JOURNAL OF GEOPHYSICAL RESEARCH VOLUME 66, No. 11 NOVEMBER 1961

Properties of the Outer Ionosphere Deduced from Nose Whistlers[1]

R. L. SMITH

Radioscience Laboratory, Stanford University
Stanford, California

Abstract. Data from a number of nose whistlers indicate that the electron density in the outer ionosphere is directly proportional to magnetic field strength. The density has an annual variation of 2:1 with a maximum near December. The average electron density is given approximately by $N = 12{,}000\ f_H$, where N is the electron density per cubic meter and f_H is the gyrofrequency in cycles per second. Local variations as large as 40 per cent are shown to exist on occasion.

Introduction. Electromagnetic energy at very low frequencies can be propagated from one hemisphere to the other along the earth's magnetic field lines in the outer ionosphere. The action of free electrons disperses the impulselike energy from a lightning discharge to form descending gliding tones called whistlers. If the highest frequency of a whistler is comparable with the minimum electron gyrofrequency along the propagation path, the dispersion at the higher frequencies may cause rising tones which are contiguous with the descending tones at lower frequencies. The whistler is then called a 'nose' whistler.

The advantages of whistlers in the determination of the ionization at great heights have been discussed by *Storey* [1958]. The time delay of a whistler from the originating discharge is proportional to the square root of ionization density, whereas other methods of determining the ionization density give quantities that are usually directly proportional to density. The contributions from regions of low ionization relative to regions of greater ionization are therefore greater in the whistler measurement. At low frequencies the weighting factor for the time delay of the whistler is inversely proportional to the square root of magnetic field strength. This factor increases the incremental time delay due to ionization at 5 earth radii by a factor of roughly 11 over the time delay from an equal amount of ionization in the lower ionospheric layers. The curved shape of the ray path gives a high value of path-length per unit height near the summit of the path relative to the lower part of the path. The result of all these factors is that the electrons at the farthest distance from the earth have the greatest effect on the dispersion characteristics of whistlers.

Nose whistlers offer the additional advantage of defining the height of maximum excursion of the ray path. The frequency of minimum time delay, called the 'nose' frequency, depends primarily on the minimum gyrofrequency along the path. The nose frequency depends only to a small extent on the ionization distribution. For example, if the ionization density in the outer ionosphere did not vary with height, the nose frequency would be about 0.35 times the minimum gyrofrequency.

The nose whistler integral. Spectrographic analysis of whistlers and nose whistlers often reveals a number of pure isolated components that have a common source. Examination of whistler data and a new theory of the propagation path have led to the hypothesis that each component represents energy from the lightning source that has been trapped in a field-aligned duct of enhanced ionization in the outer ionosphere [*Smith*, 1961]. The data indicate that the lifetime of these ducts is a few hours. The theory suggests that enhancements of 5 per cent or greater are sufficient to explain the observed whistlers [*Smith, Helliwell, and Yabroff*, 1960]. The theory further indicates that the average

[1] The major results of this paper were presented in Commission III (The Exosphere and Upper F Region Symposium) and Commission IV of the United States National Committee of the International Scientific and Radio Union (URSI), May 2–5, 1960. An abstract appears in the URSI program and in *Smith and Helliwell* [1960].

group velocity of energy trapped in a duct can be closely approximated by assuming that the energy travels along the maximum of ionization in the duct with wave normals aligned with the magnetic field [*Smith,* 1961].

The theory of whistler propagation in ducts as outlined above simplifies nose whistler analysis in two ways: (1) the propagation path for a given whistler component is independent of frequency and is known to be along a field line; (2) the effective group velocity is almost entirely independent of wave normal angle so that the simplified form of group velocity for purely longitudinal waves can be used. These simplifications would not result if the outer ionosphere did not contain field-aligned irregularities [*Yabroff,* 1959; *R. L. Smith,* 1960*b*; *Garriott,* 1958].

