 500 km of NPM, we use the Stanford Full Wave Method (FWM) [Lehtinen and Inan, 2008, 2009]. The FWM model assumes horizontally-stratified media, so it cannot currently account for Earth curvature or horizontally-varying geomagnetic field or ground conductivities. For within 500 km of the source, however, horizontal stratification is a good approximation and we can very accurately compute radiated wave electric field. This FWM model has been validated for VLF propagation into and through the ionosphere [Cohen et al., 2012; Graf et al., 2013a], and has also been used for propagation within the Earth-ionosphere waveguide [Lehtinen et al., 2010].

The vertical component of the wave electric field radiated by the NPM transmitter for within 500 km lateral distance is presented in Figure 4.3a,b. The field values were computed with the FWM model. Subplot (a) shows the fields for a vertical slice running from South to North. A null is directly above the transmitter located
at 0 km, and most of the wave energy is reflected by the lower ionosphere. We also see North/South asymmetry due to geomagnetic field orientation, with greater penetration into the ionosphere toward the South where propagation is more closely along the background geomagnetic field as opposed to across it. Subplot (b) shows the fields for a horizontal plane taken at 80 km altitude above the NPM transmitter. The source is located at the origin in the center of the image. A null is directly above the transmitter, and concentric rings form in the radiation pattern due to the mode

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**Figure 4.3:** Top Row: Vertical component of electric field radiated by NPM, as computed by the Stanford FWM model for within 500 km radius, for (a) vertical slice running from South to North, and (b) horizontal plane at 80 km altitude. Bottom Row: Ionospheric heating induced by the NPM transmitter within 500 km radial distance, as computed using electric field values computed by the Stanford FWM model. The spatial domain for plot (c) corresponds directly to that of (a), and plot (d) corresponds to (b).
structure established by wave propagating in the Earth-ionosphere waveguide below.

![NPM Electric Field](image)

**Figure 4.4:** Vertical component of electric field radiated by NPM, as computed by the DG code for distances 500 to 5000 km. Plot shown is for a 2D path extending radially from the NPM transmitter at a bearing of 300°.

The electric field radiated by the NPM transmitter for distances greater than 500 km is presented in Figures 4.4 and 4.5a. These values are computed by using the DG model. Figure 4.4 presents the vertical component of the wave electric field for along a single 2D path extending radially from the NPM transmitter at a bearing angle of 300°. The wave propagates to great distances in the Earth-ionosphere waveguide, with the limited attenuation that does occur being due to a combination of collisional losses, leakage out of the waveguide, and radial spreading of the wave energy. We note that at these greater distances from the transmitter, the wave amplitude attenuates at as little as 2 to 3 dB/Mm. This attenuation rate agrees with *Crary* [1961] and *Davies* [1990, p. 387] and indicates that significant field strengths exist even at several thousand kilometers distance from the transmitter. This 2D path of Figure 4.4 results from a single run of the DG code. We combine 36 of these 2D runs, at each 10° step in azimuthal bearing angle, to compute the electric field radiated by NPM throughout the 3D region. This approach gives relatively poor azimuthal resolution at great distances, but still clearly illustrates the structure of the radiated fields and the resolution is sufficient for the present application. The electric field values for a horizontal plane at 80 km altitude are presented in Figure 4.5a, where the values are
plotted onto a map for reference. NPM is located in Hawaii in the center of the map, and the West coast of North America is visible in the upper right.

**Figure 4.5:** (a) Vertical component of electric field radiated by NPM, as computed by the DG code for radial distances up to 5000 km. Plot shown is for 80 km altitude. Results generated by combining 36 2D radial paths, one of which is shown in Figure 4.4. (b) Map of ionospheric heating at 80 km altitude induced by the NPM transmitter. Location of the NPM transmitter is marked in the center of each image. Great circle path of the NLK to MI probe signal is drawn in blue.

While the wave amplitudes computed by the FWM model are accurate for a specified source power, the DG model assumes a source of arbitrary unit amplitude and the results require scaling. We compute a single scaling factor for the DG model results for NPM by performing a least squares fit to a combination of recorded field values at VLF receiver sites and field values estimated by the FWM model. Three VLF receiver sites were available for comparison: Midway Atoll (MI; 28.21°N, 177.38°W), Valdez, Alaska (VZ; 61.06°N, 146.02°W), and Juneau, Alaska (JU; 58.59°N, 134.90°W). All three recorded the NPM signal strength at their location throughout the keying experiments. We compute the average NPM field value recorded at each site during the times when NPM was switched on during the keying sessions and compare this to the field value predicted at each location by the DG model. Since this provides just three distant points for comparison, we include four additional points from the FWM model: one in each of the four cardinal directions at approximately 500 km radial distance where the two models merge. Model results are analyzed in the vicinity at each
of these points to ensure local extrema in field amplitude do not disrupt the scaling. After computing the universal DG model scaling factor based on these seven points of comparison, all field values computed by the DG model are scaled accordingly. A comparison of the scaled DG model results to the seven reference measurements is provided in Figure 4.6. All of the values are in close agreement, with no comparison showing more than 20% discrepancy.

![Figure 4.6: Performance of the scaled DG model for estimating fields radiated by the NPM transmitter. Comparisons are made to sample points from the FWM model in the four cardinal directions (blue) and to long-term averaged measurements from three available VLF receiver sites (red). Field values estimated by the scaled DG model show less than 20% discrepancy at each point of comparison.](image)

### 4.1.2 Ionospheric Heating

With the electric field computed at each point extending to 5000 km radial distance from the NPM transmitter, we now solve the energy balance equation for electron heating as was covered in Section 2.1. We use the energy balance equation (Equation 2.1) together with the $\vec{J} \cdot \vec{E}$ expression for electron heating (Equation 2.4) and the detailed cooling rate equations (Equation 2.8). Reproducing only the fundamental energy balance equation here, and recalling that when balancing electron heating,
$U$, with electron cooling, $L_e$, it follows that:

$$
\frac{3}{2} k_B N_e \frac{dT_e}{dt} = U - L_e,
$$

(4.1)

where $k_B$ is Boltzmann’s constant, and $N_e$ and $T_e$ are electron density and temperature, respectively.

Following the computation of wave electric field $\vec{E}$ and establishment of the ambient media parameters, the only remaining unknown in Equation 4.1 is the electron temperature $T_e$. Solving Equation 4.1 in this application is greatly simplified because we need only consider the steady state solution. Since the heating and cooling rates are both on the order of 1 msec or less, and the data sampling rate in the experiments is 20 msec, the heating process can be treated as instantaneous. Setting $dT_e/dt = 0$, we need only numerically solve the resultant equation $U = L_e$ for $T_e$. Performing this task at each point in the model produces the electron heating values presented in Figure 4.5b. In these computations, we use FWM electric field values for within 500 km radial distance and DG electric field values for 500 to 5000 km radial distance. For 80 km altitude, there is on the order of 100 K electron temperature increase near NPM (approximately a 50% change), 1 K increase at 1000 km radial distance, and 0.01 to 0.1 K increase at 3000 km radial distance. A closer look at the heating near NPM is provided in Figure 4.3c,d, where only the electric field values computed by the FWM model are required. These results for ionospheric heating near NPM compare favorably to those of Rodriguez et al. [1994], both in the general structure and intensity of the heating region.

The region of space plotted in Figure 4.3c,d compares directly to that of Figure 4.3a,b, and Figure 4.5a compares directly to Figure 4.5b. Electron heating structure roughly matches that of the wave electric field amplitude, and discrepancies arise primarily due to the anisotropic conductivity matrix. For example, in Figure 4.3a,b there is greater penetration of the wave electric into the ionosphere to the South of NPM, but in Figure 4.3c,d there is greater heating at 80 km altitude to the North. Physically, this is because propagation to the North across the geomagnetic field means the wave electric field is more closely aligned with that geomagnetic field, thus
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Taking advantage of increased electron mobility to produce greater current densities and greater electron heating. These effects go hand-in-hand: electron heating at 80 km altitude near NPM is greater to the North because more of the wave energy goes into heating the electron population in that direction, and because the wave transfers more of its energy to the electrons in that direction, wave field amplitudes at higher altitudes are correspondingly lower.

4.1.3 Probe Signal Perturbation

Now that we have estimated the ionospheric heating induced by the keyed NPM transmitter, we assess the impact of this heating on the sub-ionospherically-propagating NLK-MI probe signal. To estimate the probe signal perturbation, we propagate the NLK probe signal through the Earth-ionosphere waveguide to the MI receiver under both ambient and disturbed ionospheric conditions. The great circle path of the NLK-MI probe signal is traced in blue in Figure 4.5b. This pathway is approximately 5200 km in length and is 1750 km from the NPM transmitter at its point of closest approach. Since this full 3D region is too large to model as a single scattering problem, we focus on propagation and scattering along the 2D NLK-MI pathway. This approach ignores the effects of off-path scattering, a decision which we discuss and justify in Section 4.2.2.

To estimate the probe signal propagation under ambient conditions we employ the same ambient ionospheric profiles and geomagnetic field models used in the previous modeling steps. For propagation under disturbed conditions, we extract the change in electron temperature along the NLK-MI pathway from the ionospheric heating model results. Temperature change is proportional to collision frequency change for these low energy electrons [Budden, 1985, p. 58], and collision frequency affects the conductivity matrix which dictates wave propagation and scattering [Bittencourt, 2005, p.247]. The change in electron-neutral collision frequency along the NLK-MI pathway is presented in Figure 4.7. Changes to the collision frequency are minimal (less than 0.01%) close to the NLK transmitter along this path where it is still 3000 to 5000 km from the heating NPM transmitter, but collision frequency changes as
large as 1% occur further along the pathway where it approaches within 2000 km of NPM.

Figure 4.7: NPM-induced ionospheric heating along the NLK-MI probe signal pathway as extracted from results of the large-scale heating model. Ionospheric profiles and heating are defined above 60 km altitude, 0 to 60 km altitude is treated as free space.

For the 2D probe signal propagation and perturbation analysis, we use the SLP arrangement of the finite-difference frequency-domain (FDFD) code developed by Chevalier and Inan [2006] and Chevalier et al. [2008]. This code has been used in multiple previous studies [e.g., Chevalier et al., 2007; Marshall et al., 2008; Marshall and Inan, 2010] to estimate the perturbation to a sub-ionospherically propagating VLF probe signal induced by an ionospheric disturbance. Using this FDFD model, we propagate the NLK to MI probe signal under both ambient and disturbed conditions. The results are presented in Figure 4.8. Subplot (a) shows the propagation under ambient conditions, with the NLK transmitter located at 0 km, and the MI receiver near the right edge at 5200 km. Since the change to this probe signal under disturbed ionospheric conditions is very small, we plot the perturbation to the probe signal in subplot (b). We compute the perturbation as (disturbed)−(ambient), with red indicating a positive perturbation, blue indicating a negative perturbation, and white indicating no change. The largest probe signal perturbations occur further in the waveguide, once the wave encounters the region of more intense heating seen further in the waveguide in Figure 4.7. Since the VLF receiver MI is located on the ground, we extract the probe signal perturbation estimated along the ground and plot this in Figure 4.8c. Assuming these typical ionospheric conditions, this model estimates an
Figure 4.8: Perturbation to the NLK-MI probe signal due to NPM-induced extended lateral heating of the nighttime ionosphere. (a) Propagation of the NLK probe signal through the Earth-ionosphere waveguide to the MI receiver. (b) Perturbation to the NLK probe signal due to the NPM-induced ionospheric heating presented in Figure 4.7 for a typical ionosphere. (c) Perturbation to the NLK probe along the ground, as would be measured by a VLF receiver detecting the local transverse magnetic field component. (c) includes results for both a typical ionosphere and a tenuous ionosphere, together with the typical MI experimental observation marked in pink.

observation of $\Delta A \approx -0.1$ to $+0.1$ fT near MI. For comparison, we mark the average MI observation in pink: $\Delta A \approx -0.8$ fT at a distance of 5200 km from the NLK transmitter.

Throughout all modeling steps, we have assumed the “typical” ambient ionospheric electron density profile of Taranenko et al. [1992] and Inan et al. [1992]. For
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comparison, we also executed the entire model (NPM transmission, ionospheric heating, and probe signal propagation) with a “tenuous” ionosphere. Based on statistical bounds from in situ electron density values measured in several rocket studies [Tao et al., 2010] and following the lead of Cohen et al. [2012] and Graf et al. [2013a], we divide the “typical” electron density profile by a factor of 5 to define the “tenuous” nighttime ionosphere. While the intermediate modeling results are not presented here, we include the probe signal perturbation estimated along the ground for a tenuous ionosphere in the bottom plot of Figure 4.8. The model now predicts an observation of $\Delta A \approx -1.2$ fT at MI. Computational modeling of a “dense” ionosphere is not feasible with present resources as the increased electron density would require a finer mesh in the DG simulation, creating a simulation which would take many weeks to complete. The probe signal perturbation for a dense ionosphere should be less than that found for a typical ionosphere.

4.2 Discussion

During the NPM keying experiments, we consistently detected a perturbation on the NLK probe signal received at MI. The average perturbation was $-0.8$ fT. Computational modeling of ionospheric heating and probe signal propagation led to perturbation estimates of approximately $\pm 0.1$ fT on this probe signal for a typical ionosphere, and $-1.2$ fT for tenuous ionosphere. So within the error due to input variability, the model results agree with the experimental observations in both sign and magnitude. This agreement suggests that extended lateral heating of the nighttime ionosphere by the keyed NPM transmitter can realistically account for the observed NLK-MI probe signal perturbations. The fact that the detected perturbation is preferentially negative in both observation and model matches general intuition, but we would not expect this convenient result to be the same for all scattering geometries. An increase in electron-neutral collision frequency due to ionospheric heating increases the ionospheric reflection height of the VLF probe signal but also increases the collision frequency at that reflection height. The end result is greater attenuation of the probe signal and an overall decrease in amplitude. However, despite the overall decrease
in probe signal amplitude, the perturbation analysis in Figure 4.8b shows that there still exists patches where $\Delta A$ is positive and patches where $\Delta A$ is negative. This result is due to the impact of the ionospheric disturbance on the mode structure of the VLF probe signal propagating in the Earth-ionosphere waveguide. A slightly different scattering geometry could result in a detected perturbation that is positive or even zero at a particular receiver location.

### 4.2.1 Ionospheric & Geomagnetic Variability

While the use of typical ambient media profiles throughout the region led to an underestimation of the observed perturbation by a factor of 8 in the model, the analysis of a tenuous ionosphere showed that this discrepancy is well within the general variability of the model inputs. A tenuous ionosphere produces multiple effects which increase the probe signal perturbation: less attenuation of the NPM signal with distance, greater penetration of the NPM signal into the ionosphere, increased ionospheric heating, and greater penetration of the NLK probe signal into the ionosphere to interact with greater ionospheric disturbances. In comparison to a uniformly typical or dense ionosphere, the existence even of patches of tenuous ionosphere along the NLK-MI pathway could greatly increase the observed probe signal perturbation because heating and scattering in this tenuous region may likely dominate over the heating and scattering from nearby regions of denser ionospheric electron density. We compared the model results for typical and tenuous ionospheric electron density profiles primarily as a representative assessment of model error due to variability of the input media profiles. Ionospheric electron density is the most variable of the input media profiles, but changes to the ambient temperature, collision frequency, neutral densities, geomagnetic field, and ground conductivities may also contribute both to model error and to the variability of the observed probe signal perturbations.

In addition to the general day-to-day and place-to-place variability of ionospheric electron density, the ionosphere also experiences a known seasonal trend due to variation in local solar flux over the course of a year. In Figure 4.9, we analyze the
experimental observations for the presence of a seasonal trend. In this figure, each red dot represents the probe signal perturbation computed for a 30-minute NPM keying session on that day. The black curve marks the running mean and standard deviation of those red points. As a proxy for the seasonal trend of ionospheric electron density, we plot in blue the electron density over the course of the year from the International Reference Ionosphere (IRI) model [Bilitza and Reinisch, 2008] for a point at 85 km altitude along the NLK-MI path near the NPM heating region in the year 2007 at local midnight. For this region of the world, ionospheric electron density peaks in the late spring and early summer months. While there is significant day-to-day spread in the observed probe signal perturbation (likely due in large part to day-to-day ionospheric variability) there also appears to be a seasonal trend: the detected probe signal perturbation is lowest during the months when ionospheric electron density is highest. This result agrees with our general intuition and the computational modeling results: when the ionosphere is denser, the ionospheric heating and probe signal perturbation are both weaker.

