

EQUATORIAL GYRORESONANCE BETWEEN ELECTRONS AND MAGNETOSPHERICALLY REFLECTED WHISTLERS

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Abstract. Magnetosphericly reflected whistlers resonantly interact with energetic (of order 100 keV) electrons in a relatively narrow energy range during multiple equatorial crossings over a wide range of L -shells ($1.5 < L < 4$). Results indicate that wave energy that enters the magnetosphere at a fixed location can potentially contribute to the loss of particles over a wide range of latitudes.

1. Introduction

A large fraction of the magnetospheric wave energy from lightning discharges propagates in the nonducted (oblique) mode and is often found to multiply (up to 8-10 bounces) reflect back and forth between hemispheres [Edgar, 1976]. The reflections occur where the wave frequency matches the local lower hybrid frequency, leading to the so-called Magnetosphericly Reflected (MR) whistlers as observed on a spacecraft (Figure 1).

Little attention has been paid so far to the interaction of MR whistlers with energetic particles and their contribution to the loss of these particles from the radiation belts. Recent satellite- and ground-based data indicate that whistlers originating in lightning discharges regularly precipitate energetic electrons out of their trapped orbits [Voss *et al.*, 1984; Inan *et al.*, 1988]. While individual events are commonly associated with ducted whistlers [Inan and Carpenter, 1986] and/or lightning discharges [Inan *et al.*, 1988], the relative role of nonducted whistlers is not known. In this paper, we investigate the energies of the electrons that would undergo gyroresonance with MR whistlers near the geomagnetic equator during multiple crossings (Figure 2). We find that the resonant energy for electrons remains roughly constant as the equatorial crossings for typical rays move inward over $1.5 < L < 4$. This result indicates that wave energy entering the magnetosphere from a single location can resonantly interact with electrons of the same energy over such a wide range of L -shells. Thus VLF wave energy produced by lightning may play an important part in the precipitation of energetic electrons on magnetic field lines far removed from that of the lightning discharges.

2. Method of Calculation

The basis for our theoretical calculations to determine the wave normal angle and refractive index was the use of the Stanford VLF Raytracing program [Burtis, 1974; Inan and Bell, 1977]. Rays at selected frequencies in the range (1-32 kHz) were injected into the magnetosphere at 1000 km alti-

tude and at $L=1.5-5$ and the output parameters of ψ , ψ_{res} , ω/ω_H , $n\cos\psi$ at each step along the ray path were computed. Typical plasmaspheric conditions as represented by the equatorial electron density profile shown in Figure 3 were assumed. A sample raypath is shown in Figure 2 to establish the coordinate system for all the calculations in this paper.

For oblique whistler waves, an important characterizing parameter is the difference between the wave normal angle (ψ) and the resonance cone angle (ψ_{res}), i.e., $Y = |\psi - \psi_{res}|$. The quasi-electrostatic versus electromagnetic nature of the wave as well as the pitch angle scattering coefficient for electrons in transverse resonance with an oblique whistler depend strongly on this parameter [Inan and Bell, 1989]. The values of both ψ and ψ_{res} were directly available from the raytracing code.

With ψ and the refractive index (n) obtained from the raytracing program, the relativistic resonant electron energy for electrons in the vicinity of the loss cone (pitch angle $\alpha = \alpha_c$) was computed from

$$E = m_0 c^2 (\gamma^{-1} - 1) ; v_{||} \approx \frac{c}{\omega n \cos \psi} (\omega_H \gamma - \omega)$$

where $\gamma = \sqrt{1 - \beta^2}$, with $\beta = (v/c) = (v_{||}/c)(\cos \alpha_c)^{-1}$, $v_{||}$ is the electron velocity in the direction of the magnetic field, ω_H is the electron gyrofrequency, and ω is the wave frequency.

3. Results

In this section, we first illustrate the parameter variations along the raypath and then concentrate on the resonant interactions near the geomagnetic equator. While oblique resonance can occur with electrons encountered anywhere along

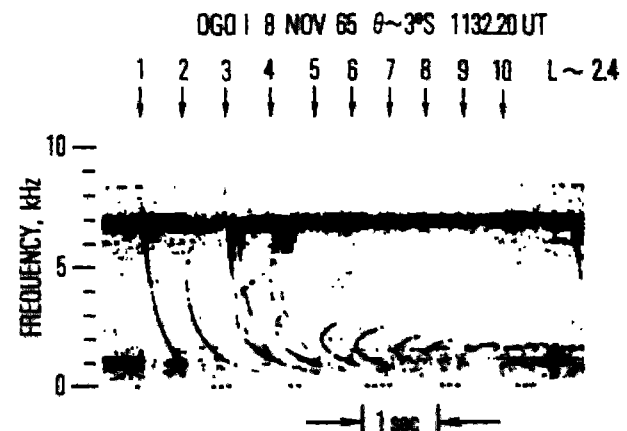


Fig. 1. Frequency-time spectrogram of an MR whistler with 10 components with well-defined upper- and lower-frequency cutoff patterns.