In expressing the path of propagation of whistlers we will use the dipole approximation to the earth's magnetic field. Recent satellite results indicate that the magnetic field is very close to the predicted field out to at least 5 earth radii from the earth's center [*E. J. Smith,* 1960]. Nearly all whistler paths fall within this distance. The higher order terms of the magnetic field are always smaller than the dipole term and decrease more rapidly with height. The frequency-time characteristics of nose whistlers are determined principally in the region of the minimum magnetic field strength. The nose whistler can be described in terms of an effective dipole field determined from the minimum field strength at the top of the field line. This effective dipole will have an effective latitude of termination at the earth's surface. The main effect of the higher order terms is to change the termination point of the effective latitude.

The nose whistler equation can be simplified by assuming that the square of the refractive index is much larger than unity. This assumption is justified by the results. The time delay is then given by

$$T = \frac{1}{2c} \int_{\text{path}} \frac{f_0 f_H}{f^{1/2}(f_H - f)^{3/2}} \, ds \qquad (1)$$

where

f_0 = plasma frequency.
f_H = gyrofrequency.
f = wave frequency.
c = velocity of light.

The 'dispersion' of a nose whistler is obtained by multiplying both sides of equation 1 by $f^{1/2}$, and is given by

$$D = Tf^{1/2} = \frac{1}{2c} \int_{\text{path}} \frac{f_0 f_H}{(f_H - f)^{3/2}} \, ds \qquad (2)$$

When f tends toward zero the dispersion tends toward a constant previously defined by *Storey* [1953] for low frequency whistlers.

The dispersion of a nose whistler can be determined experimentally as a function of frequency. Then equation 2 can be viewed as an integral equation with the plasma frequency and the gyrofrequency (or path) as unknowns. Various theoretical methods of inverting this integral have been investigated [*Storey,* 1958; *R. L. Smith,* 1960*a*], but these methods have all failed when applied to experimental data. The reason is simply that for the observed frequency range of whistlers the accuracy required for the simultaneous solution of the plasma frequency and the path is greater than the experimental errors.

Figure 1 shows the dispersion as a function of frequency for a well-defined nose whistler. Two theoretical dispersion curves are also shown for comparison. One model in which the ionization varies as exp $(2.5/R)$ is taken from *Dungey* [1954]. For the other model the ionization is directly proportional to the local gyrofrequency or magnetic field strength. The field line paths for the two models are chosen so that the resulting nose frequency is the same as that observed (14.75 kc). The paths are slightly different for the two models. The scale factors of the ionization distributions are chosen so that the dispersion at the nose frequency is the same as that observed. The experimental and theoretical dispersion curves are, within experimental error, coincident. It appears then that only two significant parameters of the dispersion can be determined from a single nose whistler trace. The two parameters which will be used here are the nose frequency and the time delay from the originating discharge to the nose.

Additional information on the ionization distribution can be determined from a study of nose whistler groups. A nose whistler group is a set of isolated nose whistler components excited by the same lightning flash. As discussed previously, each component presumably represents energy that has been trapped in a field-aligned

