# A lightning discharge producing a beam of relativistic electrons into space

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[1] Strong electric fields associated with lightning generate brief (~1 ms) but intense Terrestrial Gamma-ray Flashes (TGFs), detected by spacecrafts. A few events are thought to be the signature of a relativistic electron beam escaping the atmosphere, which is distinguishable from a TGF since the lightning discharge is along the geomagnetic field line from the spacecraft, rather than below. We refer to this event herein as a 'Terrestrial Energetic-electron Flash' (TEF), and present the first TEF with associated discharge. The TEF was detected by the Gamma-ray Burst Monitor aboard the Fermi satellite, and is correlated with a lightning discharge detected by three Stanford University AWESOME ELF/VLF receivers, a Duke University ULF receiver, and by the GLD360 lightning geolocation network. The discharge, nearly simultaneous with the generated electrons, was of intense peak current and of positive polarity, and with a modest total charge transfer, similar to TGF-associated discharges. Citation: Cohen, M. B., U. S. Inan, R. K. Said, M. S. Briggs, G. J. Fishman, V. Connaughton, and S. A. Cummer (2010), A lightning discharge producing a beam of relativistic electrons into space, Geophys. Res. Lett., 37, L18806, doi:10.1029/2010GL044481.

#### 1. Introduction

[2] Terrestrial Gamma-ray Flashes (TGFs) result from lightning, one of the most powerful natural electrical processes on Earth. TGFs, first observed serendipitously by the Compton Gamma-ray Observatory [Fishman et al., 1994], consist of a flash of  $\gamma$ -ray photons up to tens of MeV, lasting hundreds of  $\mu$ s and detected in low Earth orbit. Most TGFs are closely associated (within ~2 ms) with lightning discharges [Inan et al., 1996; Cummer et al., 2005; Inan et al., 2006; Cohen et al., 2006; Stanley et al., 2006; Cohen et al., 2010b; Shao et al., 2010; Lu et al., 2010; Connaughton et al., 2010]. TGFs are rare in that the estimated global occurrence rate of 50 per day [Smith et al., 2005] amounts to a tiny fraction of the millions of lightning flashes that occur, on average, every day.

- [3] It is widely assumed that TGF photons are emitted via bremsstrahlung when relativistic electrons collide with molecules, but the nature of the lightning that generate these electrons is not fully understood. The type of discharge is, in general, not the same type that generates sprites [Cummer et al., 2005; Inan et al., 2006], and may result from highaltitude intracloud (IC) discharges [Stanley et al., 2006; Shao et al., 2010; Lu et al., 2010].
- [4] The mechanisms by which lightning electric fields accelerate energetic electrons are not fully understood, but likely revolve around a Relativistic Runaway Electron Avalanche (RREA) in which a small number of seed MeV electrons rapidly multiply when a lightning-driven electric field overcomes the frictional losses in the atmosphere.
- [5] The RREA may also generate a beam of relativistic electrons which escapes the atmosphere [Lehtinen et al., 2001; Dwyer et al., 2008]. Since electrons are confined along the geomagnetic field line (whereas  $\gamma$ -rays travel in a straight line and spread over a wider area), the RREA beam would be observed rarely compared to TGFs, but would be longer in duration since the distribution of electron pitch angles cause the electron packet to disperse.
- [6] We refer to this event as a Terrestrial Energetic-electron Flash (TEF), since, as we will discuss, it may or may not be accompanied by a TGF. TEFs could be detected by  $\gamma$ -ray detectors due to bremsstrahlung with the spacecraft body [Smith et al., 2006; Dwyer et al., 2008], or from direct penetration of the electrons into the detector. A small number of possible TEFs have been observed by RHESSI [Smith et al., 2006] and BATSE [Dwyer et al., 2008], and SAMPEX [Carlson et al., 2009a]. However, the causative discharge has not yet been directly associated with the earlier events, leaving open the question of what kind of lightning generates them.
- [7] Since geomagnetic field lines at low latitudes are nearly horizontal, a low-Earth orbiting satellite may detect a TEF from a lightning discharge occurring many hundreds of km away from the satellite nadir point on the ground, whereas TGF detection typically occurs from lightning within ~300 km of the nadir [Cohen et al., 2010b]. Hence, ground-based geolocation of lightning can unambiguously and empirically distinguish a TGF from a TEF.
- [8] Lightning discharges radiate the bulk of their electromagnetic energy in the Extremely Low Frequency (ELF, 0.3–3 kHz) and Very Low Frequency (VLF, 3–30 kHz) ranges. The resulting radio atmospheric, or sferic, is guided efficiently (attenuation rate of a few dB/Mm) in the Earthionosphere waveguide. A globally distributed network of ELF/VLF receivers can geolocate lightning, with accuracy limited largely by the arrival time estimation [Wood and Inan, 2004; Said et al., 2010].