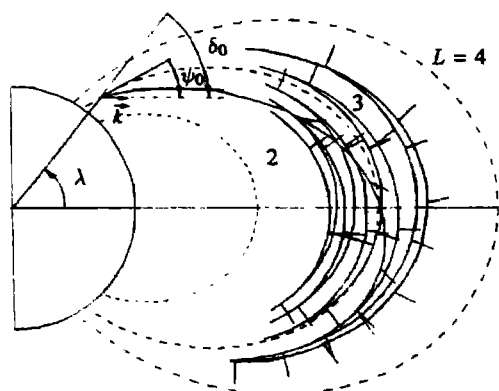


Fig. 2. Schematics showing a sample raypath in a magnetic meridional plane for a wave injected at 1000 km altitude, with frequency $f=2$ kHz and injection latitude corresponding to $L=3$. The wave normal direction is shown along the raypath, and its angle with respect to the magnetic field line and upward vertical are defined at the point of injection as ψ_0 and δ_0 respectively.

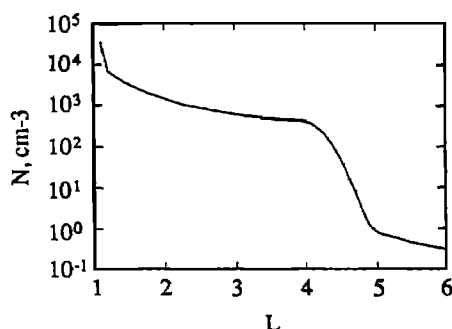


Fig. 3. Equatorial electron density profile used for raytracing calculations. The variation of the density along the field line is assumed to follow a diffusive equilibrium model [Inan and Bell, 1977].

the raypath, for typical rays the longest and most effective gyroresonance is expected to occur close to the equatorial plane, where the variations in ω_H , wave vector (k) and ψ along the field line (in the frame of the particles) are the slowest [Bell, 1986].

The equatorial plane crossings of different rays are investigated in terms of the equatorial values of Y (i.e., Y_{eq}) and E (i.e., E_{eq}) versus L for frequency range 1–32 kHz, and their dependence on injection latitude over the range $L=1.5$ –5 field lines and on initial wave-normal directions at the injection point of $\psi_0 = -80^\circ$ to $+80^\circ$.

Parameter variations along the raypath

As shown in Figure 2, a typical raypath for an MR whistler initially moves toward higher L -shells. After one complete bounce the ray begins to move toward lower L values (steady motion toward higher L values will occur if injection latitude is very low [Edgar, 1976]).

Figure 4 shows Y and E for a single selected ray at 2 kHz injected at $L=2$, vertically upwards. The '+'s represent the points along the raypath at regular intervals and are connected

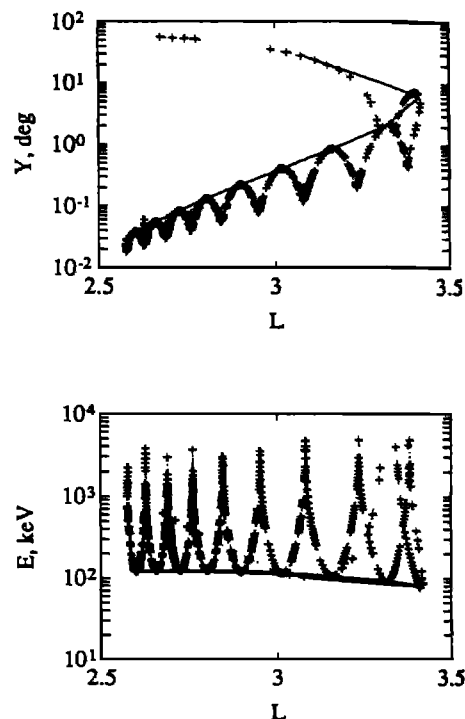


Fig. 4. The variation of Y and E along a sample raypath presented as a function of L -shell for injection vertically upward ($\delta_0 = 0^\circ$) at $L = 2$ with the frequency $f=2$ kHz. The parameter values at points are connected with a dotted line in the order of their appearance along the ray propagation path. The solid curves connect the equatorial crossings.

by the dotted lines. The solid curves connect just the equatorial crossings (Y_{eq} and E_{eq}). The variations of Y_{eq} and E_{eq} as a function of frequency are shown in Figures 5 and 6.

Along the raypath Y oscillates such that the wave normal angle gets closer to the resonance cone near the reflection points and is farthest away at the equator. As L decreases along the MR raypath Y gradually converges to zero.