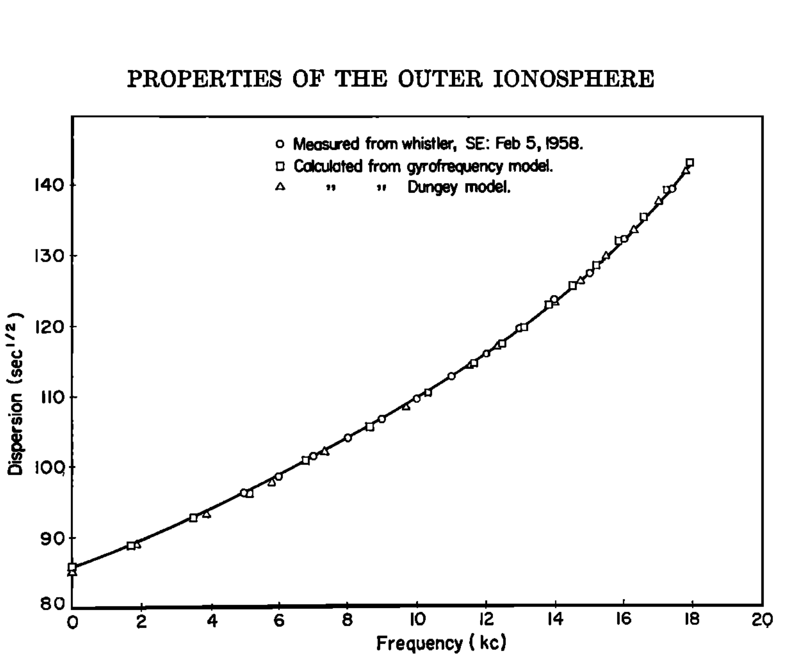

Fig. 1. Theoretical dispersion curves fitted to a nose whistler.

column of enhanced ionization. Spectrographic analysis of nose whistler groups shows that components with lower nose frequencies usually arrive later than components with higher nose frequencies. The nose frequency is approximately a constant fraction of the minimum gyrofrequency along the path. Whistlers traveling to regions of lower minimum gyrofrequency must travel over longer paths, and hence tend to suffer greater delays.

Nose whistler data. A number of nose whistlers covering the years 1955 to 1959 have been spectrographically analyzed. The data were obtained at stations in the northern and southern hemispheres and include a number of nose whistler groups.

The most difficult part of the scaling procedure is the determination of the time of occurrence of the whistler-causing atmospheric. The task is somewhat simplified when records are available from magnetically conjugate stations. *Carpenter* [1960] gives several methods of identifying the sources of whistlers. After the source has been identified, the measurement of the nose frequency and time delay to the nose are routine.

It is important to know whether a given whistler is short (one hop) or long (two hop). The classification can be determined readily by comparison of the time delays from the source to the whistler and its echoes, when well-defined echoes are present. In the absence of echoes for a given whistler, one can often find similar whistlers and associated echoes received within a few hours of the desired whistler, and thus can deduce the classification. Simultaneous records from magnetically conjugate stations can also be used. The strength or type of causative atmospherics has been found to be unreliable as an indicator of short or long whistlers. Thus many short whistlers have been found with strong preceding atmospherics. The causative atmospheric may or may not show a characteristic 'tweek' waveform, depending on the distance and the earth-ionosphere wave-guide conditions.

Almost all whistlers that show noses have been found to be short. The only long nose whistler found in the present study was obtained on May 20, 1955, at Washington, D. C. (The measured time delay was therefore divided in half for comparison with the rest of the data.)

The data are shown in Figure 2 on a log-log plot. The abbreviations on the figure indicate the station where the whistlers were received: *BO,* Boulder, Colorado; *BY,* Byrd Station, Antarctica; *CO,* College, Alaska; *DU,* Dunedin, New Zealand; *SE,* Seattle, Washington; *ST,* Stanford, California, *UN,* Unalaska, Alaska; *WA,* Washington, D. C.; *WE,* Wellington, New Zealand. It can be deduced from the data that the location of the receiving station bears no