We also compare the amplitude of the observed probe signal perturbations to geomagnetic activity and present the correlation of $|\Delta A|$ with Ap index in Figure 4.10. This analysis shows that there is no correlation between geomagnetic activity and the observations. This result is to be expected if ionospheric heating is the linking physical phenomenon. Beyond the impact of increased geomagnetic activity on ionospheric electron density (which should only have a minor effect on these mid-latitude observations), there should be no strong link between geomagnetic activity and ionospheric heating by a ground-based VLF transmitter. In addition, this lack of correlation with Ap index lends further support to the case made in Chapter 3 that the observed probe signal perturbations are not likely due to transmitter-induced precipitation of electron radiation. Geomagnetic activity is connected to the flux levels and pitch angle distributions of trapped electrons in the Earth’s radiation belts. Strong geomagnetic activity can “prime” the trapped energetic electron population, creating a “top-hat” pitch angle distribution where many particles are close to being scattered into the bounce loss-cone [Lyons and Williams, 1975]. Once the pitch angle distribution is
4.2. DISCUSSION

Figure 4.9: Seasonal variability of the NLK-MI probe signal perturbation together with seasonal variability of ionospheric electron density. Each red point marks the probe signal perturbation computed for a 30-minute NPM keying session on that day. The black curve marks their running mean and standard deviation. The blue curve marks the electron density at 85 km altitude over the course of the year as estimated by the IRI-2007 model [Bilitza and Reinisch, 2008] for a point along the NLK-MI path in the year 2007 at local midnight.

...primed in this manner, a resonant VLF wave can readily scatter more trapped energetic electrons into the bounce loss-cone, potentially producing far greater electron precipitation effects. Leyser et al. [1984] and Peter and Inan [2004] assessed this effect for the observation of lightning-induced electron precipitation (LEP) and suggested that there is a relationship between geomagnetic activity and the conditions conducive to the occurrence of detectable LEP events. We would expect a similar relationship between geomagnetic activity and the detection of transmitter-induced electron precipitation. Since the observations show no correlation with geomagnetic activity, it is unlikely that they would be due to induced electron precipitation.
Figure 4.10: Correlation of the NLK-MI probe signal perturbation with geomagnetic activity index Ap. No correlation is found.

4.2.2 Along-Path vs. Off-Path Scattering

While assembling the computational model, we made several decisions in order to model the heating and scattering with sufficient accuracy over the very large distances involved. One such decision that warrants further analysis and justification is the choice to limit the probe signal perturbation to strictly a 2D along-path scattering analysis. In reality, studying the NLK-MI probe signal during the NPM keying experiments presents a large-scale 3D scattering problem within the spherical shell of the Earth-ionosphere waveguide. A strong ionospheric disturbance exists some distance (∼1500 km) away from the probe signal pathway, and weak, diffuse ionospheric disturbance extends to great distances (∼2000 km) covering the probe signal pathway itself. The NLK probe signal scatters from the distant off-path disturbance, the extended along-path disturbances, and everything in between. In general, for different scattering geometries and ionospheric disturbances, there may be situations where the off-path scattering dominates, and situations where the along-path scattering dominates. For practical purposes, we assess the along-path scattering and the off-path scattering separately for the case in hand and then focus the modeling...
efforts on the dominant effect if we conclude that the other effect is likely inconsequential. For the NPM-NLK-MI scattering geometry, the NLK-MI great circle path is never closer than 1750 km to NPM, and the required forward-scattering angle to reach MI from NPM for the NLK probe signal is 77°. Wide-angle scattering would be required for off-path scattering to affect this probe signal. A concentrated off-path disturbance could produce this wide-angle scattering, but the NPM heating region is relatively diffuse with changes occurring gradually over the course of VLF wavelengths. To quantify this effect, Poulsen et al. [1990] presented a theoretical analysis of sub-ionospheric VLF propagation in the presence of ionospheric disturbances and showed that for a Gaussian-shaped disturbance and simplified scattering geometries, an analytic expression exists for estimation of the probe signal perturbation. Applying their Equation A13 with a heating region of 150 km effective radius as suggested by Rodriguez et al. [1994] for scattering geometries similar to the NPM-NLK-MI configuration, it can be shown that off-path scattering from the most intense heating region within 200 km of NPM falls off very rapidly as the distance to the probe signal pathway increases. For the distance of closest approach set to 100 km, the predicted amplitude perturbation is $\sim 10^{-2}$ dB. For 300 km, the predicted perturbation falls to less than $\sim 10^{-4}$ dB. For the 1750 km distance of closest approach for the NPM-NLK-MI scattering configuration, the predicted perturbation is effectively zero. This result supports our decision to ignore off-path scattering and to focus the modeling efforts on the 2D along-path scattering analysis.

While the support of basic scattering theory may sufficiently justify the decision to focus on along-path scattering, we can also provide experimental justification by analyzing the arrival azimuth of the observed probe signal perturbations. The VLF receiver located at MI records two channels, each from one of a pair of orthogonal wire-loop antennas. Typically, one of these antennas is oriented North-South (NS) and one East-West (EW). For these experiments, however, the antennas were rotated slightly to align the EW antenna more directly toward NPM. Given the 77° angle between NLK, NPM and MI, the NPM signal arrives predominantly on the EW channel of the MI receiver while the NLK signal arrives predominantly on the NS channel. If the perturbation on the probe signal were due to off-path scattering from
an ionospheric disturbance near NPM, then that perturbation should also appear on the EW channel. However, the observed perturbations appeared on the NS channel and not on the EW channel. This result suggests along-path scatter as opposed to off-path scatter. We note that the MI receiver detects the local transverse magnetic field, so for this arrival azimuth analysis to hold we require that the scattering does not generate a large longitudinal magnetic field. A large longitudinal magnetic field could occur for scattering into high order transverse electric modes, which is more common for scattering from intense, rapidly-changing disturbances. Results from 2D FDFD scattering analysis show that the perturbation to the transverse magnetic field is much larger than perturbation to the longitudinal magnetic field, so the probe signal and perturbation remain predominantly transverse magnetic and the arrival azimuth analysis discussed here remains valid.

To put the arrival azimuth analysis into a more rigorous mathematical framework, we quantify the deviation in arrival azimuth, $\theta$, of the NLK-MI probe signal during each of the NPM keying sessions. For each keying session, we first estimate the probe signal arrival azimuth for samples during the times when NPM was off. Then, we estimate the probe signal arrival azimuth for samples when NPM was on. Comparing these two values gives an estimate $\Delta \theta$ for the change in probe signal arrival azimuth due to scattering during this keying session. For along-path scatter, the scattering and probe signal perturbation arrive from the same direction as the signal itself. Thus, the perturbation should not change the probe signal arrival azimuth, and we should find $\Delta \theta = 0^\circ$ for the case of along-path scatter. If the perturbation arrives from off-path, however, then arrival azimuth $\theta$ should change. For the NLK-MI probe signal, if the perturbation arrives from the direction of NPM with a magnitude of 0.8 fT (average for the experimental observations), then we would expect to find $\Delta \theta = 0.03^\circ$. The distribution of $\Delta \theta$ is presented in Figure 4.11 for the 375 keying sessions for which a strong perturbation ($|\Delta A| > 0.8$ fT) exists. The expectation for along-path scatter is marked in blue ($\Delta \theta = 0^\circ$) and the expectation for off-path scatter is marked in green ($\Delta \theta = 0.03^\circ$). The distribution is clustered tightly near the expectation for along-path scatter, with a mean of 0.002° and a standard deviation of 0.016°. So this experimental evidence, much like the theoretical analysis, suggests that the observed
perturbations on the NLK-MI probe signal are due primarily to along-path scattering and that off-path scattering can be safely ignored.

4.2.3 Model Self-Consistency

One more modeling component that warrants discussion is the lack of strict self-consistency in computing the electron heating, together with the decision to ignore changes to electron density. Wave propagation, electron heating, and electron density changes all influence one another. Therefore, in any true calculation of one parameter, we must solve for all three together. We now consider the error incurred by solving for each effect separately. At distances greater than 1000 km from the heating transmitter NPM, we estimate changes to the electron temperature of ~1% or less. Application of an ionospheric chemistry model similar to that of Rodriguez and Inan [1994] shows that this 1% increase in electron temperature would lead to less than 0.5% change in electron density above 80 km altitude. The probe signal perturbation analysis showed
these small changes produce on the order of 0.1% change to a sub-ionospherically propagating VLF signal. In other words, the feedback effect of these minor changes appears to be negligible for our purposes. We note that this analysis is sufficient to justify application of the model toward the key conclusion of this chapter: that extended lateral heating at great distances from the NPM transmitter are able to account for the perturbations observed on the NLK-MI probe signal. Recall that the NLK-MI probe signal pathway is never closer than 1750 km to the NPM transmitter and experiences electron temperature changes of \( \sim 1\% \) or less. We further note that probe signal scattering from changing electron densities should not even be considered when comparing to the experimental observations. While electron heating and cooling occur on the order of msecs or less, electron density changes occur over tens of seconds [Glukhov et al., 1992; Rodriguez and Inan, 1994]. Since we key the NPM transmitter on/off at 0.1 Hz or faster, and since the observed perturbation onset delay is less than 50 msec, we can safely ignore the effects of changing electron densities.

While it should have little effect on comparisons to the experimental observations or on the conclusions for heating at great distances, it is worth assessing feedback nearby the NPM transmitter where field intensities and ionospheric changes are strongest. There is as much as a 50% increase in electron temperature near NPM, which would produce a \( \sim 20\% \) electron density depletion at 80 km altitude [Rodriguez and Inan, 1994]. This significant density depletion would allow increased penetration of the VLF wave into the ionosphere, which would in turn produce increased heating and positive feedback. This effect was discussed by Rodriguez and Inan [1994], and they mention that the feedback would ultimately be inhibited by a maximum in the effective three-body electron attachment rate around \( T_e = 700 \) K [Tomko, 1981, p. 163]. While they do not compute the ultimate result of this feedback, it is likely dwarfed by the general ionospheric variability between a tenuous and a dense nighttime ionosphere. Still, a complete modeling of ionospheric heating overhead a powerful VLF transmitter should account for feedback between heating, density changes, and wave propagation. For example, heat flow may also become an important factor. Such a self-consistent model for heating of the lower ionosphere by VLF transmitters is beyond the scope of this dissertation.
4.3 Summary of Sub-Ionospheric Analysis

The controlled modulation of a ground-based VLF transmitter consistently perturbs the sub-ionospherically propagating signal of a second VLF transmitter, showing a seasonal trend correlating with ionospheric electron density variation but no correlation with geomagnetic activity. Time signature analysis and laboratory testing of receiver equipment eliminates transmitter-induced precipitation of electron radiation and instrumental cross-modulation as potential causes. Experimental evidence suggests ionospheric heating by the modulated VLF transmitter to be the most probable cause. Analysis of the arrival azimuth of the perturbed signal leads to the conclusion that the perturbation is caused by extended lateral heating. Large-scale computational modeling corroborates ionospheric heating extending laterally to great distances within the Earth-ionosphere waveguide, with electron temperature increases of $\sim 0.5\%$ extending up to 2000 km from the heating transmitter. Along-path scattering of the second VLF transmitter signal (the probe signal) from this extended lateral heating can account for the experimental observations. For a mid-latitude, ground-based VLF transmitter at nighttime, $\sim 50\%$ of the total radiated power contributes to ionospheric heating within 400 km lateral distance, and $\sim 30\%$ contributes to heating and attenuation at greater distances. Due to the efficiency with which VLF signals propagate to great distances in the Earth-ionosphere waveguide, the lateral extent of ionospheric heating due to powerful VLF transmitters is several thousand kilometers, significantly greater than previously recognized.
Chapter 5

Trans-Ionospheric Attenuation of VLF Waves

In Chapters 3 and 4 we presented observations and modeling for sub-ionospheric analysis of the NPM keying experiments. As was discussed in Section 2.5, these experiments were initially performed with the goal of quantifying the effects of ground-based VLF transmitters on the Earth’s radiations belts. While sub-ionospheric VLF remote sensing failed to detect induced electron precipitation during these experiments, it succeeded in providing new insight into the phenomena of ionospheric heating and its lateral extent. We return our attention now to transmitter-induced precipitation of electron radiation and focus on the specific aspects of the process that we can now better study and explain.

When Inan et al. [2007b] initially reported detection of transmitter-induced precipitation of electron radiation during the NPM keying experiments, they provided theoretical analysis utilizing the WIPP model to show the observed perturbations to be consistent with estimates of NPM-induced precipitation under certain ionospheric and trapped radiation conditions. Two critical components of the WIPP model for this application, however, are the assumed pitch angle distribution (PAD) of the trapped radiation and the estimated magnetospheric injection of VLF waves from a ground-based transmitter. Inan et al. [2007b] assumed a square PAD, which greatly increases
the predicted precipitation flux and will be discussed in Section 6.3. For magnetospheric injection, they utilized the trans-ionospheric absorption curves of *Helliwell* [1965, Figure 3-35]. Despite the fact that quantitative estimates of trans-ionospheric attenuation of VLF waves are important for many studies in space sciences, the preeminent reference for many of these studies [e.g., *Abel and Thorne*, 1998a; *Bortnik et al.*, 2002; *Inan et al.*, 2007b; *Kulkarni et al.*, 2008b; *Starks et al.*, 2008; *Golden et al.*, 2010] has been *Helliwell’s* absorption curves. The trans-ionospheric absorption estimates of *Helliwell* [1965] were presented at the time with several known caveats, and recent in-situ satellite observations [e.g., *Starks et al.*, 2008] have further questioned their validity. In this chapter, we discuss recent improvements to trans-ionospheric attenuation estimates and assess various factors affecting those estimates. This material constituted the subject matter of *Graf et al.* [2013a].

### 5.1 Debate Over Attenuation Estimates

*Helliwell* presented the total trans-ionospheric absorption of an electromagnetic whistler mode wave through the ionosphere as a function of geomagnetic latitude for representative frequency and day/night conditions. *Helliwell* made simplifying approximations to facilitate the numerical computation of these curves, and they were originally intended only for mid- and high-latitude analysis of whistler mode waves incident upon the ionosphere with their wave normals within the cone of transmission [*Helliwell*, 1965, Section 3.7]. *Inan et al.* [1984] combined the waveguide power model developed by *Crary* [1961] with *Helliwell’s* trans-ionospheric absorption curves to estimate transmitter power above the ionosphere. *Starks et al.* [2008] combined this approach with the Air Force Research Laboratory’s VLF Propagation Code to produce a three-dimensional model for illumination of the plasmasphere by terrestrial VLF transmitters. In comparing their model to measurements from dozens of satellite passes over several VLF transmitters, *Starks et al.* [2008] concluded that *Helliwell* [1965] underestimates the 20 kHz, mid-latitude attenuation by about 10 dB in the day and 20 dB during the night. *Tao et al.* [2010], applying a full wave method for trans-ionospheric absorption but again looking at single incident plane waves with
vertical incidence, analyzed D-region electron density variation and suggested that even more discrepancy (up to 100 dB) may be present when using more realistic electron density profiles.

A series of studies attribute all or portions of this discrepancy to nonlinear effects and/or scattering from irregularities [e.g., Foust et al., 2010; Bell et al., 2011; Shao et al., 2012]. Theoretical calculations of Foust et al. [2010] attribute up to 3–6 dB of loss to scattering of whistler mode waves from magnetic field-aligned density irregularities in the F-region. Shao et al. [2012] attribute up to 9–15 dB of loss to conversion to lower hybrid waves in the D- and E-regions. The “smooth ionosphere” models of Helliwell [1965], Lehtinen and Inan [2009], and Tao et al. [2010] do not account for these effects, and it is unknown how much and how often nonlinear and scattering phenomena affect the trans-ionospheric propagation of VLF waves.