The resonant electron energy also oscillates between lower and upper limits, represented respectively by the energy curve associated with the equatorial crossings and by $E = m_0c^2(\omega_H/\omega - 1)$ (at the reflection points where $k_{\parallel} \rightarrow 0$). Our values are somewhat lower than the upper limit due to the use of a discrete set of output values.

Assuming that significant interactions can occur all along the ray path, the results shown in Figure 4 indicate that wave energy at 2 kHz entering the magnetosphere at a fixed location (e.g., $L=2$) can resonantly interact with electrons in the 100–1000 keV range as it bounces back and forth between hemispheres.

Dependence on injection latitude

The dependence of Y_{eq} and E_{eq} on injection latitude (L -shell) is shown in Figure 5. The rays injected at the lower latitudes ($L=1.5$ and 2) do not encounter the plasmapause (at $L = 4$) and exhibit relatively well defined features. The raypaths initially move towards higher L -shells and then gradually descend toward lower L -shells, with the equatorial wave normal angle getting closer to the resonance cone after each

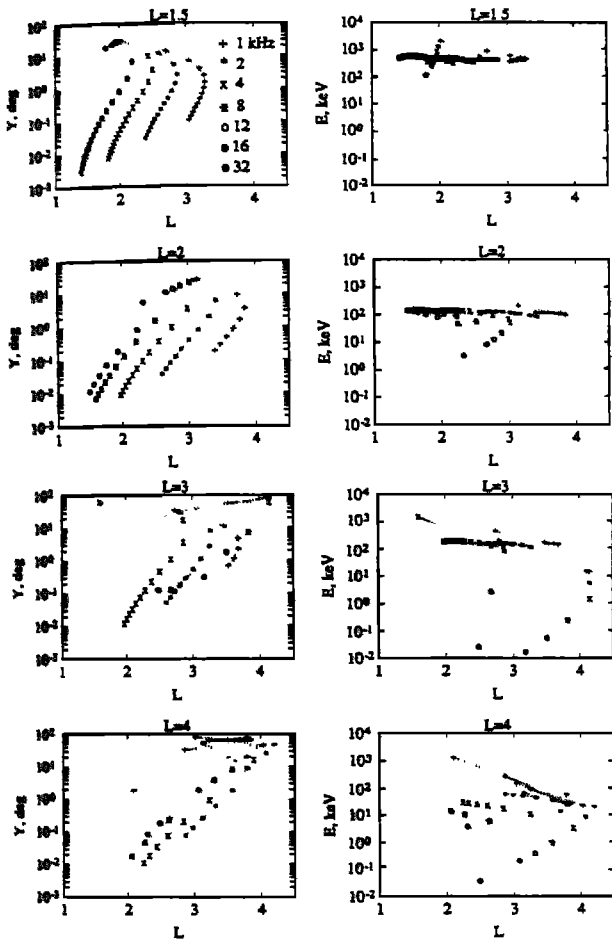


Fig. 5. Dependence on injection latitude and wave frequency of Y_{eq} and E_{eq} , where Y_{eq} is the absolute value of the difference between the equatorial wave normal angle and the resonance cone angle and E_{eq} is the equatorial resonant energy of the gyrating electron, for rays injected vertically upward ($\delta_0 = 0^\circ$) from $L = 1.5, 2, 3, 4$ (given at the top of each panel), with frequencies $f=1, 2, 4, 8, 12, 16, 32$ kHz that are denoted by different symbols. Each point represents the equatorial plane crossing of a single ray and for each frequency the successive crossings are connected with the dotted line. The L -shell at which the rays were injected is given at the top of the each panel.

reflection. The slope of these variations appears to be reasonably constant on a semilog scale. Frequency dependence is such that raypaths for higher frequency rays are shifted towards lower L -shells. We note that for wave frequencies of 16 and 32 kHz there is only one equatorial crossing since the lower hybrid resonance frequency (f_{LHR}) is always less than the wave frequency (f), and magnetospheric reflection does not occur. For 12 kHz, whether or not reflection occurs depends on the disposition of the raypath; for example the 12 kHz ray injected at $L=1.5$ arrives to the other hemisphere at latitudes where $f_{LHR} < f$ and does not reflect, whereas that injected at $L=2$ does.

The variations of the equatorial resonant energy (E_{eq}), given in the right hand panels, shows that this quantity is relatively independent of frequency, especially after the first few equatorial crossings. We note, however, that the fre-

quency dependent raypath shift towards lower L -shells for higher frequencies is still noticeable. For example, for injection at $L=2$ with $f=2$ kHz, equatorial resonant electron energy is in the range $78 \text{ keV} < E_{eq} < 122 \text{ keV}$ while for all other frequencies injected at $L=2$ we have $78 \text{ keV} < E_{eq} < 148 \text{ keV}$. Furthermore we note that this resonant energy range is not a very strong function of injection latitude (L). For injection at $L=1.5$ the energy range is $319 \text{ keV} < E_{eq} < 579 \text{ keV}$, and for $L=3$ it is $85 \text{ keV} < E_{eq} < 191 \text{ keV}$.