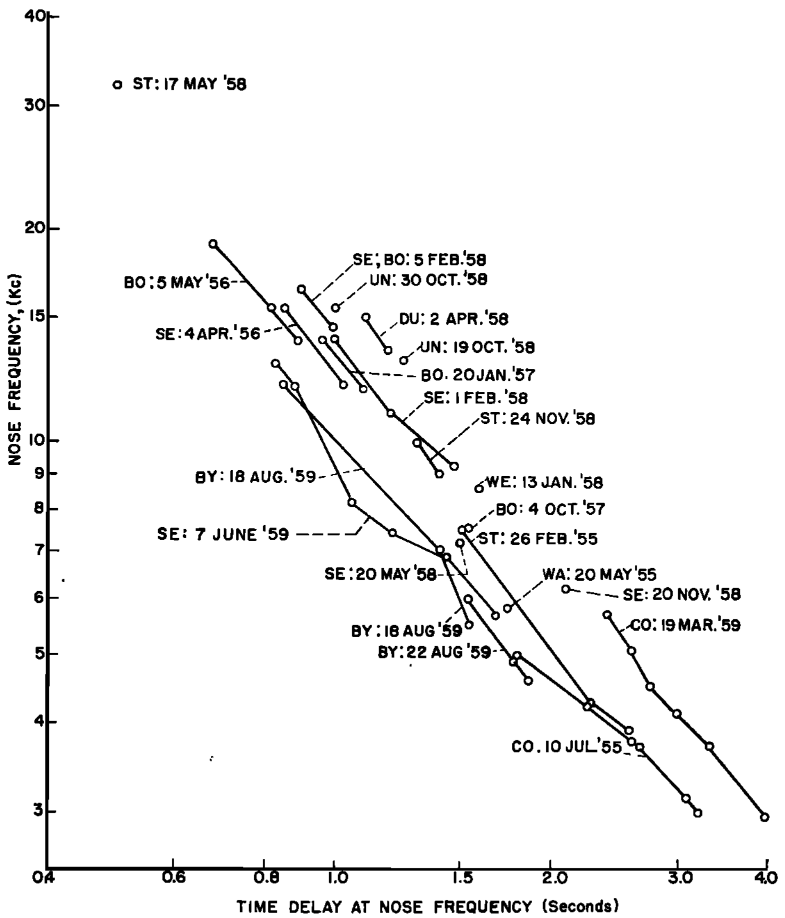

Fig. 2. Data showing the time delay and nose frequency of nose whistlers. *BO,* Boulder, Colorado; *BY,* Byrd Station, Antarctica; *CO,* College, Alaska; *DU,* Dunedin, New Zealand; *SE,* Seattle, Washington; *ST,* Stanford, California; *UN,* Unalaska, Alaska; *WA,* Washington, D. C.; *WE,* Wellington, New Zealand.

simple relation to the effective latitude of a received nose whistler component. One possible explanation is that whistler energy may, on occasion, travel for a considerable distance under the ionosphere. Another possibility is that the effective duct may terminate well above the lower ionosphere, thus allowing a small part of the total ray path to depart from a field line path. The lines that join some of the points usually indicate nose whistler groups, although they sometimes indicate a collection of different nose frequency-time delay points taken from whistlers occurring in the same day at the same station. One noticeable feature of the nose whistler groups is that the slopes of the lines are quite similar, even though the data were taken at different times at different stations. The nose whistler group data thus indicate that the background ionization in the outer ionosphere usually has a reasonably smooth distribution with only small enhancements of ionization to account for the discrete whistler components. Furthermore, the shape of the ionization distribution is less variable than the scale factor of the distribution.

The nose whistler data also strongly suggest a seasonal variation of time delay and hence ionization density. The data for the months of May through August have consistently less delay than the data for November through March. The time delay for the mid-year months is approximately the same during both sunspot minimum and maximum. The seasonal variation can be described in more detail after the model of ionization density is obtained.

Comparison of outer ionosphere models with the data. The ionization distribution cannot be obtained directly from a single nose whistler trace. However, the nose whistler data can be compared with that predicted from various theoretical models.

Dungey [1954] proposes from thermodynamical arguments that collisions cause the density of particles in the outer ionosphere to be distributed in thermal equilibrium whether or not the magnetic field is present and even though collisions are very infrequent. Dungey's model, assuming only protons and electrons at a temperature of 1500°K are present, is

$$N = N_0 \exp (2.5/R) \tag{3}$$

where N_0 is the density at infinity and R is the distance measured in earth radii from the center of the earth. *Johnson* [1960] has recently derived a similar formula, in which he has included a small correction factor to account for centrifugal force. In the region of interest here the two equations are nearly the same.

The presence of ducts of enhanced ionization suggests that the outer ionosphere does not reach thermodynamic equilibrium. In the absence of collisions, charged particles tend to spiral around a set of field lines, enclosing a constant amount of flux. There is, therefore, a tendency for the density of particles to be proportional to the magnetic field strength. We will call this model the 'gyrofrequency model' for convenience. Such a model has been proposed by *Gallet* [1959]. Storey had previously suggested this model, apparently because the resulting low frequency dispersion integral can then be directly integrated. Assuming a constant base level ionization, this model is given by

$$N = K \frac{f_H}{(1 + 3 \sin^2 \theta)^{1/2}} \tag{4}$$

where

f_H = gyrofrequency.
θ = geomagnetic latitude.

