

## Mechanism of ELF radiation from sprites

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**Abstract.** Charge and current systems associated with sprites constitute a part of the large scale atmospheric electric circuit, providing a context for physical understanding of recently discovered ELF radiation originating from currents flowing within the body of sprites. It is shown that the impulse of the electric current driven in the conducting body of the sprite by lightning generated transient quasi-electrostatic fields produces significant electromagnetic radiation in the ELF range of frequencies, comparable to that radiated by the causative lightning discharge.

### Introduction

Observations of sprite associated ELF radio atmospherics demonstrate significant ELF radiation produced by vertical currents flowing in sprites [P. Krehbiel, private communication, 1996], recently confirmed by simultaneous video, photometric and ELF measurements [Cummer *et al.*, 1998, Bell *et al.*, 1997; Reising, 1998]. In this paper we discuss the role of sprites in the electrical coupling between atmospheric regions and provide a physical picture of the associated ELF radiation.

### Sprites as Part of the Atmospheric Electric Circuit

The recent discoveries of high altitude optical emissions known as sprites provide dramatic new evidence of the electrical coupling between upper atmospheric regions. It was proposed that these spectacular glows are produced by large quasi-electrostatic (QE) fields capable of producing breakdown ionization at mesospheric altitudes above thunderstorms [Pasko *et al.*, 1997, and references therein]. Previous work on understanding the production of sprites has largely concentrated on the establishment and relaxation of the QE fields and their effects on the ambient electrons. In fact, the occurrence of sprites is but one manifestation of the overall atmospheric electric circuit. The mesospheric electric field ( $E$ ) transients following cloud-to-ground lightning discharges lead to rearrangement of the current and charge systems in the conducting atmosphere above a thunderstorm and to a first approximation can be described by a "moving capacitor plate" model proposed by Greifinger and Greifinger [1976]. This model defines a downward moving boundary  $h_i$  which separates regions of the atmosphere dominated by the conduction (above) and displacement (below) currents. For an atmospheric conductivity  $\sigma$  which increases monotonically with altitude,  $h_i$  as a function of time  $t$  is uniquely defined by the equation  $t = \epsilon_0 / \sigma(h_i)$  [e.g.,

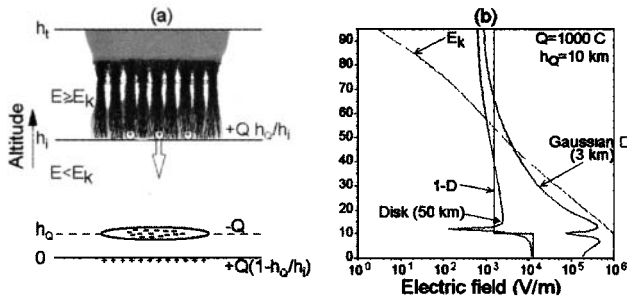
Hale and Baginski, 1987]. For conductivity profiles having regions of positive (negative) derivative as a function of altitude, several  $h_i$  downward (upward) moving boundaries separating regions of conduction and displacement currents are formed. In this paper, we use the moving capacitor plate model to place sprites in their proper context as part of the large scale atmospheric electric circuit.