5.1.1 An Experimentally-Validated Model

Applying the FWM model and measurements from DEMETER satellite, recent findings by Cohen and Inan [2012] and Cohen et al. [2012] provide the first cases of consistent agreement between modeling results and satellite-based observations for magnetospheric injection from terrestrial VLF transmitters. Cohen and Inan [2012] analyzed thousands of DEMETER satellite passes over six and a half years over each of a dozen VLF transmitters to provide radiation maps at 700 km altitude with 25 km resolution, providing significantly more averaging and spatial resolution than previous studies on this topic. Using these maps, the total power injected into the magnetosphere from each transmitter was calculated for both daytime and nighttime. Cohen et al. [2012] compared these power estimates to those of a full wave method (FWM) model described by Lehtinen and Inan [2008, 2009], finding that the model correctly reproduces the injected VLF power to within ±6 dB for both daytime and nighttime for each and every one of the twelve transmitters considered. It should be noted that the full wave model is a “smooth ionosphere” model that does not include ionospheric density irregularities, indicating that those irregularities may play a much smaller role than has been proposed. For instance, the model-data agreement was
shown not to be a function of transmitter power up to 1 MW, which does not support the suggestion by Shao et al. [2012] that transmitter-induced irregularities such as those observed by Parrot et al. [2007] and Bell et al. [2008] play a significant role in trans-ionospheric absorption.

Given the findings of Cohen and Inan [2012] and Cohen et al. [2012], in this chapter we use the FWM to compute trans-ionospheric attenuation curves for comparison to Helliwell and explain any discrepancies. We begin by assessing the importance of various factors in trans-ionospheric attenuation, such as wave polarization, incidence angle, bearing, and the ionospheric density profile, so as to better understand and apply attenuation estimates. We then provide sets of trans-ionospheric attenuation curves that are specifically applicable to the magnetospheric injection of VLF waves from terrestrial, short, vertical dipolar radiators, representative of both Navy VLF transmitters and cloud-to-ground lightning flashes. We also provide trans-ionospheric attenuation curves for the case of a single whistler mode wave vertically incident upon the ionosphere. Finally, we compare the FWM results to Helliwell’s absorption curves and rectify any apparent incongruities found with other recent studies.

5.2 Model Descriptions & Input Profiles

In this chapter, we use three different models of trans-ionospheric attenuation: (1) the absorption curves from Figure 3–35 of Helliwell [1965], (2) the FWM model detailed by Lehtinen and Inan [2008, 2009] and utilized by Cohen et al. [2012], and (3) a simplified, quicker version of that FWM model that, much like Helliwell [1965] and the model of Tao et al. [2010], considers only single incident plane waves.

5.2.1 Helliwell’s Absorption Curves

The curves shown in Figure 3–35 of Helliwell [1965] present the total trans-ionospheric absorption of an electromagnetic whistler mode wave through the ionosphere as a function of geomagnetic latitude. Helliwell presents four curves, specifying the absorption for 2 and 20 kHz, and for daytime and nighttime. To generate the curves, he
first computes the absorption as a function of wave frequency for different ionospheric conditions by numerically integrating the absorption coefficient of the wave from 60 to 1500 km altitude. Helliwell then applies multiplying factors to produce his curves of absorption as a function of geomagnetic latitude. The absorption coefficient is related to the imaginary part of the refractive index, which Helliwell calculates using the quasi-longitudinal (QL) approximation to Appleton’s equations [Ratcliffe, 1959]. He suggests that for a 20 kHz wave, this QL approximation is valid for geomagnetic latitudes above about 25° during daytime, and 45° during nighttime.

Helliwell takes the incident electromagnetic wave to be whistler mode and vertically-incident on either the base or top of a horizontally-stratified ionosphere. He mentions that one can account for coupling effects by assuming a single sharp boundary and including the one-time reflection from this boundary as an additional loss, but he does not include this loss in his absorption calculations. Helliwell suggests that if the incident wave is linearly-polarized as opposed to whistler-mode, then an additional 3 dB of attenuation should be added to his curves due to polarization mismatch between the transmitted and incident waves. Any effects due to reflection from the ground in the Earth-ionosphere waveguide are not accounted for in this model.

Tao et al. [2010] successfully reproduces Helliwell’s absorption curves by integrating the absorption coefficient for each frequency and time of day at each point in geomagnetic latitude, and we do so again here. Given this more direct approach, the media parameters specified in the computation of Helliwell’s absorption curves are effectively the background magnetic field, electron density, and collision frequency. These parameters all vary with latitude. The background magnetic field is computed using a dipole model of the Earth’s magnetic field. The electron density and collision frequency profiles are divided into two parts – one for the lower ionosphere (60 to 200 km altitude), which does not vary with latitude, and one for the upper ionosphere (200 to 1500 km altitude), which does vary with latitude. The collision frequency is the sum of his derived electron-neutral and electron-ion collision frequencies. The electron density and collision frequency profiles effectively used by Helliwell [1965] are shown in Figures 5.1a,b and 5.2a,b, respectively. The corresponding profiles from the International Reference Ionosphere (IRI), which we use with the FWM model
and explain in detail in the next subsection, are provided for comparison in Figures 5.1c,d and 5.2c,d.

5.2.2 Full Wave Method (FWM)

The FWM model described in Lehtinen and Inan [2008, 2009] intrinsically accounts for wave attenuation along with multiple reflections, polarizations, and incidence angles, even if the medium is not slowly varying. For specified source current and media parameters, the field values can be computed for any horizontal plane – whether that plane is below, in, or above the ionosphere. As with Helliwell’s approach, the configurable media parameters are the background magnetic field, electron density, and collision frequency. We once again compute the background magnetic field using a dipole model of the Earth’s magnetic field, with the value of the field on the equator at the Earth’s surface set to $B_0 = 3.12 \times 10^{-5}$ T. We specify the electron density profiles using the latest International Reference Ionosphere (IRI) model: IRI-2007 [Bilitza and Reinisch, 2008]. Our selected electron density profiles for each step in latitude are shown in Figure 5.1c,d for daytime and nighttime. To acquire the set of daytime profiles, we choose local noon on the date of 15 July 2009 at a geographic longitude of 0°, and we vary the latitude from 10° to 80° in five degree steps. We choose a summer month to ensure that the entire range in latitude is in daytime, and we choose the year 2009 to overlap with the lifespan of DEMETER – a satellite which has been critical to many recent studies [e.g., Lehtinen and Inan, 2009; Cohen and Inan, 2012; Cohen et al., 2012]. To acquire the set of nighttime profiles, we change the month to January and the local time to midnight. Compared to Helliwell’s profiles, these IRI profiles tend to show significantly lower levels of electron density. Tao et al. [2010] analyzed in situ electron density values measured in several rocket studies to provide statistical bounds for the D-region electron density profiles. Based on their analysis, and following the lead of Cohen et al. [2012], we define the IRI-2007 electron density profiles of Figure 5.1c,d as our set of “regular” profiles, and we multiply these by five for nighttime and two for daytime to estimate a “dense” ionosphere. Similarly, we divide by five for nighttime and two for daytime to estimate
Figure 5.1: Electron density profiles effectively used by Helliwell [1965] for (a) Daytime and (b) Nighttime; and drawn from the International Reference Ionosphere (IRI) for (c) Daytime and (d) Nighttime. In each panel, the electron density profile is plotted for each five degree step in latitude ranging from $10^\circ$ in red to $80^\circ$ in blue. To assist with comparing the IRI profiles to the Helliwell profiles, the IRI profiles for $45^\circ$ latitude are included in black in panels (a) and (b), and the Helliwell profiles for $45^\circ$ latitude are included in black in panels (c) and (d).
**Figure 5.2:** Collision frequency profiles effectively used by Helliwell [1965] for (a) Daytime and (b) Nighttime; and as used in our FWM model for (c) Daytime and (d) Nighttime. In each panel, the collision frequency profile is plotted for each five degree step in latitude ranging from $10^\circ$ in red to $80^\circ$ in blue. To assist with comparing the IRI profiles to the Helliwell profiles, the IRI profiles for $45^\circ$ latitude are included in black in panels (a) and (b), and the Helliwell profiles for $45^\circ$ latitude are included in black in panels (c) and (d).
a “tenuous” ionosphere. We use these dense, regular, and tenuous profiles to study typical ionospheric variation.

Similar to the approach used by Helliwell, we compute our collision frequency profiles as the sum of electron-neutral and electron-ion collisions. The electron-ion collision frequencies only add significantly to the total collision frequency above 200 km altitude, and they only add significantly to trans-ionospheric attenuation for very high electron density values in the F-region, so their omission can often be justified. Electron-ion collisions are significant for several of Helliwell’s electron density profiles, however, so we will include them in all cases here for the sake of consistency. We compute the electron-neutral collision frequency based on Swamy [1992], and we compute the electron-ion collisions as in Helliwell [1965, p. 64]. Thus, our collision frequency profiles are a function of electron density and time of day. Collision frequency profiles for each step in latitude are shown in Figure 5.2c,d for daytime and nighttime. These collision frequency profiles calculated from the IRI electron density data consistently fall below the corresponding profiles of Helliwell. If an electron density profile is modified to analyze a dense or tenuous ionosphere, then the associated collision frequency profile is recalculated.

In assessing the amount of power injected into the magnetosphere from a terrestrial VLF source, we model that source as a 1 MW, short, vertical dipole 1 m above the surface of a flat, conducting Earth of conductivity \( \sigma = 10 \text{ mS/m} \). The power above the ionosphere is computed as the upward-propagating power on a horizontal plane at 700 km altitude. We choose 700 km altitude for consistency with the DEMETER satellite observations of Cohen and Inan [2012], Cohen et al. [2012], and with the analysis of Starks et al. [2008]. While Helliwell’s absorption curves go all the way to 1500 km for the top of the ionosphere, the amount of attenuation between 700 km and 1500 km altitude is often negligible (this point will be discussed in more detail in Section 5.3.) To compute the total upward-propagating power at 700 km altitude, we integrate the upward-propagating power flux in \( k \)-space. This procedure has the advantage of accounting for all power at that altitude that is within a certain range in \( k \), and this range can easily be set to capture nearly all of the power that could be radiated from our source to that altitude. Integrating the power in \( r \)-space with
the more traditional Poynting vector only captures the power within the physical \( r \)-space limits of the simulation space. Either computation method is acceptable for a sufficiently large simulation space, but we deem the \( k \)-space estimate to be less computationally intensive for our purposes and use it here.

### 5.2.3 Quick Full Wave Method (QFWM)

The FWM model provides our most accurate estimate of the amount of power reaching a horizontal plane above the ionosphere from a specified terrestrial source. That, however, presents a very complicated picture with multiple reflections, waveguide modes, and incidence angles. For purposes of analysis, it is useful to look at simpler scenarios much like those used to produce Helliwell’s absorption curves. In the QFWM model, we consider only a single plane wave incident on the base of the ionosphere. We vary the incidence angle, bearing, and polarization of this wave, along with aspects of the background media. This procedure makes for much quicker computation, and, more importantly, allows us to isolate and analyze the extent to which specific factors affect trans-ionospheric attenuation. The full wave method of Lehtinen and Inan [2008, 2009] is still used to compute the reflection and transmission coefficients for propagation through the ionosphere. While this model considers only a single incident plane wave, we do account for the presence of multiple incoherent reflections between the Earth and the ionosphere in computing the attenuation estimate, thereby avoiding complicated Earth-ionosphere waveguide mode interference patterns. Since the portion of the incident wave that reflects from the ionosphere can subsequently reflect from the Earth and be incident once again upon the base of the ionosphere, accounting for these multiple reflections leads to increased power injected through the ionosphere. Although such multiple reflection effects decrease the apparent attenuation, the effect on the QFWM results presented in this chapter is never more than \( \sim 1 \) dB.
5.3 Reasons for Discrepancy Between Models

In this section, we first use the QFWM model to illustrate the effects of several important factors for magnetospheric injection that were not accounted for in Helliwell’s absorption curves. Then we use the FWM model to produce a set of trans-ionospheric attenuation curves which are more applicable to magnetospheric injection from a terrestrial VLF source. For the sake of clarity, the initial illustrative analysis will focus on the case of a 20 kHz wave penetrating through the nighttime ionosphere. Note, we use the term “attenuation” throughout to refer to the ratio of the total power which penetrates through the ionosphere to the power of the source. Thus, reflection and absorption both add to the wave attenuation in this context.

We begin in Figure 5.3a by reproducing Helliwell’s absorption curve. Helliwell’s absorption curve, as read directly from Figure 3–35 of Helliwell [1965], is plotted in dotted black. Our recalculation of that absorption curve is plotted in dotted red for the case of a whistler mode wave vertically incident on the base of the ionosphere, using Helliwell’s ionospheric profiles and integrating the losses from 60 km to 1500 km as did Helliwell. We do not use the QL approximation in our recalculation, and we likely handle the numerical integration differently, but this approach successfully reproduces Helliwell’s curve to within a few percent at all latitudes above 30°. The QL approximation is known to fail at low latitudes, thus accounting for the increased deviation below 30°. The dotted green curve shows the results of integrating to only 700 km altitude instead of to 1500 km, and the decrease in estimated attenuation is clearly very small. Since the decrease in the upper altitude limit makes such a small difference in the total attenuation, we proceed with the 700 km upper limit for consistency with the DEMETER observations of Cohen et al. [2012].

The four blue curves of Figure 5.3a show the effect of wave polarization on trans-ionospheric attenuation. We produce each of these curves using the QFWM model. The triangle- and asterisk-marked curves are for left hand (LH) and right hand (RH) circularly-polarized waves, respectively. Traveling upwards through the ionosphere in
Figure 5.3: Illustrative QFWM results showing the importance of various factors which affect trans-ionospheric attenuation of VLF waves: (a) wave polarization, (b) incidence angle, (c) bearing, and (d) ionosphere profile. All curves are for a 20 kHz wave at nighttime. Helliwell’s absorption curve is included as the dotted black curve in each panel, and our recalculation of Helliwell’s absorption curve is included in (a).

The Northern hemisphere, LH is the whistler mode wave while RH is mostly evanescent. The LH curve is intended to reproduce Helliwell’s absorption curve by considering a vertically-incident, whistler-mode wave. The match with Helliwell’s absorption curve is excellent for latitudes above 50°, but there is some deviation at lower latitudes, most likely due to the fact that the QFWM model accounts for reflections while Helliwell’s absorption curves do not. The deviation there is still no more than a few dB, but this does alert us to small potential deviations between the integration approach and the QFWM modeling approach at low latitudes. The last two blue
curves in this figure show attenuation for the transverse magnetic (TM) and transverse electric (TE) modes. As Helliwell suggests, changing the incident wave from whistler mode to one of these linear polarizations leads to a $\sim 3$ dB increase in the trans-ionospheric attenuation.

In Figure 5.3b, we analyze the effects of changing the incidence angle of the wave impinging on the lower ionosphere. Helliwell’s absorption curve is again reproduced in dotted black. The family of colored curves then illustrates the transition from vertical incidence ($\theta = 0^\circ$) in red to grazing incidence ($\theta = 80^\circ$) in blue. We produced this family of curves using the QFWM model, so the red ($\theta = 0^\circ$) curve in this figure matches the QFWM-Whistler curve in Figure 5.3a. All these curves are again produced using Helliwell’s ionospheric profiles and an incident whistler mode wave. The results show that incidence angle is a very important factor in trans-ionospheric attenuation. At 35° magnetic latitude, a grazing-incidence, whistler-mode wave suffers $\sim 30$ dB more attenuation than a vertically-incident, whistler-mode wave for this bearing. Vertical incidence can be a reasonable assumption for certain cases – such as for a whistler impinging from the magnetosphere upon the top of the ionosphere at high latitudes – but several important scenarios necessitate the inclusion of higher incidence angles.