The rays injected at the higher latitudes ($L = 3$ and 4) initially move towards the plasmapause and in that region exhibit irregular behaviour. After they reflect from the plasmapause and propagate inward, well defined features explained earlier are exhibited. Raytracing with a plasmapause located at higher L -shells (e.g., $L=7$) shows that rays injected at $L=3$ also exhibit well defined behaviour. Rays injected at higher latitudes inevitably encounter the plasmapause and exhibit irregular behaviour, although this also depends on the wave normal at the point of entry. The general effect of the plasmapause is thus well illustrated in Figure 5.

Dependence on wave normal angle at injection

Figure 6 shows the dependence of Y_{eq} and E_{eq} on ψ_0 for 2 kHz rays injected at 1000 km altitude and at $L=2$. Similar results are also found for other frequencies and injection latitudes.

The general behaviour of Y_{eq} and E_{eq} from Figure 6 is fairly independent of ψ_0 . The ray with $\psi_0 = +80^\circ$ tends to propagate

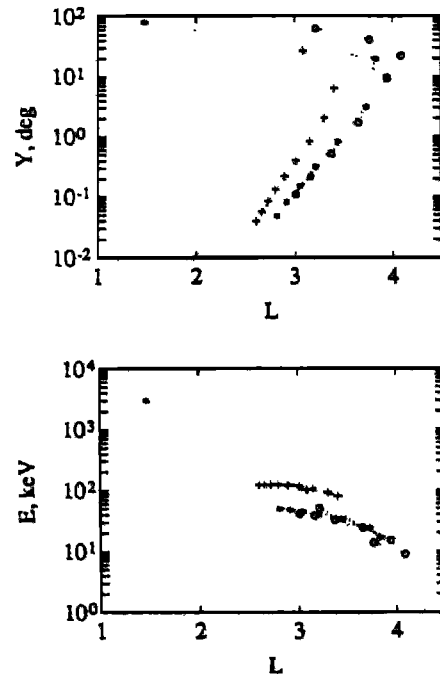


Fig. 6. Dependence on the wave normal angle at the point of injection of the absolute value of the difference between equatorial wave normal angle and resonant cone angle Y_{eq} versus L -shell value and the equatorial resonant energy of the gyrating electron E_{eq} versus L -shell value for rays injected at $L = 2$ with frequency $f=2$ kHz in three directions $\psi_0 = +80^\circ$, $\delta_0 = 0^\circ$, $\psi_0 = -80^\circ$, represented respectively with *, +, o

on lower L -shells, is far away from the plasmapause, and therefore exhibits well defined characteristics similar to the vertically injected ($\delta_0 = 0^\circ$) case. The ray with $\psi_0 = -80^\circ$ initially moves toward the plasmapause and exhibits irregular behavior until it moves to lower L -shells in a manner similar to the case of vertical injection at higher L -shells. After the fourth equatorial crossing the Y_{eq} for this ray varies in the same way as that for $\psi_0 = +80^\circ$ and is shifted somewhat towards higher L -shells compared to the $\delta_0 = 0^\circ$ case. Figure 6 shows that although the slope of the straight portion of the curve $Y_{eq} = Y_{eq}(L)$ is steeper, and equatorial resonant electron energy level is higher for $\delta_0 = 0^\circ$, dependence of these quantities on ψ_0 is rather weak.

4. Conclusions

Whistlers originating in lightning discharges, while they bounce back and forth between the hemispheres due to magnetospheric reflection, undergo gyroresonance with electrons in well defined energy ranges near the geomagnetic equator. As the wave bounces back and forth between hemispheres, the equatorial resonant electron energy (E_{eq}) remains roughly constant even though the raypaths move inward with the equatorial crossing varying over a wide range of L -shells ($1.5 < L < 4$). Typical E_{eq} are of order 100 keV, depending on the entry location of the whistler into the magnetosphere. For example, for whistlers entering the medium at $L=2$, equatorial electron resonant energy remains in the range $78 \text{ keV} < E_{eq} < 148 \text{ keV}$.

It thus appears that MR whistlers originating from a single location, initiated, for example, by isolated thunderstorm centers, could resonantly interact with and induce precipitation of electrons in relatively small energy ranges over a wide range of L -shells. The importance of this effect in terms of the loss of the particles from the radiation belts needs to be evaluated. However, we can expect the contribution of MR whistlers to this loss to be substantial, since pitch angle diffusion coefficients for oblique wave-particle interactions have been shown to be comparable to that for $\psi = 0^\circ$, especially when ψ is in the vicinity of ψ_{res} (i.e., Y small) [Inan and Bell, 1989].

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