Simple time-dependent analytical solutions for the electric field and charge density can be obtained in the one dimensional (1-D) case by considering injection at  $t=0$  of a planar charge  $Q$  (which can be viewed as a surface charge density  $\rho_s = Q / (\pi R_s^2)$  distributed over a disk with radius  $R_s$ ) at altitude  $h_Q$  and applying appropriate boundary conditions at the upper ionospheric (e.g.,  $h_i = 95$  km), lower (ground),  $h_Q$  and  $h_i$  boundaries, following a procedure similar to that outlined in Appendix B of [Pasko *et al.*, 1997]. We note that for the case when  $\sigma(h_i) \gg \sigma(h_Q)$  and  $h_i \gg h$ , where  $h$  is the altitude scale-factor of conductivity, the solution for the electric field at altitudes below  $h_i$  is  $(h_Q/h_i)(\rho_s/\epsilon_0)$  for  $h_Q < z < h_i$  and  $(h_Q/h_i - 1)(\rho_s/\epsilon_0)$  for  $0 < z < h_Q$ , where  $z$  denotes altitude. This solution has the same form as the static solution for a charge in free space between two conducting planes at  $z = h_i$  and  $z = 0$ , and does not depend on the details (magnitude and altitude profile) of conductivity above the  $h_i$  boundary. For a charge  $-Q$  effectively deposited by a positive lightning discharge at altitude  $h_Q$ , there are two induced charges,  $+Qh_Q/h_i$  on the upper ( $h_i$ ) and  $+Q(1 - h_Q/h_i)$  the lower (ground) boundaries (see Figure 1a and discussion in Hale and Baginski [1987]). The boundary at  $h_i$  decreases in altitude with time, and the source charge  $-Q$  is neutralized when  $h_i \rightarrow h_Q$ . The source of the charge on the upper boundary ( $h_i$ ) is the overlying ionosphere. Thus, the vertical current moment required to produce this charge can be evaluated at any moment of time. For a relatively weak lightning discharge, the QE fields of which are not intense enough to cause modification of the atmospheric conductivity due to breakdown ionization and for a typical ambient conductivity profile (e.g.,  $\sigma_2$  shown in Figure 2a),  $h_i$  moves only several km below 95 km altitude on a time scale  $\tau_{sf} \sim 1$  ms. For a strong sprite-associated discharge which leads to ionization and enhancement of conductivity down to  $\sim 50$  km within the same time  $\tau_{sf} \sim 1$  ms,  $h_i$  is expected to decrease in altitude by several tens of km, producing significant enhancement in the associated vertical current moment. We note that also in the case of sprites, the charge induced on the  $h_i$  boundary is insensitive to the value of conductivity above it and to the particular ionization mechanism which produced this conductivity enhancement, as long as the conductivity is large enough so as to allow the transfer of charge of amount  $+Qh_Q/h_i$  to altitude  $h_i$  in  $\tau_{sf} \sim 1$  ms.

The electric breakdown of air associated with sprites starts at the altitudes where the electric field exceeds the breakdown threshold ( $E_k$ ) (Figure 1a). Figure 1b shows an altitude scan of the electric field created by a static charge

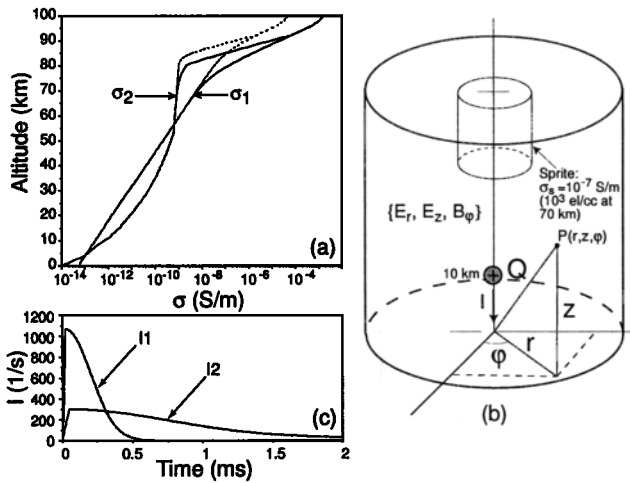
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**Figure 1.** (a) Illustration of charge systems associated with sprites; (b) Altitude scans at  $r = 0$  of the QE field for thundercloud charge  $Q=1000$  C of different geometries placed at 10 km altitude. The field  $E_k$  is the characteristic air breakdown field [e.g., Papadopoulos et al., 1993].

$Q=1000$  C of various geometries placed at  $h_Q=10$  km altitude in free space between two perfectly conducting planes at the ground and the ionosphere ( $h_i=95$  km), both assumed to be maintained at zero potential. The figure illustrates the maximum possible magnitude of the postdischarge QE field which is not distorted by the ambient atmospheric conductivity. This distribution can be linearly scaled to evaluate fields corresponding to different amounts of charge. It follows from Figure 1b that at least 1000 C of charge is required to initiate breakdown ( $E > E_k$ ) at  $\sim 50$  km altitude while only 10 C may be sufficient at altitudes  $\sim 90$  km in cases of a sufficiently low ambient conductivity, allowing effective upward penetration of the electric field to this altitude [e.g., Pasko et al., 1997].

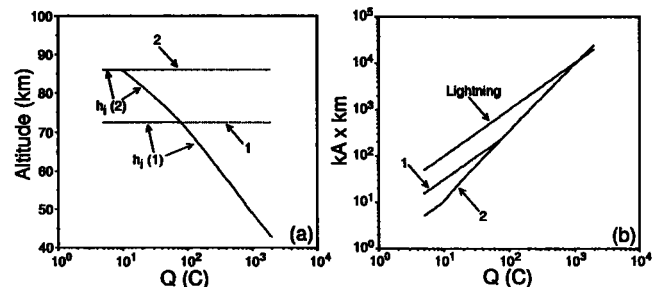


**Figure 2.** (a) Dashed lines show two altitude profiles of  $\sigma$  used in calculations, which correspond to the ionospheric electrons heated by the electric field  $\simeq E_k$ . Solid lines illustrate cold (ambient) profiles. Profile 1 is taken from [Pasko et al., 1997] and Profile 2 from [Hale, 1994]; (b) Coordinate system used in calculations; (c) Two models of the lightning current used in calculations (normalized as  $\int I dt = 1$ ). The model I1 has the linear rise time  $\tau_r=100\mu s$  and the exponential fall time  $\tau_f=1$  ms; I2:  $\tau_r=200\mu s$  and  $\tau_f=4$  ms. The model I2 includes continuing current with magnitude 1/10 of the peak current value.