For the case of radiation from a terrestrial VLF transmitter (which can be estimated as a short, vertical dipole), there is a null in the antenna radiation pattern for 0° incidence, the incident power will peak with an incidence angle around $\sim 45^\circ$, and the waves mostly approach grazing incidence at waveguide distances greater than $\sim 150$ km. Since 45° incidence is a significant contributor to magnetospheric injection from terrestrial VLF transmitters, and it is an appropriate mid-way choice between vertical and grazing incidence, we continue with analysis at this incidence angle in Figure 5.3c.

Next, we examine the effects of changes in bearing angle by considering wave propagation in the four cardinal directions. The family of curves in Figure 5.3b are all for waves headed to the North. In Figure 5.3c, we take strictly the 45° incidence angle, but vary the bearing between North, South, East, and West. An average of those four is also provided. For these four QFWM curves and their average, we again use a whistler mode wave and Helliwell’s ionospheric profiles. There is significantly
more trans-ionospheric attenuation for wave propagation to the North and West. At 35° magnetic latitude, there is \(~20\) dB more attenuation for a wave headed to the North as opposed to the South. The average over the four cardinal directions provides a rough attenuation estimate if all bearing angles are to be accounted for equally.

Finally, in Figure 5.3d, we show the effect of changing the ionospheric profile from Helliwell’s values to the set of IRI-2007 values for both electron density and collision frequency. For better comparison to Helliwell’s absorption curve in this figure, we return to considering a whistler mode wave vertically-incident upon the ionosphere. As expected, the lower electron density and collision rates of the IRI-2007 profiles lead to a significant decrease in attenuation. This result is clear despite the tendency of the QFWM-Whistler result to estimate slightly more attenuation at low latitudes compared to Helliwell’s absorption curves (as we showed in Figure 5.3a for both curves using Helliwell’s ionospheric profiles). For this case of a 20 kHz, whistler mode wave vertically incident upon a nighttime ionosphere, the switch to the IRI-2007 ionospheric profiles estimates \(~30–40\)% less attenuation outside of the equatorial region. This is similar to the change shown by Tao et al. [2010] for a transition from Helliwell’s ionospheric profiles to the IRI.

In Figure 5.4, we accumulate each of the changes we just analyzed in Figure 5.3 as we move towards the FWM results. Helliwell’s absorption curve is reproduced again in dotted black, and the vertically-incident QFWM-Whistler in purple is provided again as this model’s closest reproduction of Helliwell’s result. This is using whistler mode, vertical incidence, and Helliwell’s ionospheric profiles. Moving to the blue QFWM-TM curve, we see the effect of switching from an incident whistler mode wave to an incident linearly-polarized wave: \(~3\) dB increase in attenuation due to the non-whistler-mode, circularly-polarized component being evanescent in the ionosphere. Moving to the red curve, we keep the TM polarization, and shift from vertical incidence to a 45° incidence angle. To account for bearing here, we provide only the average of the four cardinal directions. This change again adds several dB of attenuation across all latitudes. Finally, we take this scenario and switch the ionospheric profiles from Helliwell’s to the IRI-2007 set for both electron density and collision
frequency. The result is a substantial decrease in the attenuation estimates, producing the final QFWM curve in green which predicts less attenuation at low latitudes, and slightly more at high latitudes, compared to Helliwell’s absorption curve. One additional curve is provided in this figure: the FWM results are shown with the dotted magenta curve. The FWM results, which consider the more complicated picture of radiation from a terrestrial source as opposed to considering only single incident plane waves like in the QFWM, provide our most accurate attenuation estimate for the total amount of power penetrating through the ionosphere from a specified terrestrial source. We note that the final QFWM result plotted in green agrees closely with the FWM for this case. The choices of TM polarization, 45° incidence, averaging over the four cardinal directions, and using the IRI-2007 ionospheric profiles are chosen to roughly mimic the case of magnetospheric injection from a terrestrial VLF transmitter.

![Accumulated Changes](image)

**Figure 5.4:** Accumulated changes to the trans-ionospheric attenuation of a 20 kHz wave at nighttime, illustrated using QFWM results. The effects of polarization, incidence angle and bearing, and updated ionosphere profiles are sequentially added. Also shown are Helliwell’s absorption curve in dotted black, and the FWM results in dotted magenta.
5.4 Updated Attenuation Estimates

Having assessed the importance of various factors in the trans-ionospheric attenuation of VLF waves, we provide the set of FWM attenuation curves in Figure 5.5. We generate these curves using the FWM model for a short, vertical dipole radiating 1 MW of power at 2 or 20 kHz near the surface of a flat, conducting Earth of conductivity $\sigma = 10$ mS/m, as was described in Section 5.2.2. We provide both Helliwell’s absorption curves and our FWM results for 2 kHz and 20 kHz, and for daytime and nighttime. We consider dense, regular, and tenuous ionospheres (as defined in Section 5.2.2) for each FWM result. Both Helliwell and FWM predict significantly more attenuation at lower latitudes, but the effect is less pronounced in the FWM results; in comparison to Helliwell, FWM predicts less attenuation at low latitudes, and more attenuation at high latitudes. While the discrepancy between the models grows large in the equatorial region where attenuation is high, Helliwell and FWM agree to within $\sim 10$ dB for latitudes greater than $35^\circ$ for daytime, and they agree to within $\sim 5$ dB for latitudes greater than $30^\circ$ for nighttime. The FWM daytime ionospheric variation shows a spread of $\pm 3−4$ dB at mid-latitudes for 2 kHz, and $\pm 5−8$ dB at mid-latitudes for 20 kHz. The FWM nighttime ionospheric variation shows less than 1 dB of change in attenuation between the regular and tenuous ionospheres, but the dense ionosphere adds $2−4$ dB for both 2 kHz and 20 kHz at mid-latitudes.

Dry-Earth conductivity typically varies between 3 and 30 mS/m, so our ground conductivity of 10 mS/m is a reasonable estimate for much of the non-polar land on Earth. However, sea water is $\sim 4−5$ S/m, icy regions are only $\sim 0.01−0.1$ mS/m, and localized mineral deposits or sediment composition can lead to further variations [Morgan, 1968]. Figure 5.6 compares the FWM results for two different values of ground conductivity: the 10 mS/m used to generate the FWM results of Figure 5.5, and the 0.1 mS/m used by Cohen et al. [2012]. We use only the regular ionosphere profiles for this analysis. The change in ground conductivity from 10 mS/m to 0.1 mS/m has very little effect on trans-ionospheric attenuation for a 2 kHz source, but for 20 kHz it adds $\sim 1.5$ dB attenuation to the daytime curve, and $\sim 2.5$ dB to the nighttime curve.
Estimates of trans-ionospheric attenuation of VLF waves as calculated using the FWM model. Results are provided for (a) daytime, 2 kHz and (b) nighttime, 2 kHz, and for (c) daytime, 20 kHz and (d) nighttime, 20 kHz. Dense (×2 daytime, ×5 nighttime), regular (see Figures 5.1c,d and 5.2c,d), and tenuous (÷2 daytime, ÷5 nighttime) ionospheres are considered for each case. These results are most applicable to estimating the total magnetospheric injection from a terrestrial VLF source. Helliwell’s absorption curves are included for reference.

Just as Helliwell’s absorption curves, the FWM results of Figure 5.5 only provide trans-ionospheric attenuation estimates for a specific scenario. In the scenario for Figure 5.5, the waves incident on the base of the ionosphere as radiated by a short, vertical, dipolar terrestrial VLF source are comprised of many polarizations and incidence angles. A null exists in the antenna radiation pattern for vertical (0°) incidence, and the power incident on the base of the ionosphere peaks for an incidence angle around 45°. To consider an alternate scenario that more directly updates Helliwell’s absorption curves, Figure 5.7 provides trans-ionospheric attenuation estimates.
CHAPTER 5. TRANS-IONOSPHERIC ATTENUATION

Figure 5.6: Comparison of FWM trans-ionospheric attenuation estimates for different values of ground conductivity. Results are provided for (a) 2 kHz and (b) 20 kHz, and for daytime (orange) and nighttime (blue). Results for $\sigma = 0.1 \text{ mS/m}$ are marked with circles, and results for $\sigma = 10 \text{ mS/m}$ are marked with x’s. We use the ionosphere profiles of Figures 5.1c,d and 5.2c,d for each curve.

for the case of a whistler mode plane wave vertically incident upon the ionosphere. These curves are meant to mimic the scenario used for Helliwell’s absorption curves, but simply update them with the IRI-2007 ionospheric profiles, include the effect of reflections, and remove any simplifying analytical approximations. The change in ionospheric profiles leads to a significant decrease in attenuation estimates compared to Helliwell, as we would expect.

One potential application of this direct update to Helliwell’s curves is that, in many cases, they should provide reasonable trans-ionospheric attenuation estimates for the case of whistler mode plane waves penetrating from the magnetosphere,
5.4. Updated Attenuation Estimates

Figure 5.7: Estimates of trans-ionospheric attenuation of a VLF whistler mode plane wave vertically incident upon the ionosphere. Results are provided for (a) daytime, 2 kHz and (b) nighttime, 2 kHz, and for (c) daytime, 20 kHz and (d) nighttime, 20 kHz. Dense (×2 daytime, ×5 nighttime), regular (see Figures 5.1c,d and 5.2c,d), and tenuous (÷2 daytime, ÷5 nighttime) ionospheres are considered for each case. Helliwell’s absorption curves are included for reference.

through the ionosphere, and into the Earth-ionosphere waveguide. Magneto-spherically-generated VLF emissions such as chorus and hiss are two examples of such waves which are also important to radiation belt dynamics. In the absence of satellite-based measurements, ground-based recordings of these waves can be used to estimate characteristics of the in situ distribution [Horne et al., 2005; Spasojevic and Inan, 2005; Golden et al., 2011]. Similarly, understanding this trans-ionospheric propagation from the magnetosphere into the Earth-ionosphere waveguide is an important aspect of using ground-based whistler measurements to remotely sense plasmaspheric electron densities [Carpenter, 1966; Carpenter et al., 1981]. Many of these waves
will propagate approximately along the Earth’s magnetic field while in the magnetosphere. Whether such a whistler mode wave is incident vertically or approximately field-aligned, the trans-ionospheric attenuation estimates remain very close to those provided in Figure 5.7 as long as the wave normal is within the cone of transmission at the ionospheric boundary [Helliwell, 1965, Section 3.7]. This result is tied to our bearing angle analysis. If the bearing is such as to align the whistler mode wave along the background magnetic field as opposed to across it, then the resulting attenuation is nearly the same as for vertical incidence.

5.5 Discussion of the Updated Estimates

Cohen et al. [2012] thoroughly compared the FWM model to the average of thousands of DEMETER satellite passes for magnetospheric injection from ~20 kHz terrestrial VLF transmitters, finding agreement to within ±6 dB between model and observation for every transmitter analyzed, and for both daytime and nighttime. To properly compare with DEMETER observations, Cohen et al. [2012] utilized specific ionosphere profiles and transmitter parameters in the FWM model for optimal comparison to each VLF transmitter. They also integrated the power above the ionosphere computed by the FWM model in r-space using the same integration technique as applied to DEMETER data in Cohen and Inan [2012]. Their analysis served to validate the FWM model as a means of predicting magnetospheric injection from terrestrial VLF transmitters. The simulations utilized here are identical, apart from our use of more general ionosphere profiles and transmitter frequencies, integration in k-space to compute total power, and our use of a more realistic ground conductivity of 10 mS/m as opposed to 0.1 mS/m. The impact of this ground conductivity change on the results is 1–2 dB, which actually brings the FWM model results of Cohen et al. [2012, Figure 4] into even closer alignment (±5 dB) with the DEMETER observations.

While Cohen et al. [2012] validated the FWM model for frequencies around 20 kHz, validation for the 2 kHz range is more difficult as there are no VLF transmitters operating in that frequency range. The best approach is to use natural lightning, which emits energy across the whole ELF/VLF spectrum. This comparison
5.5. DISCUSSION OF THE UPDATED ESTIMATES

of the FWM model to observation for magnetospheric injection from a lower frequency terrestrial source is ongoing. At this point in time, the FWM model is not yet experimentally-validated at 2 kHz for our specific application of estimating trans-ionospheric attenuation. We also note that Cohen et al. [2012] focuses their comparisons at mid-latitudes, with no observations made below 20° magnetic latitude or above 65° magnetic latitude. In other words, the results in the equatorial and polar regions are not currently validated by observations. Low latitude whistlers observed on the ground [Singh et al., 2012] may provide a future technique to experimentally validate the low latitude absorption models. We have no reason to believe the FWM approach will fail at lower frequencies or at equatorial or polar latitudes, however, and the method has been successfully applied in the 1 to 3 kHz frequency range in previous studies for related applications [e.g., Piddyachi et al., 2008; Cohen et al., 2010a].

With the FWM model validated to within ±6 dB by Cohen et al. [2012] for ~20 kHz emissions from terrestrial VLF transmitters, the set of curves presented in Figure 5.5 provides our most accurate estimate of trans-ionospheric attenuation using a generic set of ionospheric profiles and transmitter parameters. These provide our best estimate of trans-ionospheric attenuation for the case of total power injected into the magnetosphere from a short, vertical, monochromatic, terrestrial VLF source. The remaining ~5–6 dB of error observed by Cohen et al. [2012] may be due to ionospheric variation and/or physical limitations of our FWM model. Since the medium in our FWM model is horizontally stratified, scattering from field-aligned irregularities or coupling into quasi-electrostatic modes [Bell and Ngo, 1990] is not accounted for, and this phenomenon could add to the trans-ionospheric attenuation of VLF waves [Foust et al., 2010; Bell et al., 2011; Shao et al., 2012]. Both Shao et al. [2012] and Foust et al. [2010] can attribute several dB of additional attenuation to the interaction of VLF waves with field-aligned irregularities, and both suggest that effect is more likely to occur during nighttime. The FWM does not account for such irregularities, so although Cohen et al. [2012] significantly downplayed the global role of irregularities, both naturally present and especially generated by the VLF heating, it is possible that a few dB of attenuation should be added to our estimates for nighttime, 20 kHz
at mid-latitudes. The same may also be true for 2 kHz. Additionally, a persistent \(\sim 20\%\) reduction in electron density near 80 km altitude may exist overhead a powerful VLF transmitter due to the ionospheric heating induced by the transmitter itself [Rodriguez and Inan, 1994]. This amount of deviation is captured by our ionospheric variation analysis, but a persistent increase in electron temperature and reduction in electron density could affect our estimate of typical trans-ionospheric attenuation.

### 5.5.1 Rectifying Disparate Conclusions of Recent Studies

Comparison of the QFWM and FWM models appears capable of rectifying the disparate conclusions of Starks et al. [2008], Tao et al. [2010], and Cohen et al. [2012] with regards to trans-ionospheric absorption. For 20 kHz, daytime at mid-latitudes, Starks et al. [2008] suggests that \(\sim 10\) dB more attenuation needs to be added to Helliwell’s absorption curves to bring it into line with observations. The results of Cohen et al. [2012] suggest \(\sim 10\) dB less attenuation is needed in this scenario, not more. The FWM results presented in Figure 5.5c agree with the DEMETER observations of Cohen et al. [2012], and thus disagree with the conclusions of Starks et al. [2008]. The discrepancy is due to amount of data analyzed and incidence angle.

The data presented by Cohen and Inan [2012] are based on hundreds of satellite passes over each of a dozen different terrestrial VLF transmitters, facilitating the creation of a 25 km resolution map of each transmitter’s radiation pattern at 700 km altitude. The observations of Starks et al. [2008] consist of no more than 16 satellite passes over any given VLF transmitter. Cohen and Inan [2012] simply analyzes much more data, providing better averaging over ionospheric variation and a better view of the center of the radiation pattern where the bulk of the VLF energy is found.