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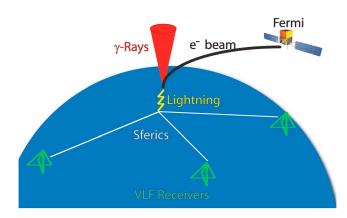
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**Figure 1.** Geometry of lightning generation of  $\gamma$ -rays and relativistic electron beams.

- [9] Connaughton et al. [2010] present a correlation of three possible TEFs to thunderstorms located close (35 km away on average) to the geomagnetic footprint, with no thunderstorm near the satellite nadir, however the associated discharge was not detected.
- [10] Figure 1 shows schematically the geometry of the  $\gamma$ -ray generation, electron beam propagation, and lightning geolocation via sferic propagation and detection. The  $\gamma$ -rays escape along a cone roughly 20–30° wide due to atmospheric absorption, whereas the electrons remain tightly confined along the geomagnetic field line.
- [11] We present here the first unambiguous evidence of a Terrestrial Energetic-electron Flash, or TEF, linked to its causative lightning discharge.

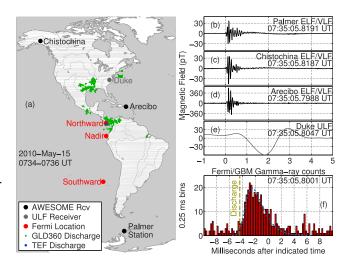
## 2. Description of Data

- [12] The Fermi Gamma-ray Space Telescope Observatory, launched in June 2008 [Meegan et al., 2008], is primarily designed to observe high-energy astrophysical objects. It consists of the Large Area Telescope, designed for observations >60 MeV, and the smaller Gamma-ray Burst Monitor (GBM), which provides  $\gamma$ -ray observations presented here. The fourteen scintillation detectors of GBM provide full-sky coverage between ~8 keV and ~40 MeV. Observations of TGFs with the GBM instrument are reported by Briggs et al. [2010].
- [13] Fermi follows a nearly circular orbit at an altitude of  $\sim$ 560 km and an inclination of 25.6°. Data download is determined by an onboard trigger algorithm, which was initially designed for longer duration  $\gamma$ -ray bursts. The detection rate of TGFs increased from  $\sim$ 1 per month to  $\sim$ 2 per week as a result of on-board software modifications, and current plans are to download all data over selected regions to further increase the detection efficiency. Absolute timing accuracy is a few $\mu$ s due to GPS synchronization, avoiding timing ambiguity that affected earlier missions [Cohen et al., 2006; Grefenstette et al., 2009].
- [14] Broadband ELF/VLF data (300 Hz–47 kHz) are acquired with the AWESOME receiver [Cohen et al., 2010a], consisting of two orthogonal air-core loop antennas, sampling magnetic field spectral densities as low as a few fT/rt-Hz with 16-bit resolution at 100 kHz, and synchronized to GPS with <100 ns timing error. Receivers utilized here are placed at Palmer Station, Antarctica (64.8°S, 64.1°W),