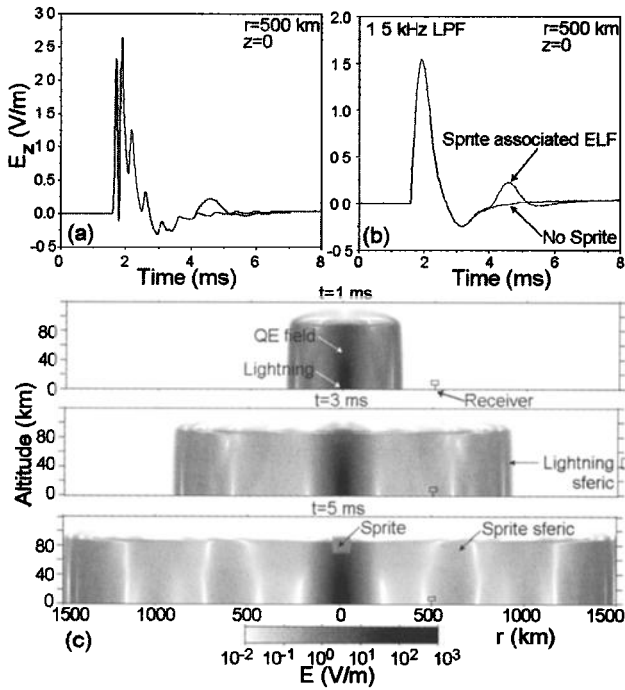
The moving capacitor plate height  $h_i$  in the case of additional ionization associated with sprites can thus be defined approximately as the altitude above which  $E > E_k$  and in the simple 1-D case can be found from the nonlinear equation  $E_k(h_i) = (h_Q/h_i)Q/(\pi R_s^2)/\epsilon_0$  which also accounts for the effect of increasing electric field with decreasing  $h_i$ . In the case when a sprite has the shape of a vertical column(s) the expression for the total charge at the lower boundary  $h_i$  remains the same,  $+Qh_Q/h_i$ , while  $h_i$  itself may extend to lower altitudes due to enhancement of the electric field around the sharp lower edges of the columns. Since sprites are electrically attached to the lower ionosphere, it is expected from conservation of vertical current that the volume-averaged conductivity in the body of the sprite does not exceed that at the point of attachment to the lower ionosphere. Figure 3a illustrates the upper (shown by dashed line) and lower ( $h_i$ ) extent of the ionized region associated with sprites, calculated for a 1-D source  $Q$ , the two different model profiles of conductivity shown in Figure 2a, and assuming that sprites are formed on a time scale of  $\tau_{sf}=1$  ms. We note that for profile  $\sigma_1$  and  $Q < 100$  C the ambient conductivity is sufficient to bring charge  $+Qh_Q/h_i$  to  $h_i \simeq 72$  km in 1 ms; therefore no additional ionization need be produced. Similarly, for profile  $\sigma_2$  and  $Q < 10$  C,  $h_i = 86$  km is reached in 1 ms. For greater values of charge  $Q$  the atmosphere must be additionally ionized in order to accumulate charge  $+Qh_Q/h_i$  at altitude  $h_i$ . The current flowing in the sprite is  $\pi R_s^2 \sigma_s E = Qh_Q/h_i/\tau_{sf}$ , where  $\sigma_s$  is the conductivity of the sprite. Since  $E = (h_Q/h_i)Q/(\pi R_s^2)/\epsilon_0$  at altitudes above  $h_Q$ , the first order estimate of minimum required value of  $\sigma_s$  is simply  $\sigma_s = \epsilon_0/\tau_{sf} \simeq 10^{-8}$  S/m, corresponding to an electron density of  $\sim 10^{12}$  cm $^{-3}$  at 70 km, in agreement with previous sprite modeling results [Pasko et al., 1997].