It is clear from the radiation patterns of Cohen and Inan [2012] and Cohen et al. [2012] that if a satellite pass is not within \(\sim 150\) km of the center of the radiation pattern, then the bulk of the peak power injected into the magnetosphere will not be observed. Starks et al. [2008], following the procedure developed by Inan et al. [1984], partially accounted for this fact by properly scaling the power of the VLF waves injected into the base of the ionosphere. However, Inan et al. (and, by extension
Figure 5.8: (a) Illustrative FWM results depicting (b) the direction and relative magnitude of the Poynting vector near the base of the ionosphere in the vicinity of a terrestrial VLF transmitter. (c) The estimated wave incidence angle, which very quickly approaches grazing incidence as the wave progresses forward in the Earth-ionosphere waveguide.

Starks et al.) did not account for change in incidence angle; they applied Helliwell’s absorption curves to estimate the trans-ionospheric attenuation at each point, which is equivalent to assuming vertical incidence at all points. As was shown in Figure 5.3b, changing from vertical incidence to grazing incidence causes a significant increase in trans-ionospheric attenuation (decrease in magnetospheric injection). In Figure 5.8, we use FWM results to analyze the wave incidence angle moving away from a terrestrial VLF transmitter. In the top panel, we provide the computed wave power density for within 300 km horizontal distance of a transmitter over an altitude range covering the base of the ionosphere. In the middle panel, we plot the Poynting vector for these results, computed as the cross-product of the total electric and magnetic...
fields at each point. The angle between this Poynting vector at 50 km altitude and vertical provides the incidence angle estimate plotted in the bottom panel. While this estimate of incidence angle does not fully account for the presence of multiple modes which each possess their own power and incidence angle, it should capture the effects of the dominant modes for this analysis. The Poynting vector shows a maximum for $\sim 45^\circ$ incidence angle, and the waves approach grazing incidence very quickly as they progress forward in the Earth-ionosphere waveguide. For waves penetrating the ionosphere even $\sim 150$ km from the transmitter, the Helliwell absorption curves will grossly underestimate the trans-ionospheric attenuation. If most of the satellite passes analyzed by Starks et al. [2008] were more than $\sim 150$ km horizontal distance ($\sim 1.3^\circ$) away from the center of the specified VLF transmitter’s radiation pattern, then direct use of Helliwell’s absorption curves to estimate trans-ionospheric attenuation would lead to the discrepancies between Starks et al. [2008] and Cohen et al. [2012].

Tao et al. [2010] present results from their own full wave method to study the variance of trans-ionospheric attenuation with changes in background electron density. Similar to our results in Figure 5.3d and 5.7, they find trans-ionospheric attenuation to be strongly dependent on the electron density of the ionosphere and analyze this particular phenomenon in more depth than done here. As both we and Tao et al. [2010] conclude, the media profiles used by Helliwell [1965] mostly overestimate the ionospheric electron density, and updating these profiles to more recent models leads to a substantial decrease in the estimated trans-ionospheric attenuation. Tao et al. [2010], however, looks strictly at single plane waves incident vertically on the base of the ionosphere. This led to the apparent discrepancy with the results of Starks et al. [2008], where the conclusions of Tao et al. [2010] were again mostly suggesting less attenuation than Helliwell as opposed to more. As we discussed above, the effect of incidence angle is likely significant enough in this scenario to reconcile the incongruity.

5.6 Effect of Updated Estimates on TIPER

Helliwell’s curves provide exactly what they claim: estimates of trans-ionospheric absorption for a whistler mode plane wave vertically incident upon the base of a
specified ionosphere, with the values incurring some error at low latitudes where the QL approximation is invalid. The ionospheric profiles used by Helliwell should be updated to contemporary models (as we did here for Figure 5.7), but otherwise his approach appears valid for the stated intentions. However, the Helliwell curves are not well applicable to estimate the magnetospheric injection of waves from a terrestrial transmitter with a short, ground-based monopole antenna. Incidence angle, bearing, wave polarization, multiple reflections, and ionospheric variation all affect that situation in ways not fully captured by Helliwell’s approach.

The new set of curves presented here in Figure 5.5 provide estimates of trans-ionospheric attenuation for the total amount of power injected into the magnetosphere from a terrestrial VLF transmitter. We generated these curves using the same FWM model which Cohen et al. [2012] shows agrees to within ±6 dB of satellite-based observations for this application. We must underscore the impact of ionospheric variation and its ability to vary these results. As Tao et al. [2010] and Cohen et al. [2012] have also shown, ionospheric variation has a significant effect on trans-ionospheric attenuation. Given how difficult it is to accurately determine the electron density profile of the ionosphere for any specific time and location, and given how much the profiles may vary, applying these results to any single observation should be done with great care. Applying them to long-term averages, however, should be more effective.

One of the goals of this work is to contribute to a complete understanding of the role terrestrial sources play in scattering magnetospheric electrons, particularly in the slot region [e.g., Abel and Thorne, 1998a,b; Kim et al., 2011]. Abel and Thorne utilized Helliwell’s trans-ionospheric absorption curves to estimate the effects of terrestrial VLF transmitters, and Kim et al. [2011] chose to scale the transmitter wave power in the magnetosphere down by a factor of 10 in comparison to Abel and Thorne based on the findings of Starks et al. [2008]. The FWM results of Figure 5.5 indicate that this factor of 10 adjustment made by Kim et al. [2011] may have been unwarranted. For daytime, 20 kHz, Helliwell actually overestimates the attenuation by 5–20 dB between 30° and 60° geomagnetic latitude, with greater overestimation at low latitudes. For nighttime, 20 kHz, Helliwell underestimates the attenuation at
mid-latitudes (30°–60°) by 0–9 dB, and overestimates the attenuation at low latitudes (≤20°) by 20–100 dB. Overall, these results suggest that the magnetospheric injection from terrestrial VLF transmitters at mid-latitudes for nighttime does not need to be drastically adjusted from the values predicted by Helliwell’s curves and utilized by Abel and Thorne [1998a]. Several dB of adjustments may be necessary, but not the factor of 10 or more suggested by recent studies [Starks et al., 2008; Kim et al., 2011].

While the trans-ionospheric attenuation curves in Figure 5.5 provide reasonable estimates for calculating the total power injected into the magnetosphere from a terrestrial VLF source, the analysis accompanying Figures 5.3, 5.4, and 5.6 highlights how limited the applicability of any single family of trans-ionospheric attenuation curves can be. Any changes to ionospheric density profile or ground conductivity affect the results. Any scenario in which the source is not a short, vertical dipole near the ground, or any scenario in which incident plane waves must be analyzed individually, requires the consideration of specific incidence angles, bearings, and wave polarizations.
Chapter 6

Satellite-Based Detection

Having covered sub-ionospheric observations recorded during the NPM keying experiments and a theoretical discussion of the trans-ionospheric propagation of VLF waves and its importance to the TIPER process, we turn now to satellite-based observations recorded during the NPM keying experiments. Previous experiments have successfully detected transmitter-induced bounce loss-cone electron precipitation in satellite-based measurements [Imhof et al., 1983; Inan et al., 1985], and others have detected aggregate scattering into the drift loss-cone by a VLF transmitter [Sauvaud et al., 2008], but the NPM keying experiments present the first opportunity for a multi-year study of induced bounce loss-cone precipitation by a keyed VLF transmitter. Wave and particle measurements were recorded onboard the DEMETER satellite, on which both lightning-induced electron precipitation [Inan et al., 2007a] and transmitter-induced electron precipitation [Sauvaud et al., 2008] events have previously been observed. Unfortunately, the quantification of induced bounce loss-cone precipitation is difficult with DEMETER data as the onboard particle detector does not directly view the bounce loss-cone particle flux local to NPM. Still, there is understanding to be gained in assessing these satellite-based observations recorded during the NPM keying experiments. Examples of coordinated transmitter-induced precipitation events detected onboard DEMETER are presented along with a description of the observation statistics from the two year study and a comparative theoretical analysis of the results. This material constituted the subject matter of Graf et al.
6.1 NPM Experiments and DEMETER Satellite

During the course of the NPM keying experiments, many transmission periods were selected to correspond to the traverses of the DEMETER satellite through the region of expected electron precipitation or its corresponding conjugate in the Southern hemisphere. During these traverses, DEMETER recorded in situ measurements of both electromagnetic field and energetic particle fluxes, which we analyze for correlations between NPM transmission bursts and particle flux bursts to identify cases of NPM-induced precipitation. The majority of the observed NPM transmissions were keyed in a 5-sec on/5-sec off format, and the VLF receiver MI stationed on Midway Atoll provided confirmation of NPM transmissions. Coordinated NPM keying experiments with DEMETER observations were possible roughly every third night between 27 March 2006 through 2 April 2008, with occasional breaks in experimentation due to high onboard memory usage in DEMETER burst mode and the need to shift emphasis to other experiments conducted with DEMETER satellite resources. The largest break occurred from 26 October 2006 through 10 April 2007, with no DEMETER recordings in burst mode being available during that time.

The predicted energetic electron precipitation region induced by NPM is shown in Figure 6.1, as determined using the WIPP model which is further discussed in Section 6.4. Note that this WIPP model utilizes Helliwell’s trans-ionospheric absorption curves for estimating the magnetospheric injection from the ground-based VLF transmitter. As was discussed in Chapter 5, these estimates should be updated in future modeling efforts. That update is beyond the scope of this dissertation, but advice for future work on the topic is provided in Chapter 7. According to the WIPP model, peak precipitation of $>100$ keV electrons is expected to occur at $L = 1.9$ with a full-width half-maximum (FWHM) of approximately 0.3 $L$ spanning the range $L = 1.7$–2.0. DEMETER passes through this precipitation region and its conjugate roughly once per day, south to north, traversing the FWHM of the precipitation region in approximately two minutes.
6.1. NPM EXPERIMENTS AND DEMETER SATELLITE

Figure 6.1: Location and magnitude of predicted NPM-induced precipitation region with the locations of the NPM transmitter and Midway Island (MI) receiver marked.

DEMETER is a microsatellite developed by the French National Center for Space Studies (CNES) with a ~700 km altitude, 98.3° inclination orbit [Parrot, 2006]. An onboard electric field instrument (ICE) measures electric field fluctuations of up to 20 kHz in burst mode, and an instrument for particle detection (IDP) [Sauvaud et al., 2006] measures 72.9 keV–2.35 MeV electrons with 8.9 keV resolution in burst mode at one sample per second. The 21.4 kHz transmission frequency of NPM places it above the cutoff of the ICE, but a powerful aliased signal is still received at 18.6 kHz. A correction factor of 2.7, which was determined from the filter characteristic of the ICE, is applied to the aliased signal to calculate the electric field strength of the NPM transmission at the location of DEMETER. The IDP collimator views ~30° FWHM perpendicular to the orbital plane with a geometric factor of 1 cm²str. During its passes through the NPM precipitation region, the IDP consistently points ~77.0° east of north. The Earth’s local magnetic field, according to IGRF/DGRF model data, is approximately \( H = 17.1 \ \mu T, \ Z = 29.3 \ \mu T, \ D = 13.6° \), where \( H \) is the horizontal component of the field, \( Z \) is the vertical component (positive downward), and \( D \) is the declination of the field (positive eastward). Given these parameters and their variations, the angle \( \theta_N \) between the IDP and the Earth’s magnetic field
at this location is typically within 0.1° of 77.9°, so the IDP views the local pitch angle range of \( \sim 62.9° - 92.9° \). In the conjugate region, the IDP points \( \sim 76.8° \) east of north and the local magnetic field is approximately \( H = 17.9 \mu \text{T}, Z = -32.8 \mu \text{T}, D = 20.6° \). The angle \( \theta_S \) between the IDP and the local magnetic field in the conjugate region is \( \sim 75.8° \), so the IDP views the local pitch angle range of \( \sim 60.8° - 90.8° \). As such, the DEMETER IDP views primarily the locally trapped particles, and does not provide a direct measurement of precipitating energetic particle flux. Nevertheless, perturbations in its measurements still serve as an indication of scattering events and can provide an estimate of precipitation flux upon additional calculations. Together, the ICE and the IDP of DEMETER provide in situ measurements of both the NPM transmissions and the energetic particle flux. We will use these measurements to study NPM-induced precipitation.

6.2 DEMETER Observations

6.2.1 Standard Formats

Case Studies

During selected few DEMETER passes through the NPM precipitation region and its conjugate with NPM transmitting in a periodic on-off format, significant bursts of energetic particle flux were detected by DEMETER in correlation with NPM on transmissions. Sample cases of detection are presented with their key features described. Analysis and Discussion of the results is reserved for Sections 6.3–6.5.

The first case of detection occurred on 29 December 2005 and is presented in Figure 6.2. On 29 December 2005 between 07:20:05 and 07:20:30 UT, DEMETER passed through the NPM precipitation region approximately 730 km east of its predicted center 10 minutes after the commencement of NPM keying. NPM transmitted in a 5-sec on/5-sec off format as confirmed by VLF data from MI. DEMETER ICE data exhibited the same NPM transmission format at its 700 km orbit and the IDP recorded two bursts of energetic particle flux near \( L = 2.0 \) in the 108.5–144.1 keV energy range. The first burst was centered at \( \sim 130 \text{ keV} \) while the second, recorded
6.2. DEMETER OBSERVATIONS

Figure 6.2: Results summary for 29 December 2005 traversing the predicted NPM precipitation region. (a) Magnetic field amplitude detected in the NPM frequency channel by the MI receiver showing the 5-sec on/5-sec off transmission format. (b) Spectrogram of electric field measured by DEMETER showing an aliased image of the NPM signal. (c) Spectrum of near loss-cone energetic electron flux as detected onboard DEMETER showing two bursts of particle flux closely following NPM on transmissions. (d) Integral flux of the energetic electron flux plotted in (c). The bursts in near loss-cone energetic electron flux correlated with NPM on transmissions suggest the detection of NPM-induced precipitation.
at a higher $L$-shell, occurred at $\sim 120$ keV. The bursts were each of 5 to 6 seconds in duration and each followed within $3 \pm 1$ seconds of the start of an NPM on transmission. The one-second time resolution of the IDP instrument did not allow for more accurate determination of delay times. DEMETER started recording burst mode data for this pass at 07:20:05 UT, which is the start time for the plots of Figure 6.2. The significant energetic particle flux which was measured in the opening seconds of this record period may have been due to the NPM on transmission which ended at 07:20:05 UT, but this cannot be stated definitively since data for that preceding transmission period were not captured by our data set. After the 25 second window shown here, no other such bursts of energetic particle flux were detected during this pass.

A similar case occurred on 3 September 2007 between 10:05:45 and 10:06:10 UT; this time in the conjugate precipitation region with DEMETER passing within 50 km of the center of the predicted region 21 minutes into the 30 minute NPM keying session. This case is presented in Figure 6.3. NPM once again transmitted a 5-sec on/5-sec off format. This NPM transmission was detected in DEMETER ICE data in the form of two pulses at two distinct frequencies for each 5-sec pulse, and the transmission was confirmed in the VLF data from MI. The persistent noise received at 18.6 kHz by the ICE in the southern hemisphere on this day was likely due in part to the 18.6 kHz transmitter NST located in Woodside, Australia ($38.5^\circ$S, 146.9$^\circ$E; $L = 2.34$). The multi-pulse configuration of the received NPM signal was a manifestation of Doppler shift resulting from the satellite motion [Starks et al., 2009]. One of the pulses was the signal which first propagated in the Earth-ionosphere waveguide to the southern hemisphere and leaked upward to the satellite altitude therein. The other pulse was the signal which entered the magnetosphere in the North, and then propagated to the southern hemisphere in a non-ducted, nearly field-aligned path. This second pulse would arrive in the conjugate region with a relatively high wave normal angle (and thus a high refractive index), oriented nearly horizontal along the satellite trajectory, thus leading to a large Doppler shift. Doppler shifted pulses were also visible in Figure 6.2, but the Doppler shifted pulse was much weaker in that case because detection was taking place in the northern hemisphere. Near $L = 2.0$, two
Figure 6.3: Results summary for 3 September 2007 traversing the predicted NPM conjugate precipitation region. (a) Magnetic field amplitude detected in the NPM frequency channel by the MI receiver showing the 5-sec on/5-sec off transmission format. (b) Spectrogram of electric field measured by DEMETER in the conjugate region showing a Doppler shifted aliased image of the NPM signal. (c) Spectrum of near loss-cone energetic electron flux as detected onboard DEMETER showing two bursts of particle flux closely following NPM on transmissions. (d) Integral flux of the energetic electron flux plotted in (c). The bursts in near loss-cone energetic electron flux correlated with NPM on transmissions suggest the detection of NPM-induced precipitation.
bursts of energetic particle flux were detected in the 188.6–242 keV energy range, with no noticeable change in energy spectrum occurring between the two bursts. The bursts were each of 3 to 5 seconds in duration and each followed within 3±1 seconds of the start of an NPM on transmission. Outside of the 25 second window discussed here, no other such bursts of energetic particle flux were detected during this pass.