Johnson [1959] and the author have independently suggested a combination model in which the ionization is proportional to the product of the magnetic field strength and an exponential factor. Again, to keep the base level constant, we will use the formula

$$N = K \frac{f_H}{(1 + 3 \sin^2 \theta)^{1/2}} \exp (2.5/R) \tag{5}$$

Johnson has since rejected this model in favor of the one discussed previously.

The above three models were used to compute theoretical nose frequency-time delay curves. A small correction ($\Delta D = 5$) was applied to account for the additional dispersion in the F layer. The theoretical curves are compared to the nose whistler group data in Figure 3. (The factor $(1 + 3 \sin^2 \theta)^{-1/2}$ should be included in the labels for the curves in which the factor f_H appears.) A change in the scale factor for the theoretical models would shift the curves to the right or left, but the slopes would remain substantially unchanged.

The slope of the nose frequency-time delay characteristic for the Dungey model clearly does not fit the majority of the data. Each nose whistler trace can give only a weighted integral of plasma frequency along one set of field lines. The ionization is distributed much more rapidly along the field lines than across the field lines. One might ask if the Dungey model could fit the

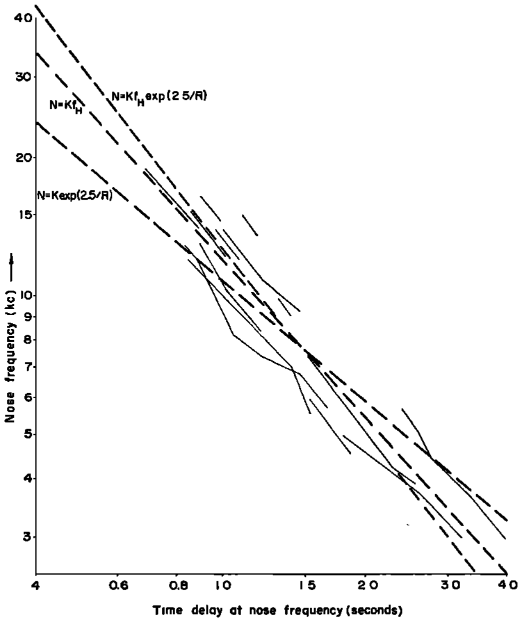

Fig. 3. Comparison of nose whistler data with the time delay as a function of frequency predicted from three models of the outer ionosphere.

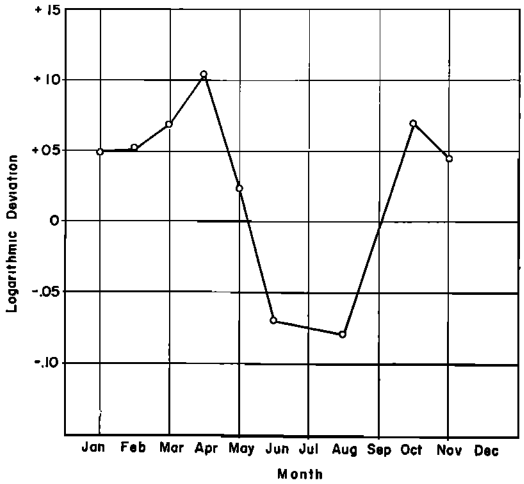

Fig. 4. Annual variation of nose frequency time delays.

whistler data if the base level of ionization were allowed to vary with latitude. The nose whistler data cover effective base latitudes from about 50° to 65°. Over this 15° range the base level would have to change by a factor of 4 or more to bring the model into reasonable agreement with the nose whistler data. There is no reason to expect such a large change in the base level.