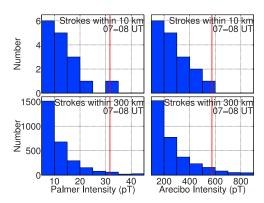
- Chistochina, Alaska (62.6°N, 144.6°W), and Arecibo, Puerto Rico (18.3°N, 66.8°W), as shown in Figure 2a. In particular, the receiver at Palmer Station is extremely sensitive, and can detect <3 kA cloud-to-ground (CG) lightning discharges from 10 Mm distance [*Inan et al.*, 2006].
- [15] Lightning geolocations are determined by the new GLD360 network, which utilizes time of arrival and magnetic direction finding of sferics at a number of globally spaced AWESOME receivers (independent from the aforementioned Stanford-operated sites) to produce lightning geolocations. GLD360 utilizes geolocation methods developed by *Said et al.* [2010] to greatly improve the accuracy (<5 km), detection efficiency (~70% of CG discharges, plus a large number of IC discharges) over existing global lightning networks, and provide discharge polarity and peak current measurements [*Said et al.*, 2010].
- [16] ULF magnetic field data are sampled at 2.5 kHz by a pair of sensors at Duke University (35.98°N, 79.10°W) with a flat response from 0.1–400 Hz. These data provide unambiguous measurement of the polarity of the charge transfer in the discharge and also detect any slower (>1 ms) processes associated with the discharge.

#### 3. Observations

[17] On 15-May-2010, the Fermi spacecraft detected a sharp rise in photon count rate, and a gradual decline over a few ms. The Fermi  $\gamma$ -ray observations are shown in Figure 2f. The dashed curve shows a log-normal fit to the data using equation (2) of *Briggs et al.* [2010]. The duration is consistent with past predictions regarding TEF events [*Dwyer et al.*,



**Figure 2.** (a) Map with locations of Fermi (nadir, northern geomagnetic footprint, and southern geomagnetic footprint), GLD360 strokes (green dots), AWESOME receivers (black dots), ULF receiver (gray dot) and the TEF-associated discharge (blue dot) (b–d) Stanford AWESOME data at three receivers showing the TEF-associated radio atmospherics, (e) Duke ULF data showing the slower magnetic field development from the discharge. (Data are filtered to remove some low frequency interference below ~100 Hz.), (f) Fermi/GBM  $\gamma$ -ray data, in 0.25 ms bins, with the time of the lightning discharge (after correcting for 3.77 ms speed of light propagation delay) indicated with yellow stripe.



**Figure 3.** Histogram of VLF sferic intensities of sferics at (left) Palmer Station and (right) Arecibo for discharges within (top) 10 km and (bottom) 300 km of the TEF-associated discharge for the period 07–08 UT. The red line is the intensity of the TEF-associated sferic.

2008; Carlson et al., 2009a]. The first electrons arrive at the spacecraft at  $\sim$ 07:35:05.79585 UT  $\pm$  0.25 ms. A brief spike at +5 ms in Figure 2f is likely connected to a cosmic ray.

[18] Figure 2a shows (red dots) the location of the Fermi satellite nadir (0.98°S, 81.71°W), the northern geomagnetic footprint (7.48°N, 81.85°W), and the southern geomagnetic footprint (31.12°S, 83.99°W). The geomagnetic footprint is taken at 35 km, since below this altitude the atmosphere is too collisional to confine relativistic electrons along the geomagnetic field.

[19] The green dots show lightning discharges detected by the GLD360 network during the period 0734–0736 UT. There is no detected discharge within 400 km of either the satellite nadir point or the southern geomagnetic footprint, but there are 130 discharges within 300 km of the northern geomagnetic footprint. Over a one-hour period (07–08 UT), more than 400 discharges are detected within 100 km of the northern geomagnetic footprint.

[20] The TEF-associated discharge is detected at the location  $6.84^{\circ}$ N,  $81.62^{\circ}$ W, at  $07:35:05.7920 \pm 0.05$  ms. The  $\sim$ 75 km horizontal displacement from the northern magnetic footprint of Fermi would put it near the edge of a predicted beam of relativistic electrons emerging from the atmosphere [Dwyer et al., 2008; Carlson et al., 2009a], and detected by Fermi prior to reaching the geomagnetic equator. Given the GLD360 detection rate of sferies within 100 km of the northern magnetic footprint over the one-hour period, the probability of chance occurrence of a discharge within  $\pm 5$  ms of the TEF is  $\sim 0.001$ .