Statistics

This type of on-off transmission format and associated detection technique comprised the majority of the NPM keying experiments during DEMETER passes. DEMETER traversed the precipitation region too rapidly with the background energetic particle flux varying too significantly for superposed epoch or Fourier analysis of the IDP data to be effective. The one-second time resolution of the IDP made detection more difficult with faster transmission formats like 1-sec on/1-sec off, and the quick pass through the precipitation region made the slower transmission formats such as 10-sec on/10-sec off less effective. As a result, 194 of the 211 passes utilized the 5-sec on/5-sec off format. Of these 194 passes, 91 were through the precipitation region and 103 were through its conjugate in the southern hemisphere.

Table 6.1: Results of analyzing the 194 DEMETER passes when NPM was transmitting in a 5-sec on/5-sec off format for NPM-correlated bursts of energetic particle flux.

<table>
<thead>
<tr>
<th>Number of Occurrences</th>
<th>Number of Occurrences</th>
</tr>
</thead>
<tbody>
<tr>
<td>in Precipitation Region</td>
<td>in Conjugate Region</td>
</tr>
<tr>
<td>2 Correlated Bursts</td>
<td>3</td>
</tr>
<tr>
<td>1 Correlated Burst</td>
<td>9</td>
</tr>
<tr>
<td>No Bursts Detected</td>
<td>73</td>
</tr>
<tr>
<td>1 Uncorrelated Burst</td>
<td>5</td>
</tr>
<tr>
<td>2 Uncorrelated Bursts</td>
<td>1</td>
</tr>
<tr>
<td>Total Number of Passes:</td>
<td>91</td>
</tr>
</tbody>
</table>

All passes were analyzed for potential signatures of NPM-induced precipitation using the detection technique detailed for the two cases above. If a significant burst in energetic particle flux lasted for 3 to 6 seconds and started within 3 seconds of the
start of an NPM on transmission, it was counted as correlated with NPM transmission and qualified as potentially NPM-induced precipitation. If such a burst in energetic particle flux started 4 to 9 seconds after the start of an NPM on transmission, it was counted as uncorrelated with NPM transmissions and was not considered to have been potentially caused by NPM. The results of this analysis are presented in Table 6.1. The majority of the cases of detection occurred for energetic electrons in the 100–200 keV range, with detection occasionally occurring in the 200–250 keV range. For the sake of completeness, the data were also analyzed for the correlation of flux decreases with NPM transmissions, but no instances of such correlation were found.

6.2.2 Special Formats

There were a number of other transmission formats attempted over the course of the experiments that were not included in the overall results discussed above. One of these was a 1-min off/5-min on format designed to allow the drift loss-cone to empty and subsequently turn NPM on just as DEMETER passed through the precipitation region. This type of transmission format was used during six DEMETER passes, and data from two of the six exhibited correlation of an increase in energetic particle flux with NPM on transmission. One of these occurred on 20 February 2008 and is presented in Figure 6.4.

On 20 February 2008 between 07:19:30 and 07:22:30 UT, DEMETER passed through the NPM precipitation region approximately 450 km east of its predicted center. NPM transmitted a 1-min off/5-min on format as detected both with the VLF receiver at MI and onboard DEMETER with the ICE. NPM turned from on to off when DEMETER was near $L = 1.65$ and a slight, but not statistically significant, decrease in energetic particle flux in the 99.6–144.1 keV energy range followed within thirty seconds. NPM turned back on when DEMETER was near $L = 1.8$ and a significant increase in energetic particle flux immediately followed. The energy of this flux enhancement proceeded to decrease with increasing $L$. 
Figure 6.4: Results Summary for 20 February 2008. (a) Magnetic field amplitude detected in the NPM frequency channel by the MI receiver showing the 1-min off/5-min on transmission format. (b) Spectrogram of electric field measured by DEMETER showing an aliased image of the NPM signal. (c) Spectrum of near loss-cone energetic electron flux as detected onboard DEMETER showing an increase correlated with NPM turning on at 07:21:00 UT. (d) Integral flux of the energetic electron flux plotted in (c). The increase in near loss-cone energetic electron flux correlated with NPM turning on suggests the detection of NPM-induced precipitation.
6.3 Analysis of DEMETER IDP Viewing Window

An analysis of the IDP viewing window in relation to the trapped and precipitating radiation belt particles is critical for proper interpretation of the experimental results. The IDP possesses a FWHM of $\sim 30^\circ$, and the angle between the IDP and the local magnetic field of the Earth is $\theta_N = 77.9^\circ$ in the northern hemisphere NPM precipitation region and $\theta_S = 75.8^\circ$ in the conjugate region. These two configurations are close enough that an in-depth analysis of just the northern precipitation region is sufficient for an understanding of both. Comparisons are drawn between this analysis and the clearest case of DEMETER measurements of 29 December 2005. For these purposes, it is shown in Figure 6.5 that the differential flux in the 130.75 keV energy bin increases from an average of 6.6 cm$^{-2}$s$^{-1}$str$^{-1}$keV$^{-1}$ for NPM off to 10.2 cm$^{-2}$s$^{-1}$str$^{-1}$keV$^{-1}$ for NPM on. These correspond to IDP counting rates of 59.0 s$^{-1}$ and 90.5 s$^{-1}$, respectively.

Figure 6.5: (a) Energy spectra of near loss-cone energetic electron flux as detected onboard DEMETER on 29 December 2005. (b) An estimate of the NPM-correlated flux increase computed by subtracting the average of the adjacent off windows of (a) from the on window and converting to cm$^{-2}$s$^{-1}$keV$^{-1}$. 
In the precipitation region, the IDP detects particles of local pitch angles $\sim 62.9^\circ - 92.9^\circ$. By combining the expression for a dipole magnetic field with the first adiabatic invariant, these local pitch angles are related to equatorial pitch angles through:

$$
\alpha_{eq} = \sin^{-1}\sqrt{\frac{\sin^2 \alpha \cos^6 \lambda}{\sqrt{1 + 3 \sin^2 \lambda}}}
$$

where $\lambda$ is the local geomagnetic latitude, $\alpha$ is the local pitch angle, and $\alpha_{eq}$ is the equatorial pitch angle. The corrected geomagnetic latitude at the location of DEMETER is $41.49^\circ$ for $L = 2.0$, and it is determined that the IDP thus views equatorial pitch angles $\sim 17.7^\circ - 19.9^\circ$. Given that the bounce loss-cone and drift loss-cone angles at the geomagnetic longitude of NPM are $\sim 16.86^\circ$ and $\sim 23.5^\circ$ respectively, it is clear that DEMETER in fact measures particles that are still trapped, but which are destined to precipitate at the South Atlantic Anomaly. In other words, particles detected by DEMETER at 700 km altitude possess pitch angles such that they mirror prior to interacting with the denser regions of the ionosphere. While this presents a background flux which can hinder the detection of precipitation events, the detection of bursts of energetic particle flux on DEMETER is still an indicator of pitch angle scattering and eventual precipitation, as is shown with our modeling below.

The illustrative model presented here consists of three main steps: 1) The evaluation of the counting rate (CR) integral for the IDP for the case of a typical equatorial differential directional flux $j(\alpha_{eq}, E)$. 2) The simulation of scattering by perturbing $j(\alpha_{eq}, E)$. 3) The recalculation of the CR integral and the determination of the precipitated flux for the scattered $j(\alpha_{eq}, E)$. Additional steps appear in the conversions between local and equatorial pitch angles and in scaling to match experiment and established models. In order to simplify the procedure, $j(\alpha_{eq}, E)$ is approximated as a scalable pitch angle distribution $j(\alpha_{eq})$. This approximation is valid for comparisons to our experimental results because the flux measurements appear in discrete energy bins of 8.9 keV resolution, so we can interpret $j$ as $j(\alpha_{eq}, E_{\text{min}} < E < E_{\text{max}}) = j(\alpha_{eq})$ by assuming the distribution to be uniform over our chosen energy bin.

Four equatorial pitch angle distributions (PADs) are presented: square, sine,
6.3. ANALYSIS OF DEMETER IDP VIEWING WINDOW

anisotropic, and shifted. Respectively, these are defined as

\[ j_1(\alpha_{eq}) = a_0 \rho_1 u(\alpha_{eq} - \alpha_{eq}^{lc}) \] (6.2)
\[ j_2(\alpha_{eq}) = a_0 \rho_2 u(\alpha_{eq} - \alpha_{eq}^{lc}) \sin(\gamma_1) \] (6.3)
\[ j_3(\alpha_{eq}) = a_0 \rho_3 \sin(\alpha_{eq}) \left[ 0.2 \sin^{0.4}(\gamma_1) + 0.8 \sin^{10}(\gamma_1) \right] \] (6.4)
\[ j_4(\alpha_{eq}) = a_0 \rho_4 \{ 10^{-4} u(\alpha_{eq} - \alpha_{eq}^{lc}) \sin^{0.2}(\gamma_1) \\
+ u(\alpha_{eq} - \alpha_{eq}^{c}) \left[ 0.46 \sin^{0.57}(\gamma_2) + 0.14 \sin^{12}(\gamma_2) \right] \} \] (6.5)

where \( \alpha_{eq}^{lc} = 16.86^\circ \) is the bounce loss-cone angle, \( u(\cdot) \) is the unit step function, \( a_0 \) is a scalable constant determined by the differential flux, and each \( \rho \) is a constant chosen such that \( \int_{0}^{\pi} \int_{0}^{\pi} j(\alpha_{eq}) u(\alpha_{eq}) d\alpha_{eq} = 1 \). Therefore, \( \rho_1 \simeq 0.52 \), \( \rho_2 \simeq 0.72 \), \( \rho_3 \simeq 1.23 \), and \( \rho_4 \simeq 1.30 \). In (6.5), \( \alpha_{eq}^{c} \simeq 19.92^\circ \) provides a shifted cutoff just outside the IDP viewing window. These equatorial PADs are shown in the top row of each panel in Figure 6.6. The first three are representative PADs, close variants of which commonly appear in radiation belt analysis [e.g., Anderson, 1976; Inan, 1977; Inan et al., 1978]. Most quiet time PADs tend to fall somewhere between the sine and the anisotropic distributions [Lyons and Williams, 1975]. The structure of the shifted PAD is designed to fall in between those of the sine and the anisotropic distributions, but its key feature is that its primary cutoff is shifted from \( \alpha_{eq}^{lc} \) to just beyond the IDP viewing window with only a very small tail extending to \( \alpha_{eq}^{lc} \). This shifted PAD is designed to match the experimental results discussed here and is representative of a PAD whose drift loss-cone is relatively empty. All distributions are assumed to be azimuthally invariant.

For comparison to the experimental results of 29 December 2005, the case of 130 keV particles at \( L = 2.0 \) is considered. According to the AE8 radiation belt model, the equatorial, omnidirectional differential flux is \( 5.62 \times 10^5 \text{ cm}^{-2}\text{s}^{-1}\text{keV}^{-1} \) for these parameters at solar minimum. Since this value must match \( \int_{0}^{2\pi} \int_{0}^{\pi} j(\alpha_{eq}) \sin(\alpha_{eq}) d\alpha d\phi \),
Figure 6.6: Four modeled pitch angle distributions (PADs) - square, sine, anisotropic and shifted - with pitch angles detected by DEMETER highlighted, plotted both at the equator and at the $L = 2.0$, $\lambda = 41.49^\circ$ location of DEMETER in the precipitation region. Scattering is simulated by convolving the equatorial PAD with a Gaussian distribution whose width is defined by the rms pitch angle scatter.
it is determined that $a_0 \simeq 89.4 \times 10^3 \text{ cm}^{-2}\text{s}^{-1}\text{str}^{-1}\text{keV}^{-1}$.

With the PADs defined and the $\alpha \leftrightarrow \alpha_{eq}$ relation of (6.1), the CR of the IDP located at $L = 2.0$, $\lambda = 41.49^\circ$ with orientation $\theta_N = 77.9^\circ$ is calculated following the formulation of Walt [1994]:

$$\text{CR} = E_{\text{bin}} \int_0^{2\pi} \int_0^\beta j(\alpha) A \cos(\eta) \sin(\eta) d\eta d\psi$$

where, $A = 4.67 \text{ cm}^2$ is the IDP area, $E_{\text{bin}} = 8.9 \text{ keV}$ is the energy resolution, $\beta = 15^\circ$ is the IDP half-width half-maximum, and the coordinate system has been transformed from that which is oriented along the magnetic field vector to that which is orientated along the direction of the IDP through use of the cosine law for spherical triangles. In this new coordinate system, $\eta$ is the polar angle measured from the IDP vector and $\psi$ is the azimuthal angle.

Next, the resultant pitch angle scattering is approximately estimated by convolving the equatorial PAD with a normalized Gaussian distribution. The standard deviation of the Gaussian effectively represents the root-mean-square (rms) pitch angle change $\sqrt{\langle(\Delta\alpha)^2\rangle}$. By estimating the wave parameters of the NPM signal in the magnetosphere as calculated in the model of Section 6.4 and following the formulations of Inan [1987] for scattering by coherent waves, the rms pitch angle change is found to be $\sim 0.001^\circ$.

Once a scattered equatorial PAD $j_{\text{scat}}(\alpha_{eq})$ is calculated, it is transformed to the detector location using (6.1) and the CR integral is recalculated using (6.6). The precipitation flux can be calculated by transforming $j_{\text{scat}}(\alpha_{eq})$ to the height of the upper ionosphere and calculating the downward-propagating flux at that location. Alternatively, the precipitation flux can be calculated directly from the equatorial distribution by following the formulations of Ristić-Djurović et al. [1998] and Lauben et al. [2001] which adjust for solid angle and flux tube compression:

$$N = 2\pi \sin^{-2}(\alpha^\text{lc}_{eq}) \int_0^{\alpha^\text{lc}_{eq}} j_{\text{scat}}(\alpha_{eq}) \cos(\alpha_{eq}) \sin(\alpha_{eq}) d\alpha_{eq}$$

The basic functionality of this illustrative model has been validated in two ways:
1) Precipitation flux is calculated using both methods discussed above in order to verify agreement and confirm that the coordinate transformations and numerical integrations required for the counting rate integrals were implemented properly. 2) For the case of a sine PAD with NPM transmitting, inducing an estimated 0.001° rms pitch angle change, the results are directly compared to the results of the precipitation model of Kulkarni et al. [2008b] which is discussed in Section 6.4. At the $L=2.0$, $E=130$ keV for which the scaling factors of this section have been calibrated, both models predict the precipitation flux to be $\sim 10^{-4}$ cm$^{-2}$s$^{-1}$keV$^{-1}$.