The exponent in the Dungey model is proportional to atomic weight and inversely proportional to temperature. Some calculations were made using larger exponents for the Dungey model. The model

$$N = N_0 \exp(8/R) \qquad (6)$$

fits the data quite well. However, the larger exponent would imply a temperature of 500°K if hydrogen is the main constituent in the outer ionosphere, or a temperature of 7000°K if nitrogen or oxygen were the main constituent. These temperatures differ considerably from temperatures found in satellite drag data (1250°K) [*Harris and Jastrow,* 1959] or helium escape calculations (1500°K) [*Spitzer,* 1948].

Of the two remaining models, the gyrofrequency model appears to fit the data somewhat better, both on the basis of the slope measurements and the over-all data, including the single nose whistler data points. An average model of the outer ionosphere density is then found to be

$$N = 12{,}000\, f_H \qquad (7)$$

where N is the electron density per cubic meter and f_H is the gyrofrequency in cycles per second.

Having obtained an average model we can now estimate the annual variation in electron density. The monthly average logarithmic deviation of time delays to the nose are shown in Figure 4. These data cover the years 1957–1959 only. The nose whistler data for 1955–1956 are probably not statistically significant, but the nose whistler recorded on February 26, 1955, indicates that the seasonal variation during sunspot minimum is much smaller than during sunspot maximum. Supporting data, which also indicate an annual variation in the ionization density, are the monthly dispersion averages shown in Figure 5 for the year 1958. The dispersion data alone would not be sufficient evidence since it could be argued that the variations in dispersion could result from seasonal movements in the ducts of enhanced ionization. Since the time delays at a given frequency and the dispersion values are proportional to the square root of electron density, the data from Figure 4 indicate an annual variation of electron density of 2:1. Possible factors which may be related to the annual variation are discussed by *Helliwell* [1961].

Other work. The first attempt to obtain the electron distribution in the outer ionosphere from whistler data was made by *Allcock* [1959] without using nose whistlers. Average whistler time delays were aurally determined at a number of stations. The whistlers were assumed to travel along field lines terminating at the receiving station. By using these data and further assuming a radially symmetric distribution, All-

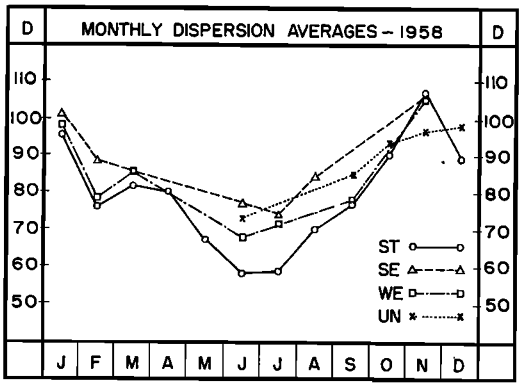

Fig. 5. Annual variation of whistler dispersion.

cock was able to derive an electron-density profile. The resulting densities are somewhat higher than those obtained in the present report. There are two principal reasons for the higher values. As we have seen, the geomagnetic latitude of a station does not necessarily represent the effective geomagnetic latitude of a received whistler. At a given station, whistlers can be expected to have a range of effective latitudes around the station latitude. Since the whistler rate increases rapidly with latitude for the stations used by Allcock, the average whistler time delay at a station would correspond to the time delay at a higher geomagnetic latitude. Allcock discusses another source of error, the nose whistler effect, but does not correct for it.

On the basis of records obtained at College on one day, March 19, 1959, *Pope* [1961] deduces that the ionization distribution is similar to that given in equation 5. The data from this day are included in Figures 2 and 3. It can be seen from the figures that the slope of nose frequency versus time delay for this day is not typical of the majority of data. It might be noted that the gyrofrequency model would be consistent with the data from March 19, 1959, if the base level ionization were made to decrease slightly with latitude.

Variations of density across field lines. The nose whistler group recorded at Seattle on June 7, 1959, shows large deviations from a simple ionosphere model. No simple model could account for the irregular variation of nose frequency as a function of time delay. The time delays at nose frequencies of 5.65, 6.82, 11.9, and 13.0 kc are consistent with the simple gyrofrequency model. The time delays of the two components with nose frequencies of 7.35 and 8.1 kc are, however, less than the expected delays by approximately 18 per cent. This indicates a 40 per cent reduction in ionization density along the paths followed by these two nose whistler components. Thus large variations of ionization density may exist across the field lines. The variations noted here suggest that ducts of enhanced ionization of the size sufficient to trap whistler energy are easily obtainable.