### Physical Mechanism of ELF Radiation from Sprites

Figure 3b illustrates vertical current moments associated with sprites evaluated as  $(h_i - h_i)Qh_Q/h_i/\tau_{sf}$  and causative lightning  $Qh_Q/\tau_{sf}$  assuming that charge  $Q$  is removed in  $\tau_{sf}=1$  ms. We note a change in slope of the current moment at  $Q \simeq 100$  C for case  $\sigma_1$  and  $Q \simeq 10$  C for  $\sigma_2$  which corresponds to downward extension of  $h_i$  due to ionization, as discussed in the previous section. For dense ambient con-



**Figure 3.** Illustration of the upper (dashed line) and lower  $h_i$  boundaries of the ionization regions associated with sprites (a), and sprite and lightning current moments (b), for the range of  $Q$  values and two ambient  $\sigma$  profiles marked as 1 and 2. Shown  $h_i$  corresponds to the moment of time  $t = \tau_{sf} = 1$  ms.



**Figure 4.** (a) The electric field dynamics at the ground 500 km away from the causative lightning discharge; (b) The same as (a) but low pass filtered with cut off frequency 1.5 kHz; (c) A cross-sectional view of the distribution of the absolute values of the electric field at selected instants of time.

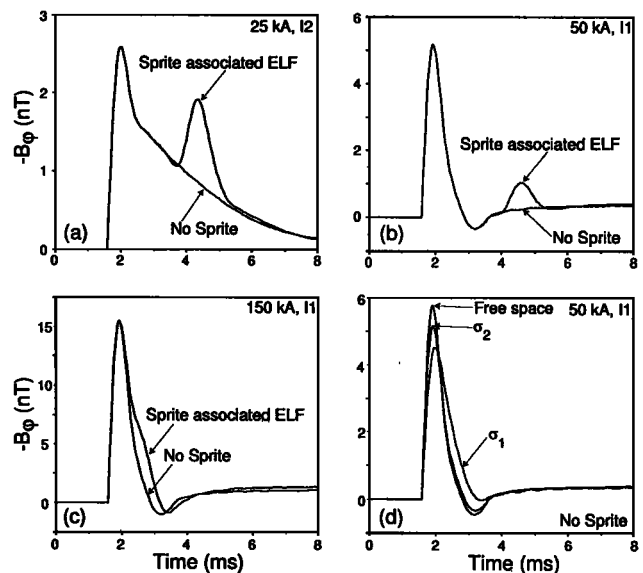
ductivity ( $\sigma_1$ ) the current moment associated with currents in the conducting atmosphere is within a factor of 3 below that associated with the lightning itself, even without additional ionization (for  $Q < 100$  C).

The physical effects of enhancement in conductivity associated with sprites are the acceleration of the relaxation of the electric field in regions where breakdown ionization is produced, and the support of sprite currents required to transfer the charge  $+Qh_Q/h_i$  from the ionosphere to altitude  $h_i$ . Since sprites are formed within  $\tau_{\text{eff}} \sim 1$  ms, these processes may manifest themselves in significant current moments, radiating in the ELF frequency range. For a quantitative discussion, we use a two-dimensional cylindrically symmetric numerical model (Figure 2b) to calculate the radiation and propagation in the Earth-ionosphere waveguide of electromagnetic fields generated by lightning and sprite currents. The model solves the full set of Maxwell's equations using space and time centered finite differences [e.g., *Birdsall and Langdon, 1985, p. 341*], accounts for the vertical lightning current  $I(t)$  with given source functions (Figure 2c), removing the thundercloud charge from altitude 10 km (Figure 2b), and accounts for currents flowing in a conducting atmosphere with realistic conductivity profiles (Figure 2a). For simplicity, the model does not account for effects of the external geomagnetic field on conductivity and treats only the TM (including TEM) component ( $E_r, E_z, B_\phi$ ) of the electromagnetic field. We model the sprite as a cylinder with conductivity  $\sigma_s = 10^{-7}$  S/m and radius  $R_s = 50$  km, the lower boundary of which ( $h_i$ ) is defined self-consistently as a function of the electric field during the charge removal by

lightning. To illustrate the experimentally observed delay in sprite appearance until first several tens of coulombs of charge are removed from cloud to ground [*Cummer et al., 1998; Reising, 1998*], we assume that  $\sigma_s$  grows linearly to  $10^{-7}$  S/m value in  $\tau_{\text{eff}} = 1$  ms, as soon as the first 50 C of charge are removed.