[21] ELF/VLF Data from the three AWESOME receivers are plotted in Figures 2b–2d, after processing with an adaptive filter to remove periodic 60-Hz power-line interference [Cohen et al., 2010c] and a subtraction technique to remove VLF transmitter interference [Said, 2009, chap. 3].

[22] The ULF signature of the discharge is detected at Duke University (filtered to remove <100 Hz interference), and shown in Figure 2e, where t=0 corresponds to the arrival time at the sensor for an Earth-ionosphere waveguide propagation group velocity of 0.85c. The discharge has polarity consistent with +CG or vertical +IC, consistent with previous measurements [Cummer et al., 2005; Stanley et al., 2006] and with a small number of NLDN-identified polarities [Cohen et al., 2010b; Lu et al., 2010].

[23] Since at least three AWESOME receivers observed the TEF-associated sferic, independent verification of the GLD360 geolocation can be achieved using older triangulation techniques applied by *Cohen et al.* [2010b] for sferics at longer distances. The calculated discharge location from these three sferic arrival times (<30 km error) is within ~11 km of the GLD360 geolocation.

[24] Tracing the magnetic field line from Fermi's location to 35 km altitude (1115 km arc length), then vertically downward to 20 km altitude, gives a speed-of-light propagation delay of  $\sim 3.77$  ms. The first electrons arrived at Fermi  $3.85 \pm 0.25$  ms after the lightning discharge, or  $0.08 \pm 0.30$  ms after the propagation-shifted discharge time. The timing is therefore consistent with the electron avalanche occurring nearly simultaneous with the discharge, and the earliest arriving electrons propagating close to the speed of light with a low pitch angle.

## 4. Discussion

[25] For a given thunderstorm, the VLF intensity of a sferic is roughly proportional to the peak current of the source discharge. Figure 3 shows a histogram of VLF intensities from all discharges within 10 km of the TEF-associated discharge (Figure 3, top), and 300 km (Figure 3, bottom) of the TEF-associated discharge, in an hour-long period (07–08 UT) around the TEF. The histogram is shown separately for Palmer Station (Figure 3, left) and Arecibo (Figure 3, right). The red line shows the intensity of the TEF-associated sferic.

[26] The VLF intensity of the TEF-associated discharge was the strongest of all sferics originating from within 10 km over the one-hour period, and stronger than 90–95% of sferics from within 300 km, as detected by GLD360. The TEF-associated sferic saturated the receiver at Chistochina, and quite nearly at Palmer Station, despite propagating ~8000 km in the Earth-ionosphere waveguide.

[27] The intensity of this discharge is consistent with past observations [Inan et al., 2006] as well as recent predictions of RREA in the lightning channel which requires higher peak currents [Carlson et al., 2009b]. On the other hand, based on the ULF signature, the impulsive charge moment of the discharge is estimated at 26 C km  $\pm$  20% (using the approach of Cummer and Lyons [2005]), consistent with the modest charge moment measurements of TGF-associated discharges [Cummer et al., 2005].

[28] It is quite possible that a given TEF event simply represents another manifestation of the same physical phenomenon that produce TGFs. More specifically, Compton scattering from TGF photons as they reach the end of the atmosphere may yield an electron beam even if the RREA occurred closer to the cloud top (15–20 km), and some earlier modeling [*Dwyer et al.*, 2008; *Carlson et al.*, 2009a] supports this possibility and agrees with earlier satellite TEF observations.

[29] However, we cannot establish empirically that there was a TGF event along with the TEF, since both gamma rays and electrons have not yet been independently detected for the same event. For instance, RREA at higher altitudes (40–60 km), where the atmosphere is thinner, would occur in the absence of significant Compton scattering, implying many fewer  $\gamma$ -rays.