Figure 6.6 shows the four PADs before and after scattering, plotted both at the equator and at the location of DEMETER. For illustrative purposes in the plots, the standard deviation used to define the Gaussian for scattering the square, sine, and anisotropic PADs is 3.0°. The more realistic value of 0.001° is used for scattering the shifted distribution. The square and sine PADs illustrate the effects which pitch angle scattering can have on the IDP measurements. The square PAD clearly leads to the highest precipitation flux, producing $\sim 18$ times the precipitation of the sine PAD for $\sqrt{\langle (\Delta\alpha)^2 \rangle} \simeq 3.0°$, but it actually leads to a decrease in the Counting Rate Integral. Recall that the IDP views local pitch angles $\sim 62.9°–92.9°$, corresponding to equatorial pitch angles $\sim 17.7°–19.9°$, and particles are scattered away from these angles for a square PAD leading to a decrease in CR by 21%. For a sine PAD, the particles scattered into the near loss-cone region increase the CR by 15%. As was pointed out by Inan et al. [1978], the anisotropic PAD behaves approximately like a scaled square PAD near the loss-cone, and its CR decreases by 13%. Note that these CR changes are for $\sqrt{\langle (\Delta\alpha)^2 \rangle} \simeq 3.0°$; if this value is instead set to 0.001°, all of these CR changes fall to significantly less than 1%. These small percentage changes in CR would not be detectable over normal fluctuations without substantial averaging. The histogram of Figure 6.7 shows that the CR for about half of the IDP measurements is less than 36 s$^{-1}$. Even a 15% change in this CR would be less than the standard deviation of the measurement. Considering that the actual change is likely to be much less than 1%, only the most extreme of the observed cases would produce a positive detection for either of these PADs.

The fact that not one of PADs (6.2)–(6.4) with their edges at the bounce loss-cone
can produce a positive detection on DEMETER is in agreement with the low rate of detection seen in the experimental results. However, these distributions are flawed in that they produce significantly higher CR levels than are measured by DEMETER. For example, the sine PAD, scaled to match the AE8 model for 130 keV electrons as described above, produces a CR of $29 \times 10^3 \, \text{s}^{-1}$. The square PAD, $408 \times 10^3 \, \text{s}^{-1}$. These values are roughly $10^3$ and $10^4$ times greater than a typical CR. The shifted PAD corrects this issue by having a nearly square tail scaled by $10^{-4}$ extend to the bounce loss-cone while the bulk of the distribution is shifted to just beyond the viewing window of the IDP facilitating a significant CR increase following scattering. For the shifted PAD in Panel (d) of Figure 6.6, the CR changes from 55.9 s$^{-1}$ before scattering to 85.5 s$^{-1}$ after scattering. These CR values are generated using the realistic $\sqrt{\langle (\Delta \alpha)^2 \rangle} \approx 0.001^\circ$ and they closely mirror the measured CR values for 29 December 2005, which were 59.0 s$^{-1}$ for NPM off and 90.5 s$^{-1}$ for NPM on.

It is clear from this analysis that the detection of NPM-induced pitch angle scattering onboard DEMETER is possible, but requires a very specific PAD. The detection of pitch angle scattering at the angles viewed by DEMETER would suggest comparable pitch angle scattering of near loss-cone particles, meaning that some particles

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{counting_rate_distribution.png}
\caption{Distribution of average counting rates measured by DEMETER while traversing $L = 1.9 - 2.0$ of the NPM precipitation region. All outliers beyond the maximum value of the plot are grouped in the final bin.}
\end{figure}
would precipitate if near loss-cone particles are present in the PAD. Such detection of pitch angle scattering also requires favorable wave propagation considering that the wave magnetic field strength used in the rms pitch angle change calculation is a relatively high estimate, according to recent modeling by Lehtinen and Inan [2008]. To summarize, typical scattering events in the presence of most PADs would produce precipitation without causing a noticeable change in the flux measurements of the DEMETER IDP for the precipitation regions considered here. Therefore, the rarity of the observations on DEMETER is largely attributed to the orientation of the IDP, which primarily views the trapped particle population and is thus only capable of detecting a scattering event in the presence of particular PADs.

6.4 Comparison to WIPP Model

While the experimental results were compared to an illustrative model in the previous section in order to gain insight into DEMETER IDP measurements and demonstrate the critical importance of the PAD, in this section we compare the results to a much more detailed model. As is mentioned in Inan et al. [2007b], and is discussed in more detail in Kulkarni et al. [2008b], a model of whistler-induced particle precipitation (WIPP) is used to model the precipitation induced by the NPM transmitter. In this WIPP model, whistler wave propagation in the magnetosphere is simulated using the Stanford ray tracing code [Inan and Bell, 1977; Golden et al., 2010], including Landau damping effects in accordance with the theoretical formulation of Brinca [1972]. The plasmaspheric cold plasma density is based on Carpenter and Anderson [1992], while the energetic particle populations (with a sine pitch angle distribution) are based on the (solar minimum) AE8 fluxes. Pitch angle scattering of energetic particles into the loss-cone by the whistler wave is calculated according to the work of Bortnik et al. [2006], and yields precipitated flux as a function of energy, $L$-shell, longitude, and time. Parameters of the NPM transmitter as discussed in Section 6.1 serve as inputs to the model, and the bandwidth of the signal is estimated to be $\sim 3$ Hz based on DEMETER measurements. The results of the model are summarized in Figure 6.8 and are briefly presented here. A more thorough discussion of the model and its results
are available in Kulkarni et al. [2008b] and references therein. The WIPP model utilizes the outdated Helliwell trans-ionospheric absorption curves. While a proper update to the WIPP model is beyond the scope of this dissertation, recommendations for its improvement will be provided in Chapter 7. For the present analysis, we note that it may affect the magnitude of precipitation flux, but is unlikely to affect the distribution of that precipitation in either energy or time. Also, the influence here of PAD and trapped particle flux levels should be much greater than the error incurred due to outdated trans-ionospheric absorption estimates.

Figure 6.8: WIPP simulation results. (a) Predicted distribution of NPM-induced precipitation of >100 keV electrons L. (b) Predicted precipitation at L = 1.9 as a function of energy and time for a 1-second burst of transmission by the NPM transmitter starting at time t = 0. (c) Predicted precipitation versus energy plotted for L = 1.8, 1.9, and 2.0 showing peak energies and flux levels.

According to the model, peak precipitation of >100 keV electrons occurs at L = 1.9 with an FWHM of approximately 0.3 L spanning L = 1.7–2.0. The precipitation
peaks near a resonant energy which decreases with increasing $L$-shell, occurring near 157 keV at $L = 1.8$, 98 keV at $L = 1.9$, and 59 keV at $L = 2.0$. At $L = 1.9$, the peak precipitation flux is $1.17 \times 10^{-3}$ cm$^{-2}$s$^{-1}$keV$^{-1}$. While these results are for a sine PAD with AE8 scaling, results from this same model were presented in Inan et al. [2007b] for a square PAD and for energetic particle populations based on observations from the POLAR spacecraft [Bell et al., 2002]. The use of a square PAD leads to higher values of precipitation flux, but the distribution of that flux in energy, $L$-shell, longitude, and time remains largely unchanged.

As discussed in Section 6.3, the results of this model cannot be directly compared to experiment because DEMETER does not directly measure the flux of precipitating particles. In order to attempt an indirect comparison, the shifted PAD of Figure 6.6 was concocted to simultaneously reproduce the counting rate measurements of DEMETER and produce an estimate of the induced precipitation at a specific energy and location. This exercise was nominally successful, but the results cannot be used as a check against the magnitude of the precipitation flux predicted by the model of this section. The reason is that even if a PAD is constructed to reproduce the DEMETER measurements, the shape and absolute level of its near loss-cone distribution can still be manipulated to give a wide range of precipitation fluxes. This behavior is greatly accentuated by a low rms pitch angle change like our $0.001\degree$. For example, two feasible near loss-cone edges for the shifted PAD are $10^{-4}\sin^{0.2}(\gamma_1)$ and $10^{-3}\sin(\gamma_1)$. While both of these reproduce the background CR typically measured by DEMETER, the former produces $\sim$600 times the precipitation flux for our rms pitch angle change. The shifted PAD used in Section 6.3 produced a precipitation flux of $\sim10^{-4}$ cm$^{-2}$s$^{-1}$keV$^{-1}$, which is in approximate agreement with the model of this section for the same $L = 2.0$, $E = 130$ keV, but this agreement is mostly coincidental.

Even though the limitations of the experiment prevent us from determining the actual magnitude of the precipitation flux, DEMETER observations can still provide inputs on its key determining factors, which are the rms pitch angle change and the near loss-cone pitch angle distribution. The rms pitch angle change calculated using the formulations of Inan et al. [1978] and used by the model of this section reproduce the DEMETER measurements. While this agreement is still closely tied to the use of
a configurable PAD and outdated trans-ionospheric attenuation estimates, this result is nevertheless encouraging. The other key factor, the near loss-cone distribution, can be restricted in scale and shape to those combinations which can produce the range of counting rates measured by DEMETER. As discussed in Section 6.3, the typical IDP counting rates presented in Figure 6.7 suggest that a square PAD should be scaled by $\sim 10^{-4}$ relative to AE8, and that a sine PAD should be scaled by $\sim 10^{-3}$. This result can be directly compared to the model predictions presented in Inan et al. [2007b], which used a square PAD with scaling based on Bell et al. [2002]. For $L=2$ and energies near 100 keV, the scaling of Bell et al. [2002] is about 100 times less than that of AE8. Therefore, the square PAD of Inan et al. [2007b] is effectively a steep, scaled-down near loss-cone distribution much like the tale of the shifted PAD of Figure 6.6, Panel (d). Since it is only the edge of the loss-cone that matters for the purposes of absolute value of precipitation when the rms pitch angle change is low, this choice is reasonable. However, a square PAD should be scaled by $\sim 10^{-4}$ relative to AE8, not just $10^{-2}$, to represent a typical near loss-cone edge. This scaling means that the results of Inan et al. [2007b] are more applicable to the $\sim 3\%$ of the days for which near loss-cone flux is markedly higher and a $10^{-2}$ scaling with respect to AE8 would agree with DEMETER measurements of background flux densities in the NPM precipitation region.

While the above analysis of the magnitude of the precipitation flux is very convoluted and limited, the energy spectrum of the precipitation bursts detected by DEMETER can be compared more directly to the WIPP model. The energy spectra of 29 December 2005 were presented in Figure 6.5 for periods near $L = 1.97$. Panel (b) of Figure 6.5 shows an NPM-induced increase of 0.12 cm$^{-2}$s$^{-1}$keV$^{-1}$ near 130 keV. The flux bursts detected by DEMETER are at energies higher than the model predicts. This result may suggest higher numbers of more energetic electrons in the trapped electron energy spectra, or that the cold plasma density is lower than modeled leading to an increase in resonant energy. For example, if the cold plasma density is simply scaled by $2/3$, then the energy of peak precipitation for $L = 1.9$ changes from 98 keV to 136 keV. These scenarios could explain the detection of a 130 keV precipitation peak near $L = 1.97$ as seen on 29 December 2005, and the
detection of a 220 keV precipitation peak near $L=2.0$ as seen on 3 September 2007. The 3 September 2007 case, however, should not be compared directly to the model results because it is a case of detection in the conjugate region while the model results are specific to the northern precipitation region. While the precise energies appear to be high in these cases, it should be noted that detection at lower energies is largely prevented by the presence of high background flux. Additionally, it is encouraging that the energies of the energetic electron bursts decrease with increasing $L$ for the cases of 29 December 2005 and 20 February 2008. The model predicts a similar rate of decrease in energy. The electron bursts detected on 3 September 2007, which did not change in energy by a discernible amount, did not follow this trend. However, considering the 8.9 keV energy resolution of the electron flux measurements and the expected change of $\sim 10-20$ keV over the $\sim 0.04$ change in $L$, the lack of discernible change in energy is within the error of this comparison.

The model predicts that the onset delay of precipitation following the commencement of an NPM on transmission should be just less than a quarter of a second for the northern hemisphere, and another quarter of a second for the conjugate. These delays closely reflect the approximate quarter second bounce period of a 100 keV electron at $L=2.0$, and the group travel time of the NPM signal to the geomagnetic equatorial region where most of the pitch angle scattering occurs. The one second time resolution of the IDP prevents a precise measurement of this delay, but most bursts tend to appear with a 1-3 second delay. The lengthy delay may be the result of pitch angle scattering requiring multiple interactions to fill the IDP viewing window to the point that detection can occur. Since the IDP views trapped particles, as particles at higher pitch angles outside the IDP viewing window undergo multiple bounces and are scattered multiple times, the IDP viewing window may be gradually filled until detection finally occurs at a delayed time. This is one possible explanation for the increase in onset delay, but by itself it is still insufficient because the bursts in energetic particle flux that DEMETER detects do not appear gradually, but rather are delayed and then appear abruptly. This observed behavior would suggest a PAD which is shifted slightly further from the IDP viewing window than the simulated PAD in Panel (d) of Figure 6.6. For such a PAD, multiple resonant interactions
would be required before the pitch angle scattered electrons appear in the IDP viewing window. Another explanation – one that could explain both the high energy and the lengthy onset delay – is that the detected flux bursts are the result of pitch angle scatter by the wave that has already reflected off the conjugate ionosphere and is propagating back along the magnetic field line. This reflected wave would have a smaller component of its wave vector parallel to the magnetic field line, leading to a higher resonance energy, and would also interact with particles at a later time than would the initial wave. In agreement with this possibility is the fact that Doppler shifted NPM pulses were observed on both the 29 December 2005 and 3 September 2007 cases when significant onset delay occurred, but a Doppler shifted NPM pulse was not observed on 20 February 2008 when the onset delay was much shorter.

Despite the minor disagreements with the best-case experimental results, the model is still in agreement with the general result that NPM-induced precipitation should only rarely be detected onboard DEMETER. The minimum energy particle detectable by DEMETER is 72.9 keV, and the background flux detected in the 72.9-100 keV energy range is consistently very high, effectively preventing the analysis of small perturbations of precipitation bursts below 100 keV. Additionally, for a precipitation event to be detected by DEMETER in the 100-200 keV range near $L=2.0$, an increase in energetic electron flux on the order of $1 \text{ cm}^{-2}\text{s}^{-1}\text{str}^{-1}\text{keV}^{-1}$ or greater is required. Such an increase requires either very specific PADs or transmitter-induced precipitation of quantities much higher than those predicted by simulation, and thus should only occur on rare occasions. Given that even a single burst of such precipitation events is observed on less than 15% of the passes, it is likely that the majority of the scattering events induced by NPM are simply below the level of detection for the IDP instrument, due largely to the orientation and viewing window of that instrument.

6.5 Further Discussion of DEMETER Observations

The detection of consecutive bursts of near loss-cone energetic electron flux which correlate with NPM transmissions suggest that NPM induced significant precipitation
at those times and that DEMETER successfully detected the signatures of those events. Counting only two-burst events, detection occurred on only 2.6% of the DEMETER passes. If both one-burst and two-burst events are counted, the detection rate increases to 13.9%. Despite the low rate of detection, the results are in general agreement with model predictions. Even if NPM routinely induced precipitation, DEMETER would only detect the event in the cases of a particular initial PAD, and only if the energies and levels of the pitch angle scattering were at slightly greater values than those predicted by the WIPP model.

The reason for the low detection rate of NPM-induced precipitation has been explained by the low levels and energies to be expected for NPM-induced precipitation as discussed in Section 6.4, in conjunction with the confounding factor of the IDP viewing window as discussed in Section 6.3. An additional factor is the large differential between the bounce and drift loss-cones at the longitude of NPM. This differential means that few particles may reside near the loss-cone and significant cumulative scattering may be required in order to induce detectable flux increases. Instances of geomagnetic activity could populate this region of the pitch angle distribution, but no significant correlation was found between geomagnetic activity and instances of precipitation. (It should be noted, however, that very little significant geomagnetic activity occurred during the course of these experiments, and that the Kp and Dst indices used for comparison may not be localized enough for the required analysis.)

It is unclear what facilitated detection in the few cases that it did occur. Increased coupling of VLF wave power into the magnetosphere could increase pitch angle scattering, but the power of the NPM signal as detected onboard DEMETER was no higher than usual for the cases of detection. Geomagnetic activity or lightning west of NPM could prime the PAD for detection, but, based on the Kp and Dst indices and samples of lightning activity from LIS (Lightning Imaging Sensor) data [Christian et al., 1999; Boccippio et al., 2002; Christian et al., 2003], there was nothing unique about the times when detection did occur. The detection of a Doppler shifted NPM pulse in the northern region would suggest that the wave has reflected off the conjugate ionosphere and traversed the magnetosphere with increased wave normal
angle. As was suggested in Section 6.4, this reflected wave may induce the cases of pitch angle scattering which are detected by DEMETER with a lengthy onset delay. However, such Doppler shifted pulses are detected on nearly every pass, so this is not a characteristic that can be unique to the cases of detection. Since the shape of the PAD near the IDP viewing window is so critical to detection, it is suspected that the cases of detection benefited from a favorable PAD, but it is unclear what would have established those conditions.