Results. The electron-density distribution deduced from the present study is shown in Figure 6. (This figure is the same as that shown in *Smith and Helliwell* [1960].) The annual

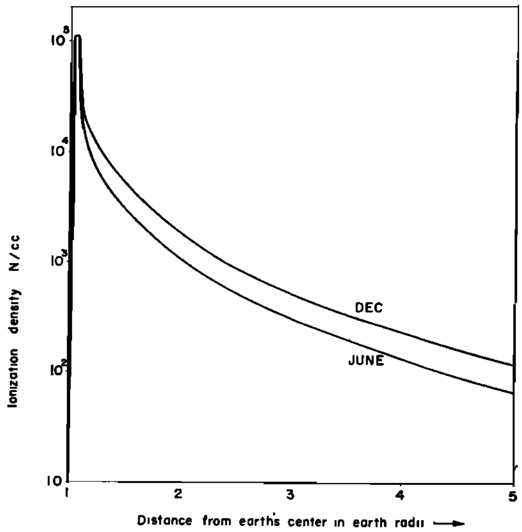

Fig. 6. Gyrofrequency model of the outer ionosphere.

variation is indicated by the separate curves for June and December. The distributions are faired to a simple parabolic approximation of the lower ionosphere with a maximum plasma frequency of 3 Mc, corresponding roughly to the nighttime F_2 layer critical frequency. The average density at 5 earth radii, about 100 per cc, corresponds very closely to the density deduced by *Obayashi* [1958] from geomagnetic fluctuation data. However the interpretation of these data has since been refuted by *Jacobs* [1960].

The densities deduced from whistlers are most accurate near the top of the field line path. The present data thus apply best to the region from 2½ to 5 earth radii. By comparing the data with other models we can deduce that the gyrofrequency model gives the correct density at any place in this region within a factor of 2. The uncertainty in the integrated ionization in this region should be much less.

From recent rocket flights, *Berning* [1960] has measured ionization densities to 1500 km. He deduces an ionization density of 10^4/cc at this height, but indicates that the true value may be as low as 5×10^3/cc. The nose whistler data, using the gyrofrequency model, gives values between 7.8×10^3 and 1.5×10^4/cc. It is worth mentioning that reasonably accurate values of ionization densities to heights of 2000 km, deduced perhaps by rocket or satellite measurements, will greatly increase the accuracy of whistler measurements at far greater distances.

The present data indicate a large annual variation in electron density in the outer ionosphere during sunspot maximum. There is insufficient data to determine the variations near sunspot minimum. An adequate amount of data will probably be collected during the coming sunspot minimum to answer this question. Present indications from nose whistler and dispersion data indicate that the annual variations will decrease in amplitude in the next few years.

The data also show that the ionization density does not vary in an entirely smooth and simple manner with position in the outer ionosphere. The ducts of enhanced ionization have lifetimes of only a few hours. Attempts to explain the distribution of ionization in the outer ionosphere must be based on dynamic considerations.

Acknowledgments. I wish to express my appreciation to Dr. R. A. Helliwell for his guidance and stimulating counsel during the course of this work. I am grateful to the many people and agencies who have supplied data for this project. The bulk of the data was obtained from the Whistlers-West project during IGY and IGC by the National Science Foundation grants Y-6.10/20 and NSF-G 8839, and cooperating agencies. Pre-IGY data at Stanford were obtained under Air Force contract AF19(604)-795. Nose whistlers from Byrd Station, Antarctica, were analyzed under contract NSF-G 13465. Nose whistlers from the Geophysical Institute, College, Alaska, were supplied by J. Pope. Nose whistlers from Washington, D. C., were supplied by H. Dinger. Other useful data and help were supplied by G. McK. Allcock, L. R. O. Storey, R. Barrington, W. C. Johnson, Don Jones, J. Katsufrakis, D. Carpenter, and J. Crary.