- [30] Two proposed theories of  $\gamma$ -ray generation via RREA bremsstrahlung occur at high altitudes. The electromagnetic pulse (EMP) mechanism [Inan and Lehtinen, 2005] from a fast and intense return stroke may not allow escaping electrons since the electrons are often (depending on the geomagnetic field orientation) accelerated orthogonal to the geomagnetic field. The quasi-electrostatic (QES) field mechanism from extremely high charge transfer lightning allows electron escape [Lehtinen et al., 2001].
- [31] Although the charge moments predicted for the QES model are inconsistent with past observations of TGFassociated sferics [Cummer et al., 2005], the threshold charge moment for generating a detectable electron beam is significantly lower than that for generating detectable  $\gamma$ -ray flux.
- [32] For instance, according to Figure 3 by Lehtinen et al. [2001], a relativistic electron density of  $10^{-4} \text{m}^{-3}$  (which implies 15 electrons passing through a 0.1 m<sup>2</sup> area over 5 ms) may be achievable with charge moments of only 1000-2000 C km, lower than the 6000 C km required for  $\gamma$ -ray generation by OES fields. It is therefore possible for a TEF event to exist in the absence of a TGF, although the 26 C km  $\pm$ 20% measured for this event is well short of 1000-2000 C km. Further modeling and observation are needed on the causative mechanisms of TEF events.

## 5. Conclusion

- [33] We have presented the first unambiguous evidence of a relativistic electron beam associated with a lightning discharge. The herein named Terrestrial Energetic-electron Flash (TEF) was detected by the Fermi GBM.
- [34] The TEF-associated lightning discharge was geolocated by the GLD360 network, and characterized by three AWESOME ELF/VLF receivers and a ULF receiver. The source lightning discharge occurred ~75 km laterally away from the location where the magnetic field line passing through Fermi reaches the collisional atmosphere. After correcting for propagation delays, the electron generation appears to have been simultaneous with the lightning discharge ( $\pm 0.25$  ms). The lightning discharge was among the most intense from its thunderstorm (by peak current), but did not have an unusually large charge removal, in line with TGF-associated sferics.
- [35] Acknowledgments. This work was supported by NSF grant OPP-0233955 to Stanford University, and also by the DARPA NIMBUS program under grant HR0011-10-1-0058. GLD360 lightning discharge data are provided under a cooperative agreement with Vaisala, Inc. The Fermi satellite is supported by NASA. Duke University effort is supported by NSF grant ATM-0642757. We thank Brant Carlson and Nikolai Lehtinen for helpful discussions.

#### References

- Briggs, M. S., et al. (2010), First results on terrestrial gamma ray flashes from the Fermi Gamma-ray Burst Monitor, J. Geophys. Res., 115, A07323, doi:10.1029/2009JA015242.
- Carlson, B. E., N. G. Lehtinen, and U. Inan (2009a), Observations of terrestrial gamma-ray flash electrons, in Coupling of Thunderstorms and Lightning Discharges to Near-Earth Space: Proceedings of the Workshop, AIP Conf. Proc., 1118, 84-91, doi:10.1063/1.3137717
- Carlson, B. E., N. G. Lehtinen, and U. S. Inan (2009b), Terrestrial gamma ray flash production by lightning current pulses, J. Geophys. Res., 114, A00E08, doi:10.1029/2009JA014531.
- Cohen, M. B., U. S. Inan, and G. Fishman (2006), Terrestrial gamma ray flashes observed aboard the Compton Gamma Ray Observatory/Burst