The one-shot detection case of 20 February 2008, where NPM turned on after a full minute of being off and a significant increase in energetic particle flux immediately followed, may illustrate the behavior near the loss-cone in the PAD. Even when NPM is not transmitting a specific on-off format for these experiments, the transmitter is typically on transmitting modulated signals for its regular message traffic so that the near loss-cone region may always be populated by the resultant scattering. The minute of off time on this day may have allowed this region to empty so that a significant increase was witnessed when NPM was turned back on. On the other hand, the NPM signal typically transmits in an MSK (Minimum Shift Keying) format when it is not being keyed for these experiments, and even though NPM is on prior to the initiation of a keying session, its signal may be less effective at pitch angle scattering.
Chapter 7

Conclusions and Future Work

In this chapter we present a summary of our results as organized within each of the scientific contributions of this dissertation. Then we conclude with a discussion of future work to be done both on the topic of ionospheric heating and on the topic of transmitter-induced precipitation of electron radiation.

7.1 Scientific Contributions & Summary of Results

**Contribution #1:** Determined that heating of the ionosphere by VLF transmitters is the cause of modulation observed on probe signals during controlled experiments.

The naval VLF transmitter NPM was modulated in periodic on/off keying formats between the dates of 25 August 2005 and 2 April 2008 with the goal of better quantifying its effects on the Earth’s radiation belts. During the NPM keying sessions, a perturbation of the same periodicity and phase regularly appeared on the sub-ionospherically propagating VLF probe signal NLK received at MI. The periodic perturbation was not present on the probe signal at times when NPM was not modulated. The NLK-MI probe signal pathway passes ~1750 km north of the NPM transmitter, directly through the theoretical location of the NPM-induced bounce loss-cone energetic electron precipitation region. *Inan et al.* [2007b] initially reported these observed probe
signal perturbations as sub-ionospheric VLF remote sensing of an ionospheric disturbance generated by NPM-induced precipitation of electron radiation. The initial reports proved incorrect, however, when improved signal processing illuminated the less than 20 msec onset delay of the perturbations. This lack of onset delay eliminated transmitter-induced precipitation as a possible physical cause because that process would require at least 200 msec to occur. Thorough testing of the MI receiver eliminated the possibility of instrumental cross-modulation as a confounding variable in these experiments, which left direct ionospheric heating by the NPM transmitter as the most probable physical explanation. This physical process would occur instantaneously compared to the narrowband data sampling rate at MI, so it matched the time signature of the observed perturbations. An observed seasonal variation in the detected perturbation magnitude, but no correlation with geomagnetic activity, also suggested ionospheric heating could be the cause.

Contribution #2: Established experimentally that the lateral extent of ionospheric heating due to VLF transmitters is several thousand kilometers, significantly greater than previously recognized.

Contribution #3: Developed a large-scale modeling framework to confirm theoretically that ionospheric heating can account for the observed probe signal modulations.

The determination of ionospheric heating as the most probable physical cause of the observed probe signal perturbations left two effects to consider: off-path scattering of the NLK-MI probe signal from the intense ionospheric heating near NPM, and along-path scattering of the NLK-MI probe signal from the relatively weak extended lateral ionospheric heating of NPM. Arrival azimuth and theoretical analyses both indicated off-path scattering were insignificant in this situation; the NPM transmitter was too distant from the NLK-MI pathway and the required scattering angle too wide for the intense ionospheric heating near NPM to have an effect. A large scale computational model was assembled to estimate the extended lateral heating generated by NPM and the effects that heating would have on the NLK-MI probe signal. This model showed
electron temperature increases of \(\sim0.5\%\) in the D-region of the ionosphere extending up to 2000 km from the heating transmitter. It was confirmed that along-path scattering of the NLK-MI probe signal from this ionospheric heating, which would extend over portions of the NLK-MI pathway, could theoretically have produced the sub-ionospheric VLF perturbations observed during the NPM keying sessions. This confirmation illuminated a pair of conclusions which had not been previously recognized in full for these experiments: 1) the lateral extent of ionospheric heating due to powerful VLF transmitters is several thousand kilometers, and 2) sub-ionospherically propagating VLF signals are remarkably sensitive to D-region conductivity changes, which becomes particularly noteworthy when averaging facilitates improvement in signal to noise ratio.

**Contribution #4:** Identified the causes for discrepancy between observations and theoretical estimates of trans-ionospheric attenuation of VLF waves and provided an updated set of estimates based on full-wave modeling.

While sub-ionospheric VLF remote sensing was successfully used only in detecting the effects of ionospheric heating, theoretical analysis and satellite-based measurements provided improved understanding of aspects of the transmitter-induced precipitation process. The trans-ionospheric attenuation of VLF waves has only just recently been estimated accurately with an experimentally-validated model \([\text{Cohen and Inan, 2012; Cohen et al., 2012}]\). This attenuation is a critical component in the TIPER process, with some past reports suggesting up to 20 or even 100 dB discrepancy between observations and previously-used estimates \([\text{Starks et al., 2008; Tao et al., 2010}]\). The numerous factors for these discrepancies were assessed, with the incidence angle of the VLF wave on the base of the ionosphere proving to be the most explanatory factor in these discussions. Updated estimates for trans-ionospheric attenuation of VLF waves have been provided both for the case of total magnetospheric injection from a ground-based VLF transmitter, and for the case of a VLF whistler mode plane wave vertically incident upon the ionosphere.
CHAPTER 7. CONCLUSIONS AND FUTURE WORK

Additional Results

Coordinated satellite-based observations recorded onboard DEMETER during the NPM keying experiments provided evidence of NPM-induced precipitation, but instrumental shortcomings for this application prevented robust quantification of those effects. DEMETER satellite has been used with great success to detect the aggregate effects of scattering into the drift loss-cone [e.g., Sauvaud et al., 2008; Gemelos et al., 2009], but the alignment and viewing window of its instrument for particle detection make the quantification of bounce loss-cone precipitation difficult. Another complication in assessing bounce loss-cone effects proved to be the pitch angle distribution of the trapped electron population. Long-term averaged studies may successfully assume a typical PAD based on past satellite observations and geomagnetic conditions, but any bounce loss-cone case study analysis requires knowledge of the immediate PAD to reach a reliable conclusion. Even small changes to the PAD can change the detected precipitating particle flux by an order of magnitude.

Conclusion

While we have not quantified the effects of VLF transmitters on the Earth’s radiation belts, we have improved understanding and estimates for portions of that process in addition to better illuminating the far-reaching effects a VLF transmitter has upon the ionosphere. For a powerful ground-based VLF transmitter at mid-latitudes, \( \sim 20\% \) of the total radiated power penetrates through the ionosphere into the magnetosphere where it can influence radiation belt dynamics [Graf et al., 2013a]. \( \sim 50\% \) of the power heats the ionosphere within 400 km lateral distance, generating D-region electron temperature increases of up to 200\% and electron density changes of up to 30\% [Rodriguez and Inan, 1994; Rodriguez et al., 1994; Graf et al., 2013b]. These are significant changes which can influence radio wave propagation both below and through the ionosphere. The final \( \sim 30\% \) of the total power radiated by a ground-based VLF transmitter attenuates at distances greater than 400 km, generating \( \lesssim 1\% \) changes to electron temperature and density. This extended lateral heating can have a minor influence on radio wave propagation, but only for signals that are very sensitive to
7.2 Future Work on Ionospheric Heating

While the topic of ionospheric heating by VLF transmitters has now been analyzed from multiple perspectives [e.g., Galejs, 1972; Inan, 1990; Barr and Stubbe, 1992; Taranenko et al., 1993; Rodriguez et al., 1994; Graf et al., 2013b] and its general form seems established, there remains significant work to done. The most immediate issue to address is the construction of a fully self-consistent ionospheric heating model. Wave propagation, ionospheric heating and electron density changes all influence one another. Any true calculation of one parameter must solve for all three together, but computational limitations have forced most modeling efforts either to consider the components as separate steps, or to make compromises in modeling one aspect or the other. Most models do not even consider the electron density changes despite the fact that the 30% change nearby a VLF transmitter could significantly alter wave propagation. (We did not account for electron density changes in our model of Chapter 4, but that was because those changes would occur too slowly to affect our observations.) Positive feedback may even exist between the ionospheric changes and penetration of VLF waves into the ionosphere. Ionospheric heating produces electron density depletion through most of the D-region, which can allow for increased penetration of VLF waves into the ionosphere and thus more ionospheric changes. Rodriguez and Inan [1994] mention that the feedback would ultimately be inhibited by a maximum in the effective three-body electron attachment rate around $T_e = 700$ K [Tomko, 1981, p. 163], but it is unclear at what point between the ambient $\sim 200$ K temperature and that 700 K threshold the feedback would settle. For greater temperature increases and longer time scales, heat flow may also become an important factor. Efforts to build upon the time-domain lightning electromagnetic pulse model of Marshall [2012] have shown promise in creating a self-consistent wave propagation and ionospheric heating model, but that work is not yet complete.

It is also worth considering any possible connection between this lower ionosphere heating and the heating observed at 700 km altitude overhead VLF transmitters [Bell
et al., 2011]. The higher altitude heating is yet to be modeled, but Bell et al. [2011] suggest that high altitude heating is most likely separate from the lower ionosphere heating, and that it is due primarily to heating between the E-region and 700 km altitude. Also note that any changes to ionospheric electron temperatures and densities would have subsequent effects on VLF propagation, most notably upon the trans-ionospheric propagation of VLF waves discussed recently by Cohen and Inan [2012], Cohen et al. [2012], Graf et al. [2013a], and here in Chapter 5.

7.3 Future Work on Transmitter-Induced Precipitation

While numerous examples of transmitter-induced electron pitch angle scattering and drift loss-cone precipitation exist [e.g., Imhof et al., 1983; Sauvaud et al., 2008; Selesnick et al., 2013], efforts at directly detecting bounce loss-cone precipitation could be improved. Attempting to detect TIPER with NPM between 2006 and 2008 via sub-ionospheric remote sensing in the same hemisphere as NPM was not ideal. Ionospheric heating proved to be a confounding variable capable of confusing detection even at several thousand kilometers distance, so sub-ionospheric detection in the conjugate hemisphere would be preferred. Also, the dates of experimentation spanned only an extended solar minimum, meaning minimal geomagnetic activity and changes in trapped energetic particle distributions likely limited the opportunities for detecting bounce loss-cone precipitation. Experiments executed during solar maximum may prove more fruitful. In addition, the location of the transmitter NPM is not ideal for inducing particle precipitation. A VLF transmitter located at a slightly higher L-shell should be more effective [Kulkarni et al., 2008b], and the longitudinal dependency discussed by Cotts et al. [2011] should also be considered when picking the transmitter and hemisphere of detection. The transmitter NWC appears to be the most appealing candidate for detecting particle precipitation among present VLF transmitters. Keying experiments have been performed with NWC, but an effective sub-ionospheric detection network did not yet exist in the conjugate precipitation
region at that time. Finally, if satellite-based detection is possible, then direct detection of bounce loss-cone precipitation would benefit from particle detectors specifically designed and aligned to quantify precipitating particle fluxes.

While these modified experiments should prove superior in directly detecting bounce loss-cone precipitation, that alone may serve little purpose. First of all, periodic keying experiments like the one performed with NPM may produce a rather confusing effect on the trapped particle population that would be difficult to interpret. Naval VLF transmitters are typically communicating transmissions 24 hours a day, only rarely turning off for maintenance purposes. Then an experimental keying session begins, and the on-off modulation of the transmitted signal takes place. The “modulation” in this case is actually from the nominal “on” state to “off,” not the other way around. Most resonant-energy particles existing on the edge of the bounce loss-cone would have been drained by the preceding transmissions during regular transmitter operation. Drifting and scattering of particles to refill that loss-cone edge happens continuously, but may require several minutes to significantly accumulate under quiet geomagnetic conditions. In a sense, the induced precipitation “system” may well be at a point of saturation. The ionization of the ionosphere in the precipitation region can exhibit similar characteristics, though likely over shorter time scales. Modulating the transmitter off for seconds at a time may only produce a very minor effect under these conditions. This possibility motivated some unique transmission formats during our experiments featuring several minutes of off time, but those efforts did not have the advantage of periodic averaging.

Second, even the direct detection of bounce loss-cone precipitation may provide very little information if the concurrent PAD of the local particle population is not also known. As was discussed in Section 6.3, even small changes to the PAD can have large effects on the precipitating particle flux. Case study measurements of bounce loss-cone precipitation could not reliably quantify the pitch angle scattering effects without detailed knowledge of the local PAD in each case. Averaging over many observations may be able to rely on average PADs for analysis, but it is possible that only certain well-suited distributions would even facilitate detection. There may actually be more promise in ignoring these direct bounce loss-cone measurements
altogether and focusing instead on increased analysis of long-term averaged drift loss-cone measurements. Selesnick et al. [2013] is a recent example of such an effort, providing enough information on the effects of ground-based VLF transmitters to theoretically estimate their influence on the radiation belts even without directly measuring their induced precipitation.

Even without the consistent direct detection and quantification of transmitter-induced bounce loss-cone precipitation, the tools now exist to estimate the illumination of the plasmasphere by ground-based VLF transmitters and the relative significance of those transmitters in inducing energetic electron losses from the Earth’s radiation belts. With the FWM model now providing experimentally-validated estimates for the trans-ionospheric propagation of VLF waves, the complete field map of magnetospheric injection overhead any existing or hypothetical ground-based VLF transmitters can now be computed. Combining these field maps with the Stanford VLF 3-D ray tracer developed by F. R. Foust [Golden et al., 2010] could estimate the total three-dimensional illumination of the plasmasphere by ground-based VLF transmitters, including spatial and temporal variation of wave power, spectral properties, propagation angle, and occurrence rate. The WIPP model could be modified to use these updated wave parameters to better estimate bounce loss-cone precipitation. For better analyzing global effects on the trapped radiation population, however, the updated wave parameters could also directly update inputs to the time-dependent 3-D Versatile Electron Radiation Belt (VERB) code [Shprits et al., 2008; Subbotin and Shprits, 2009] which solves the modified Fokker-Planck equation for the drift- and bounce-averaged phase space density of energetic particles within the magnetosphere.

Model results for both VLF wave parameters and energetic particle fluxes could be directly compared to wave and particle measurements from the recently-launched Radiation Belt Storm Probes (RBSP) spacecraft [Stratton et al., 2013] which targets study of the very high energy electrons and ions magnetically trapped in the Earth’s radiation belts. RBSP records a versatile data set that provides both the wave and particle measurements necessary for improved understanding of the Earth’s radiation belts and the dominant factors affecting their evolution [Mauk et al., 2012;
7.3. FUTURE WORK ON TRANSMITTER-INDUCED PRECIPITATION

Kessel et al., 2012; Stratton et al., 2013]. Not only would RBSP provide wave and energetic particle measurements for direct comparison to model, but the improved measurements of suprathermal particle distributions provided by the mission would facilitate more accurate calculation of Landau damping [Bell et al., 2002; Golden et al., 2010; Foust et al., 2011a] and thus more accurate estimates of VLF wave intensity throughout the plasmasphere. The end result of this analysis could quantify the relative importance of ground-based VLF transmitters in inducing pitch angle scattering losses from the Earth’s radiation belts across different $L$-shells, energies, pitch angles, and space weather conditions. This quantification would provide an update to the pertinent portions of Abel and Thorne [1998a,b] and assess the feasibility of relying upon ground-based VLF transmitters for the purposes of radiation belt remediation.
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