Figure 4a,b illustrate the electric field measured on the ground at 500 km distance from the source lightning, using current model I1 (Figure 2c) with peak current 50 kA, and assuming ambient conductivity  $\sigma_2$  (Figure 2a). Figure 4b shows the result of low pass filtering at 1.5 kHz of the spheric waveform in Figure 4a. Calculations were performed with and without the sprite to illustrate the contribution of the sprite current to the total wave field. The physical mechanism of ELF radiation from the sprite is interpreted in terms of considerations discussed above: the formation of the region of enhanced conductivity placed in the external QE field leads to an impulse of current which generates a charge at the bottom/around it and effectively expels the electric field from the conductor. This effect is illustrated in Figure 4c at selected instants of time. One can see the reduction of the electric field within the body of the sprite as well as the portion of the electric field radiated by the sprite current which propagates in the earth-ionosphere waveguide. Calculations (not shown) indicate only weak dependence on the values of  $\sigma_s$  and  $R_s$ , except for a general narrowing in time and enhancement in magnitude of the sprite's ELF radiation with increasing  $\sigma_s$ .

Figure 5 illustrates magnetic field waveforms for different input parameters. Since it is assumed that conductivity in the sprite region does not increase until 50C have been removed from the cloud, for low peak currents (25 kA, Figure 5a), radiation from sprites can be significantly delayed in time (several ms) with respect to the onset of lightning. In spite of this the radiation has ELF magnitude compara-



**Figure 5.** The magnetic field ( $B_\phi$ ) dynamics at the ground 500 km away from the causative lightning discharge for different models of the lightning current: (a) Model I2 (Figure 2c) with peak current 25 kA; (b) I1, 50 kA; (c) I1, 50 kA and for three models of ambient conductivity discussed in the text.

ble to that of the causative lightning discharge (Figure 5a). On the other hand, a lightning discharge with larger peak current (150 kA, Figure 5c) triggers a sprite within the first millisecond and does not show a separate ELF spheric peak associated with sprites, because the causative lightning and sprite radiate almost simultaneously in time. These results are consistent with the conclusions of *Bell et al.* [1997]. Figure 5d illustrates the ELF radiation from the upper atmosphere due to the ambient conductivity (i.e., without the enhancement due to the sprite). The solution marked "free space" corresponds to the solution between two conducting planes (upper placed at 95 km altitude) with zero conductivity between them. Although it is difficult to compare the magnitudes of spherics in these cases (since part of the electromagnetic energy is absorbed during reflections from the lower ionosphere) one can observe extension of the spheric in time for conductivity  $\sigma_1$ , as compared to the free space and  $\sigma_2$  cases. This signal lasts longer than the spheric in free space because it is generated by the  $h_i$  layer which moves to altitude  $\simeq 72$  km in 1 ms as illustrated in Figure 3a.

## Discussion

Recent experimental findings and theoretical analysis [*Pasko et al.*, 1998; and references therein] indicate that the spatial structure of sprite conductivity is a complicated function of altitude, exhibiting in some altitude ranges a highly localized filamentary structure (e.g., lateral extents  $\simeq 10$  m at 70 km altitude). The simultaneous modeling of these localized structures and sprite ELF radiation is not computationally possible at present due to differences in spatial and time scales involved. In this paper we deal with the volume average sprite conductivity, which is an input parameter in our modeling. We thus do not account for any particular mechanism of production of sprite ionization, with the breakdown field  $E_k$  being the only physical parameter which defines the lower extent of our model sprites. An advantage of this approach is that the volume average sprite conductivity can be varied and, in fact, determined, by comparing our model results with experimentally observed sprite associated spherics [e.g., *Cummer et al.*, 1998; *Bell et al.*, 1997; *Reising*, 1998]. In particular, we found that sprite conductivity  $\sigma_s \simeq 10^{-7}$  S/m (corresponding to the electron number density  $10^3$  cm $^{-3}$  at 70 km altitude) agrees well with typical magnitudes and durations of the observed sprite associated ELF radiation [*Krehbiel*, private communication, 1996; *Cummer et al.*, 1998; *Bell et al.*, 1997; *Reising*, 1998].

## Summary

Sprites are considered as an essential part of the large scale atmospheric electric circuit. Sprites simply serve for

acceleration of the electric field relaxation in regions of the atmosphere dominated by the breakdown ionization and lead to downward transfer of positive charge from the ionosphere in order to satisfy the boundary conditions for the QE fields at the lower extents of the ionized regions. The associated impulse of electric current flowing in the conducting body of a model sprite due to transient QE fields is shown to produce significant electromagnetic radiation in the ELF frequency range, comparable in magnitude to that radiated by the causative lightning discharge. The volume average sprite conductivity is estimated to be  $\sigma_s \simeq 10^{-7}$  S/m corresponding to the electron number density  $\simeq 10^3$  cm $^{-3}$  at 70 km altitude.

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