- and Transient Source Experiment and ELF/VLF radio atmospherics, J. Geophys. Res., 111, D24109, doi:10.1029/2005JD006987
- Cohen, M. B., U. S. Inan, and E. P. Paschal (2010a), Sensitive broadband ELF/VLF radio reception with the AWESOME instrument, IEEE Trans. Geosci. Remote Sens., 48(1), 3-17, doi:10.1109/TGRS.2009.2028,334.
- Cohen, M. B., U. S. Inan, R. K. Said, and T. Gjestland (2010b), Geolocation of terrestrial gamma-ray flash source lightning, Geophys. Res. Lett., 37, L02801, doi:10.1029/2009GL041753
- Cohen, M. B., R. K. Said, and U. S. Inan (2010c), Mitigation of 50/60 hz power-line interference in geophysical data, Radio Sci., 45, doi:10.1029/ 2010RS004420, in press
- Connaughton, V., et al. (2010), Associations between Fermi GBM terrestrial gamma-ray flashes and sferics from the WWLLN, J. Geophys. Res., doi:10.1029/2010JA015681, in press.
- Cummer, S. A., and W. A. Lyons (2005), Implications of lightning charge moment changes for sprite initiation, J. Geophys. Res., 110, A04304, doi:10.1029/2004JA010812.
- Cummer, S. A., Y. Zhai, W. Hu, D. M. Smith, L. I. Lopez, and M. A. Stanley (2005), Measurements and implications of the relationship between lightning and terrestrial gamma ray flashes, Geophys. Res. Lett., 32, L08811, doi:10.1029/2005GL022778.
- Dwyer, J. R., B. W. Grefenstette, and D. M. Smith (2008), High-energy electron beams launched into space by thunderstorms, Geophys. Res. Lett., 35, L02815, doi:10.1029/2007GL032430.
- Fishman, G. J., et al. (1994), Discovery of intense gamma-ray flashes of atmospheric origin, Science, 264, 1313.
- Grefenstette, B. W., D. M. Smith, B. J. Hazelton, and L. I. Lopez (2009), First RHESSI terrestrial gamma ray flash catalog, J. Geophys. Res., 114, A02314, doi:10.1029/2008JA013721.
- Inan, U. S., and N. G. Lehtinen (2005), Production of terrestrial gamma-ray flashes by an electromagnetic pulse from a lightning return stroke, Geophys. Res. Lett., 32, L19818, doi:10.1029/2005GL023702
- Inan, U. S., S. C. Reising, G. J. Fishman, and J. M. Horack (1996), On the association of terrestrial gamma-ray bursts with lightning and implications for sprites, Geophys. Res. Lett., 23(9), 1017-1020.
- Inan, U. S., M. B. Cohen, R. K. Said, D. M. Smith, and L. I. Lopez (2006), Terrestrial gamma ray flashes and lightning discharges, Geophys. Res. Lett., 33, L18802, doi:10.1029/2006GL027085.
- Lehtinen, N. G., U. S. Inan, and T. F. Bell (2001), Effects of thunderstormdriven runaway electrons in the conjugate hemisphere: Purple sprites, ionization enhancements, and gamma rays, J. Geophys. Res., 106 (A12), 28,841-28,856.
- Lu, G., R. J. Blakeslee, J. Li, D. M. Smith, X.-M. Shao, E. W. McCaul, D. E. Buechler, H. J. Christian, J. M. Hall, and S. A. Cummer (2010), Lightning mapping observation of a terrestrial gamma-ray flash, Geophys. Res. Lett., 37, £11806, doi:10.1029/2010GL043494.
- Meegan, C., et al. (2008), The Fermi gamma-ray burst monitor, Astrophys. J., 702(1), 791, doi:10.1088/0004-637X/702/1/791.
- Said, R. K. (2009), Accurate and efficient long-range lightning geo-location using a VLF radio atmospheric waveform bank, Ph.D. thesis, Stanford Univ., Stanford, Calif.
- Said, R. K., U. S. Inan, and K. L. Cummins (2010), Long-range lightning geo-location using a VLF radio atmospheric waveform bank, doi:10.1029/ 2010JD013863, in press.
- Shao, X.-M., T. Hamlin, and D. M. Smith (2010), A closer examination of terrestrial gamma-ray flash-related lightning processes, J. Geophys. Res., 115, A00E30, doi:10.1029/2009JA014835.
- Smith, D. M., et al. (2006), The anomalous terrestrial gamma-ray flash of 17 January 2004, Eos Trans. AGU, 87(52), Fall Meet. Suppl., Abstract AE31A-1040.
- Smith, D. M., L. I. Lopez, R. P. Lin, and C. P. Barrington-Leigh (2005), Ter-
- restrial gamma-ray flashes observed up to 20 MeV, *Science*, 308, 1085. Stanley, M. A., X.-M. Shao, D. M. Smith, L. I. Lopez, M. B. Pongratz, J. D. Harlin, M. Stock, and A. Regan (2006), A link between terrestrial gammaray flashes and intracloud lightning discharges, Geophys. Res. Lett., 33, L06803, doi:10.1029/2005GL025537.
- Wood, T. G., and U. S. Inan (2004), Localization of individual lightning discharges via directional and temporal triangulation of sferic measurements at two distant sites, J. Geophys. Res., 109, D21109, doi:10.1029/2004JD005204.
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