The research was sponsored in part by the Air Force Office of Scientific Research under contract AF18(603)-126.

References

Allcock, G. McK., The electron density distribution in the outer ionosphere derived from whistler data, *J. Atmos. Terrest. Phys., 14,* 1959.

Berning, W. W., A sounding rocket measurement of electron densities to 1500 kilometers, *J. Geophys. Research, 65,* 2589–2594, 1960.

Carpenter, D. L., Identification of whistler sources on visual records and a method of routine whistler analysis, *Tech. Rept. no. 5,* contract AF18(603)-126, Radioscience Laboratory, Stanford University, March 15, 1960.

Dungey, J. W., Electrodynamics of the outer atmosphere, *Sci. Rept. no. 69,* contract AF19 (122)-144, Ionospheric Research Laboratory, Pennsylvania State University, September 15, 1954.

Gallet, R. M., The very low frequency emissions generated in the ionosphere, *Proc. IRE, 47,* 211–231, 1959.

Garriott, O. K., A note on whistler propagation in regions of very low electron density, *J. Geophys. Research, 63,* 862–865, 1958.

Harris, I., and R. Jastrow, An interim atmosphere derived from rocket and satellite data, *Planet. Space Sci., 1,* 20–26, 1959.

Helliwell, R. A., Exospheric electron density variations deduced from whistlers, *Ann. géophys., 17* (1), 1961.

Jacobs, J. A., as reported in *Trans. Am. Geophys. Union, 41,* p. 627, 1960.

Johnson, F. S., The structure of the outer atmosphere including the ion distribution above the F_2 maximum, *Tech. Rept.* no. LMSD-49719, Lockheed Missiles and Space Division, Sunnyvale, California, April 28, 1959.

Johnson, F. S., The ion distribution above the F_2 maximum, *J. Geophys. Research, 65,* 577–584, 1960.

Obayashi, Y., Geomagnetic storms and the earth's outer atmosphere, *Rept. Ionospheric Research Japan, 12,* 301–335, 1958.

Pope, J. H., An estimate of electron densities in the exosphere by means of nose whistlers, *J. Geophys. Research, 66,* 67–75, 1961.

Smith, E. J., Characteristics of the extraterrestrial current system: Explorer VI and Pioneer V, *J. Geophys. Research, 65,* 1858–1861, 1960.

Smith, R. L., The use of nose whistlers in the study of the outer ionosphere, *Tech. Rept. no. 6,* contract AF18 (603)-126, Radioscience Laboratory, AFOSR TN-60-861, July 11, 1960*a*.

Smith, R. L., Guiding of whistlers in a homogeneous medium, *J. Research NBS, 64D* (5), 1960*b*.

Smith, R. L., Propagation characteristics of whistlers trapped in field-aligned columns of enhanced ionization, *J. Geophys. Research, 66,* 3699–3707, 1961.

Smith, R. L., and R. A. Helliwell, Electron densities to 5 earth radii deduced from nose whistlers, *J. Geophys. Research, 65,* 2583, 1960.

Smith, R. L., R. A. Helliwell, and I. W. Yabroff, A theory of trapping of whistlers in field-aligned columns of enhanced ionization, *J. Geophys. Research, 65,* 815–823, 1960.

Spitzer, L., The terrestrial atmosphere above 300 km., Chapt. VII of *The Atmospheres of the Earth and Planets,* Chicago University Press, pp. 213–249, 1948.

Storey, L. R. O., An investigation of whistling atmospherics, *Phil. Trans. Roy. Soc. London, A., 246,* 113–141, 1953.

Storey, L. R. O., A method to interpret the dispersion curves of whistlers, RPL 23-4-1, DRTE, Ottawa, Canada, April, 1958.

Yabroff, I. W., Computation of whistler ray-paths, AFOSR TN-60-71, December 1959.

(Manuscript received May 17, 1961; revised August 29, 1961.)